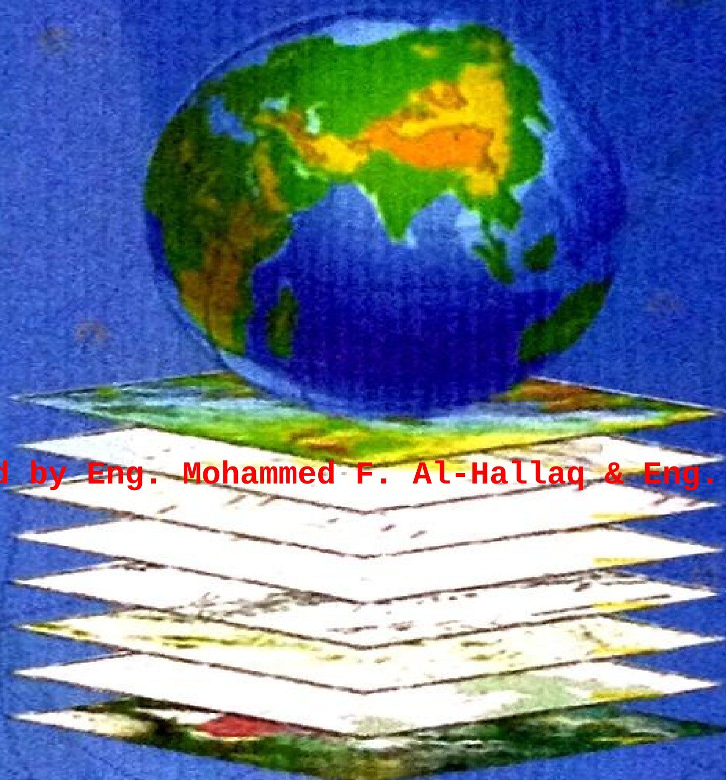
SURVEYING FOR ENGINEERS

SECOND EDITION



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SURVEYING FOR ENGINEERS

SECOND EDITION

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Surveying for Engineers: Second Edition.

PREFACE

This book aims to provide the reader with a concise modern book on surveying principles, techniques and equipment. Many books around the world have been written which deal with the subject of surveying; however, two main factors have given me the motivation to add another book to the library. First, the need to re-present the material in a way that copes better with the recent developments in surveying instrumentation, computer technology and surveying practice. As an academic and a licensed surveyor with more than six years of practical field experience, I included a description of several easy and some new techniques for performing surveying operations that I did not see in other surveying books (example: layout of transition-circular-transition curves in section 9.2.2.5). Second, the realized need to provide the surveying and engineering students at the Palestinian and Arabic universities with an accessible inexpensive surveying book written in the English language, given that the teaching language in engineering in most of these institutions is still English. Students, especially in Palestine are not capable of supporting and buying an imported expensive book due to the difficult economic situation. Moreover, this locally written book is more oriented towards the practice and application of the surveying profession, laws and units of measurement in these countries.

The subject material of this book is divided into thirteen chapters. Chapter one gives an introduction to surveying and explains its importance to people. It also discusses the historical connection between surveying and civil engineering, figure of the earth, units of measurement, scales of surveys, as well as, the basic geometric principles of traditional surveying.

Due to the harmful nature of measurement errors, and the dangerous effects that they might have on the results, it is very important that the reader learn about them so that these errors might be avoided, corrected or minimized. Therefore, this subject is dealt with early in the book in Chapter 2. Consequently, the reader is made aware of the different types of measurement errors that might occur in the measurements that are explained afterwards.

The first edition of this book has been refereed through
The Deanship of Scientific Research
At
An-Najah National University.

الطبعة الأولى لهذا الكتاب تم تحكيمها واعتمادها من خلال
عمادة البحث العلمي
في
جامعة النجاح الوطنية

Chapters three, four, five and six discuss the different types of ground surveying instruments and techniques starting with the basic use of the chain, going through the use of tapes, levels and theodolites, and ending with the use of the sophisticated total stations, as well as, their applications in making planimetric and topographic maps. Chapter seven deals with coordinate geometry and traverse surveying. It describes the different techniques and procedures used to calculate the position (coordinates) of points from measurements done using theodolites and electronic distance measuring equipment, as well as, other basic surveying instruments. Chapter eight explains the most commonly used methods for area and volume calculation, both computational techniques and mechanical ones using the planimeter. Chapter nine deals with route surveying. This includes the planning, design and layout of both vertical and horizontal curves (circular and transition). Chapter ten discusses the subject of horizontal control surveys and methods used to provide and establish horizontal control points. Chapter eleven gives an introduction to photogrammetry and its importance in topographic mapping as an alternative way of surveying. And finally, this second edition has an additional two chapters: global positioning systems – GPS (Chapter 12) and geographic information systems – GIS (Chapter 13) which are considered very hot subjects and advancements in the area of Geomatics. GPS deals with position measurement and mapping using satellite technology, while GIS deals with the computerized input, manipulation, analysis and presentation of spatial data.

In addition to the two new chapters (12 & 13) to this second edition of the book, several major revisions have been made. These include fixing typing mistakes, rephrasing many sections to become simpler and easier to read and understand, and adding new solved examples, illustrating figures and pictures as well as adding new problems at the end of most of the chapters. The word civil has been dropped from the previous title of the book given that it is suitable for all engineers who need to deal with surveying at some stage and not only civil engineers.

As a textbook, and from the author's perspective and teaching experience, this book is recommended to be taught in two undergraduate level courses. The first course includes chapters one through eight, while the second course includes the remaining five chapters.

I specifically tried to make this book as concise and easy to read and understand as possible. I hope that it will be useful to the readers and certainly welcome any comments that will help improve it in the future.

The author

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3. CHAIN SURVEYING (TAPE MEASUREMENTS)

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13. GEOGRAPHIC INFORMATION SYSTEMS (GIS)

1 INTRODUCTION

1.1 DEFINITION OF SURVEYING

In a traditional sense, surveying may be defined as the art and science of making and analyzing measurements made on, above or below the surface of the earth, and the processing of these measurements into some positional form such as maps and coordinates. It also includes the opposite activities that involve the establishment of the positions on, above or below the surface of the earth of points which have been previously located on a plan or a map by a design engineer. The measurements include both angles and distances performed in the horizontal, as well as, the vertical direction. They also include positional data (coordinates) obtained using modern positioning technology such as Global Positioning Systems.

In a broader sense, surveying is defined as the art and science of obtaining reliable quantified and qualified measurements; the interpretation of those measurements, and the meaningful presentation of the results.

1.2 DEFINITION OF A SURVEYOR

The International Federation of Surveyors defined the surveyor as a professional person with the academic qualifications and technical expertise to practice the science of measurement; to assemble and assess land and geographic related information; to use that information for the purpose of planning and implementing the efficient administration of the land, the sea and structures thereon; and to instigate the advancement and development of such practices.

Practice of the surveyor's profession may involve one or more of the following activities, which may occur, on, above or below the surface of the land or the sea and may be carried out in association with other professionals.

1. The determination of the size and shape of the earth and the measurement of all data needed to define the size, position, shape and contour of any part of the earth's surface.

2. The positioning of objects in space and the positioning and monitoring of physical features, structures and engineering works on, above or below the surface of the earth.

3. The determination of the position of the boundaries of public or private land, including national and international boundaries, and the registration of those lands with the appropriate authorities.

4. The design, establishment and administration of land and geographic information systems and the collection, storage, analysis and management of data within those systems.

5. The study of the natural and social environment, the measurement of land and marine resources and the use of the data in the planning of development in urban, rural and regional areas.

6. The planning, development and redevelopment of property, whether urban or rural or whether land or buildings.

7. The assessment of value and the management of property, whether urban or rural and whether land or buildings.

8. The planning, measurement and management of construction works including the estimation costs.

9. The production of plans, maps, files, charts and reports.

In the application of the foregoing activities, surveyors take into account the relevant legal, economic, environmental and social aspects affecting each project.

1.3 IMPORTANCE AND USES OF SURVEYING

Surveying is a subject of high importance because it affects many aspects of the human activities. Since old times, it has been closely related to the prosperity and welfare of mankind. Briefly, it deals with:

- Locating and describing property boundaries for area measurement, dispute resolution between neighbors, etc.
- The preparation of plans associated with the work of the civil engineer, architect, builder and town planner.
- The making of maps and plans for military, geographical, geological, agricultural and other purposes.

1.4 HISTORICAL CONNECTION BETWEEN CIVIL ENGINEERING AND SURVEYING

All civil engineering activities require the use of measured data, both in the field and in the office. For example, a typical civil engineering project first requires that a map or other scaled representation be made of a portion of the earth's surface. The land or engineering surveyor makes the field measurements for such a map. After the design of the project by a civil engineer, it must be "staked" by surveyors or surveying technicians so as to guide the contractor through the construction phase. Most civil engineering projects require the location of real property boundaries, and this requires the services of a land surveyor. Therefore, the tasks of civil engineering and surveying are very closely connected.

1.5 THE FIGURE OF THE EARTH AND ITS RELATION TO SURVEY MEASUREMENTS

As mentioned in the introduction, surveying can be simply defined earlier as earth measurements, and as such, an understanding of the size and shape of the earth is necessary.

The ancient Greeks were among the first to take interest in the dimensions of the earth. In the 6th century (BC), Pythagoras suggested that the earth is spherical. Eratosthenes, on the other hand, was perhaps the first one to take measurements and calculate the size of this assumed sphere. His method can be summarized as follows (Figure 1.1):

a. The distance between the cities of Aswan and Alexandria in Egypt was estimated to be 5000 Stadia (each stadia ≈ 185.2 m). This was based on the fact that a camel will cross the distance between the two cities in 50 days, with an average speed of 100 stadia/day.

b. Eratosthenes measured the central angle opposite to this distance as follows:

   1) On the longest day of the year, he noticed that sun rays at noon are perpendicular at a well located in Aswan. This means that the extension of these rays would go through the center of the assumed spherical earth.

   2) At the same time during the longest day of the next year, he measured the shadow (s) of a perpendicular pole of height h (Figure 1.1) which is located in Alexandria. To make sure that it was the same time, he watched the shadow of the pole around noon until it became the shortest, and measured it. Using Equation 1.1, he then calculated angle α, and found it to be 7.2°.

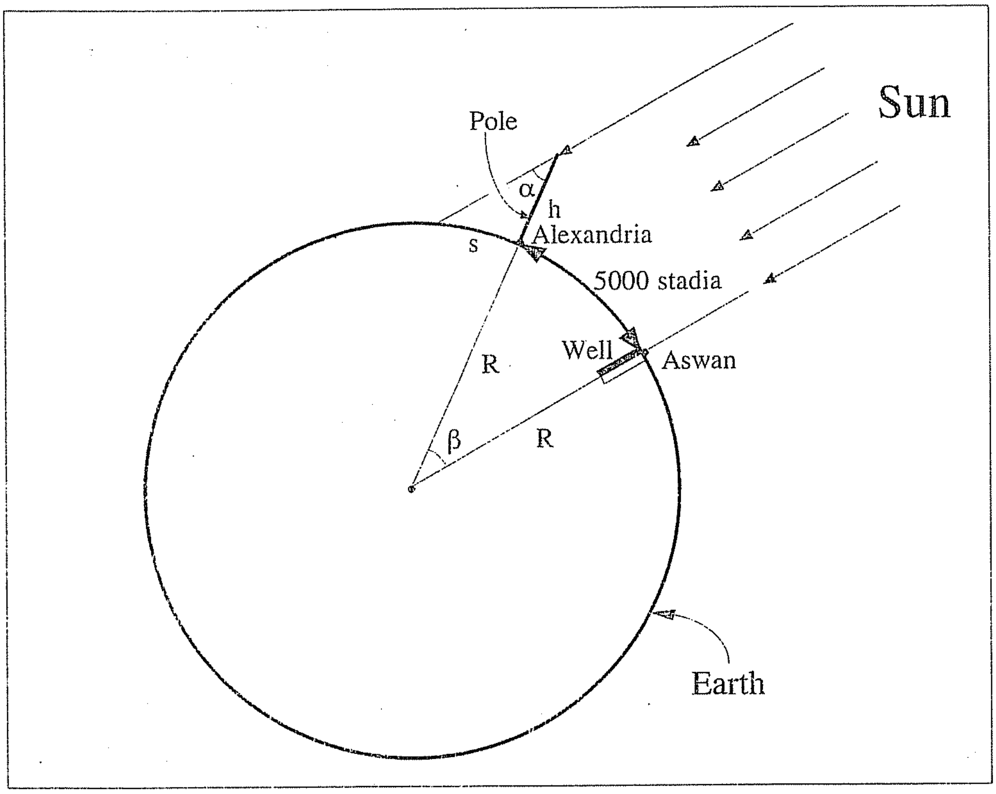


FIGURE 1.1: Eratosthenes method for measuring the earth's perimeter.

$$\tan \alpha = \frac{s}{h} \qquad \text{.......... (1.1)}$$

3) Under the assumption that sun rays are parallel, the central angle β would be equal to the angle $\alpha = 7.2°$, and from Figure 1.1:

$$\frac{\beta}{360} = \frac{5000 \text{ stadia}}{L} \qquad \text{.......... (1.2)}$$

Where L is the earth's perimeter. From this equation Eratosthenes found that $L = 250{,}000$ stadia ($\approx$ 46,300 km). This value, which was reached by elementary measurements, is only 16% larger than the value known today using advanced and accurate technology.

After that, the idea of flat earth dominated until the 15th century, when Columbus supported again the idea of the spherical earth. In the 16th century Magellan proved this hypothesis by sailing around the world. Recent accurate measurements showed that the earth has an ellipsoidal shape.

The surface of the earth is irregular and impossible to be represented by a simple mathematical model. If it is assumed that the earth masses above the mean sea level are removed, and the land areas below mean sea level (e.g., Jerico) are filled, the resulting surface is termed ***Geoid***. This surface has equal gravitational potential at all its points with the direction of gravity being perpendicular to the surface at all these points.

If the earth had a uniform density and the topographic variations did not exist, the Geoid would have the shape of an ellipsoid of revolution. However, it does not. The effects of density on the Geoid are summarized in Figure 1.2.

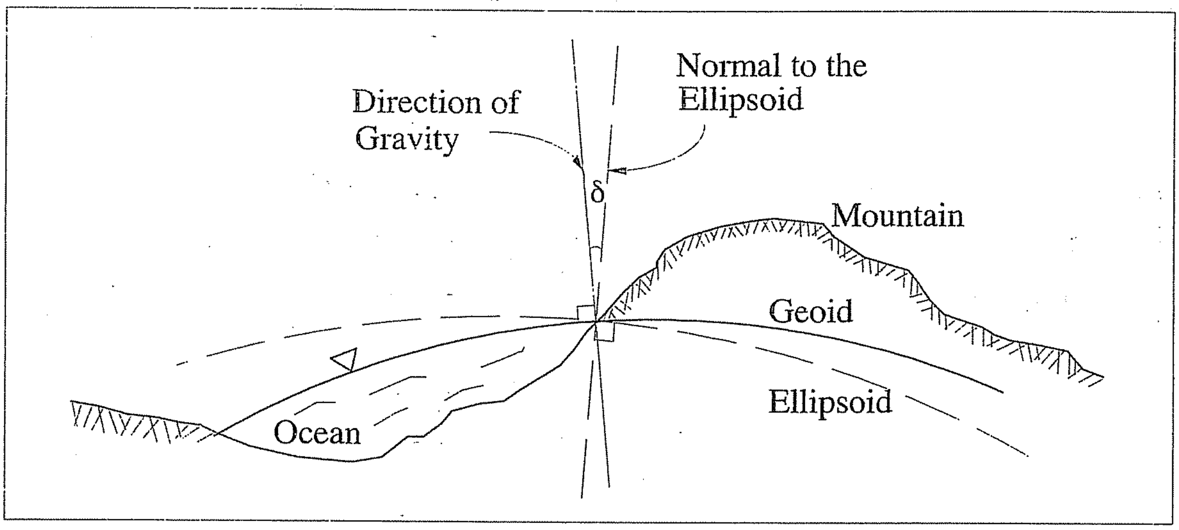


FIGURE 1.2: Relationship between actual ground surface, Geoid and Ellipsoid.

The Geoid, in turn, is also difficult to be represented by a simple mathematical model (equation). However, a mathematical reference frame is necessary to carry out computations of positions, distances and directions. Therefore depending on the area to be surveyed, the following systems are used:

1) For areas > 500 km^2 (especially in Geodetic Surveying), the ellipsoid is used for reference. This ellipsoid has the following properties (Figure 1.3):

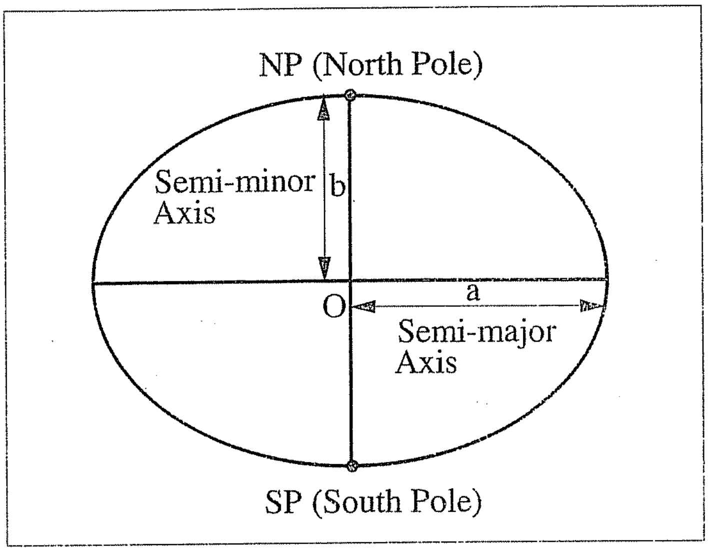


FIGURE 1.3: Ellipsoid of revolution.

- a: Semi-major axis
- b: Semi-minor axis
- f: flattening $= \dfrac{a - b}{a}$ ....(1.3)
- e: eccentricity, where:

$$e^2 = \left(1 - \frac{b^2}{a^2}\right) = 2f - f^2 \qquad \text{.................(1.4)}$$

Typical values of these variables are:
$a = 6378135$ m, $b = 6356750.5$ m,
$f = 1/298.26$, $e = 0.08181881066$

2) $50\ km^2 <$ Areas $< 500 km^2$: Since the value of the flattening (f) is too small, the ellipsoid is replaced by a sphere which has an average radius of $R = 6372200$ m.

3) For areas less than $50 km^2$, it is accurate enough to neglect the curvature of the earth and replace the curved earth surface with a plane.

1.6 TYPES OF SURVEYING

Several different types of surveying can be identified which, although they all use the same basic principles and instrumentation, vary in the intention and end products. These can be divided into three main categories depending on the size of the survey, the method of surveying, or the purpose of surveying.

a) Size of the survey area:

Depending on the area to be surveyed, surveying can be divided in two main types:

1) Geodetic Surveying. When the area to be surveyed is large, such as the surveys of countries, the effect of earth curvature must be taken into consideration and, as a result, knowledge of spherical geometry is required. In this type of surveying, the measurements are done with great care and accuracy, and the scale of the resulting map is usually small (1:100,000 or smaller).

2) Plane Surveying. When the area to be measured is small, the effect of earth curvature is ignored, and the plotted measurements will represent the projection on a horizontal plane of the surveyed area. A horizontal plane is one, which is normal to the direction of gravity, as defined by a plumb bob at a point, and owing to the curvature of the earth, such a plane will in fact be tangential to the earth's surface at that point.

b) Method of surveying:

When the method of surveying is of concern, surveying can be divided into two types:

1) Ground Surveying. In this branch of surveying, the features to be surveyed and mapped are directly measured by physically touching them. Equipment like tapes, ranging rods, levels, theodolites, etc. are used.

2) Remote Surveying. In this type of surveying, the features are first photographed or sensed using a camera or a sensor, and then information about them is collected indirectly by doing measurements on the photographs or images.

c) Purpose of surveying:

Depending on the purpose and the end product of the surveying process, surveying can be divided into several types. These include:

1) Cadastral or Land Surveying. This branch of surveying deals with property boundaries, areas, as well as, land subdivision and consolidation.

2) Topographic Surveying. This includes all the operations leading to the production of topographic maps. A topographic map is one, which portrays the shape and elevation of the terrain. This includes both natural and man-made features such as drainage features, natural vegetation, transportation facilities, rivers, lakes, cities, towns, as well as, any other features that may be of interest to the map users.

3) Hydrographic Surveying. The principal purpose of this type of surveying is to gather information about water bodies. This information is essential for the preparation of nautical charts which, in turn, delineate the submarine topography or bathymetry of a given water area and portray other significant features, as well as, those of the shore area.

4) Route Surveying. This includes all the surveying and mapping activities that are performed for the planning, design and construction of any route of transportation, such as highways, railroads, canals, etc.

5) Construction Surveying. These are the operations performed to layout, locate and monitor public and private engineering projects.

6) Mine Surveying. This type of surveying deals with the control, location and mapping of underground and surface works related to mining operations.

1.7 UNITS OF MEASUREMENT

In surveying, units of measurement are those that are used for the representation of measured lengths, areas, volumes and angles. These units are:

A) *Units of Length Measurement:*

1 - English System: inch (in, "), foot (ft, '), yard (yd), and mile

1 foot = 12 inches
1 yard = 3 ft
1 mile = 5280 ft

2 - Metric System: centimeter (cm), meter (m) and kilometer (km)

1 meter = 100 cm
1 kilometer = 1000 m

B) *Units of Area Measurement:*

1 - English System: ft^2, acre

1 acre = 43560 ft^2

2 - Metric System: m^2, donum, hectare (ha) and km^2

1 donum = 1,000 m^2
1 hectare = 10,000 m^2
1 km^2 = 1,000,000 m^2 = 100 ha

C) *Units of Volume Measurement:*

1 - English System: in^3, ft^3, yd^3

1 ft^3 = 1728 in^3
1 yd^3 = 27 ft^3

2 - Metric System: cm^3, liter, m^3

1 m^3 = 1,000,000 cm^3
1 m^3 = 1,000 liters

D) *Units of Angle Measurement:*

Three systems or units for angle measurement are used in surveying. These are:

1. The Sexagesimal System (النظام الستيني). In this system, the circle is divided into 360°, each degree (°) is divided into 60 minutes ('), and each minute is divided into 60 seconds (") $\Rightarrow$ 1° = 60' = 3600". Example: 213° 24' 47"

2. The Decimal or Centesimal System (النظام العشري أو المئوي). In this system, the circle is divided into 400 equal parts called grads (g). Each grad is then divided into 100 simal minutes (c), and each simal minute is divided into 100 centesimal seconds (cc) $\Rightarrow$ $1^g = 100^c = 10{,}000^{cc}$. Example: $128^g\ 60^c\ 81^{cc}$

3. The Radian System (نظام الراديان). The radian is defined as the central angle opposite to a circular arc whose length is equal to its radius. The whole perimeter of the circle will be opposite to a central angle equal to 2π ($\pi = 3.141592654$). Therefore, $2\pi = 360° = 400^g$.

Relation between the units of measurement:

1 inch = 2.54 cm
1 ft = 30.48 cm = 0.3048 m

International system of units:

For simplicity, the following international units are used.
- Meter (m) for distances
- Square meter (m^2) for areas
- Cubic meter (m^3) for volumes
- Radian for angles.

EXAMPLE 1.1:

How many kilometers are there in a mile?

SOLUTION:

$$1 \text{ mile} = 5280 \text{ ft} = 5280 \times 30.48 \text{ cm}$$

$$= \frac{5280 \times 30.48}{100 \times 1000} = 1.609344 \text{ km}$$

EXAMPLE 1.2:

How many acres are there in a hectare?

SOLUTION:

$$1 \text{ hectare} = 10000 \text{ m}^2 = 10000 \times \left(\frac{100}{30.48}\right)^2 \text{ ft}^2$$

$$= 10000 \times \left(\frac{100}{30.48}\right)^2 \times \frac{1}{43560} = 2.471 \text{ acres}$$

EXAMPLE 1.3:

Change the angle 1.5 radians into its equivalent values in both the sexagesimal and centesimal systems.

SOLUTION:

a) Sexagesimal system:

$$2\pi = 360°$$

$$1.5 \text{ radians} = ? \Rightarrow \theta = \frac{1.5 \times 360}{2\pi} = 85° \, 56' \, 37''$$

b) Centesimal system:

$$2\pi = 400^g$$

$$1.5 \text{ radians} = ? \Rightarrow \theta = \frac{1.5 \times 400}{2\pi} = 95^g \, 49^c \, 30^{cc}$$

1.8 SCALE OF SURVEYS

A map or plan might be defined as a reproduction, at a reduced ratio, of an orthographic projection of the terrain onto a reference horizontal datum plane. This reduced ratio is what is called "scale", and is used because it is physically and economically not possible and inconvenient to represent the measured lengths on a big sheet of paper as they are without reduction. The scale might differ from one plan to another, but should be uniform throughout the same plan. In a simplified form, scale might be defined as the ratio between a length on a map or a plan to its equivalent length in reality.

Three factors affect the choice of the appropriate scale. These are: the size of area to be mapped, the size of the paper sheet to be used for drawing, and the purpose for which the plan is to be used for. Basic scales may range from 1/10 to 1/1,000,000. The former is appropriate for certain detail drawings, and the latter for small-scale mapping. Some examples on scales and their uses are:

1. Architectural works, building works, location drawings: 1/50, 1/100 and 1/200.

2. Site plans, civil engineering works: 1/500, 1/1,000, 1/1250, 1/2,000 and 1/2,500.

3. Town surveys, highway surveys: 1/1250, 1/2,000, 1/2,500, 1/5,000, 1/10,000, 1/20,000 and 1/50,000.

4. Mapping: 1/25,000, 1/50,000, 1/100,000, 1/200,000, 1/500,000 and 1/1,000,000.

1.9 BASIC GEOMETRIC PRINCIPLES OF SURVEYING

The two traditional geometric principles of Surveying are:

1) Working from the whole to the part. This means that when surveying a large area, a set of control points, which are relatively far from each other, are chosen, and the positions of these points are located with a high degree of accuracy. These points are then used to add more inner points to the network through a process called densification. This working from the whole to the part helps minimize and control the measurement errors (Figure 1.4)

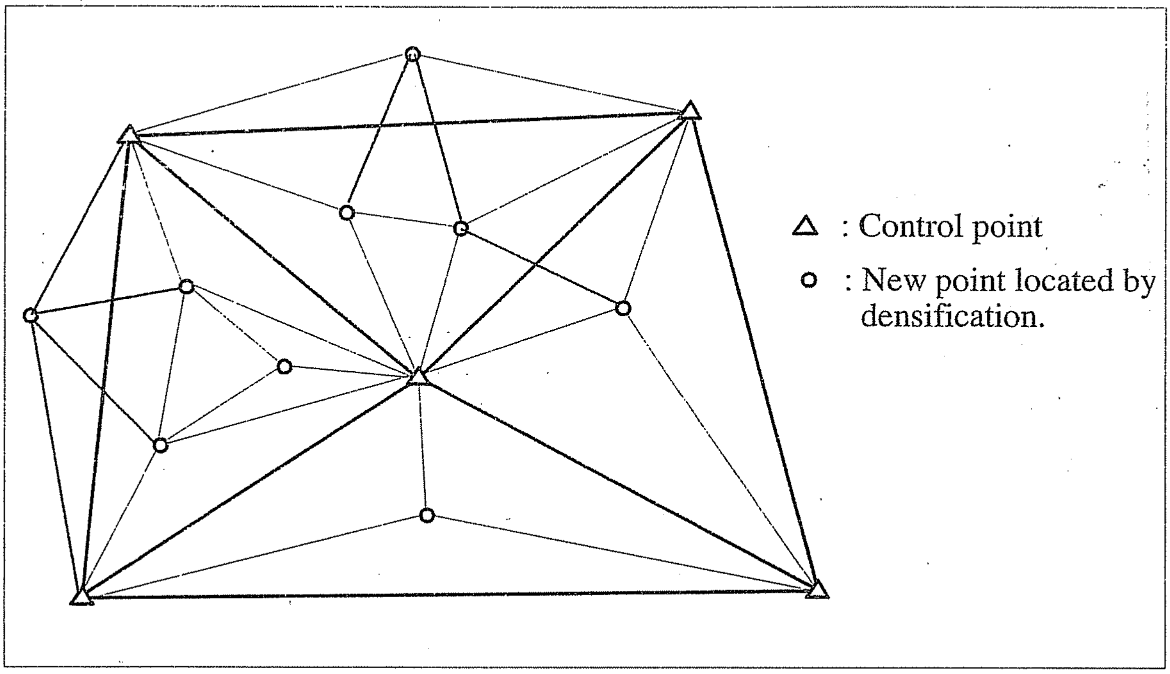


FIGURE 1.4: Working from the whole to the part.

2) The position of an unknown point relative to a known line is located by at least two independent measurements. These measurements can be: an angle and a distance (Figure 1.5a), two angles (Figure 1.5b), or two distances. The two distances could be two direct distances (ties) from two points on the line to the unknown point (Figure 1.5c), or a distance and offset to the unknown point (Figure 1.5d).

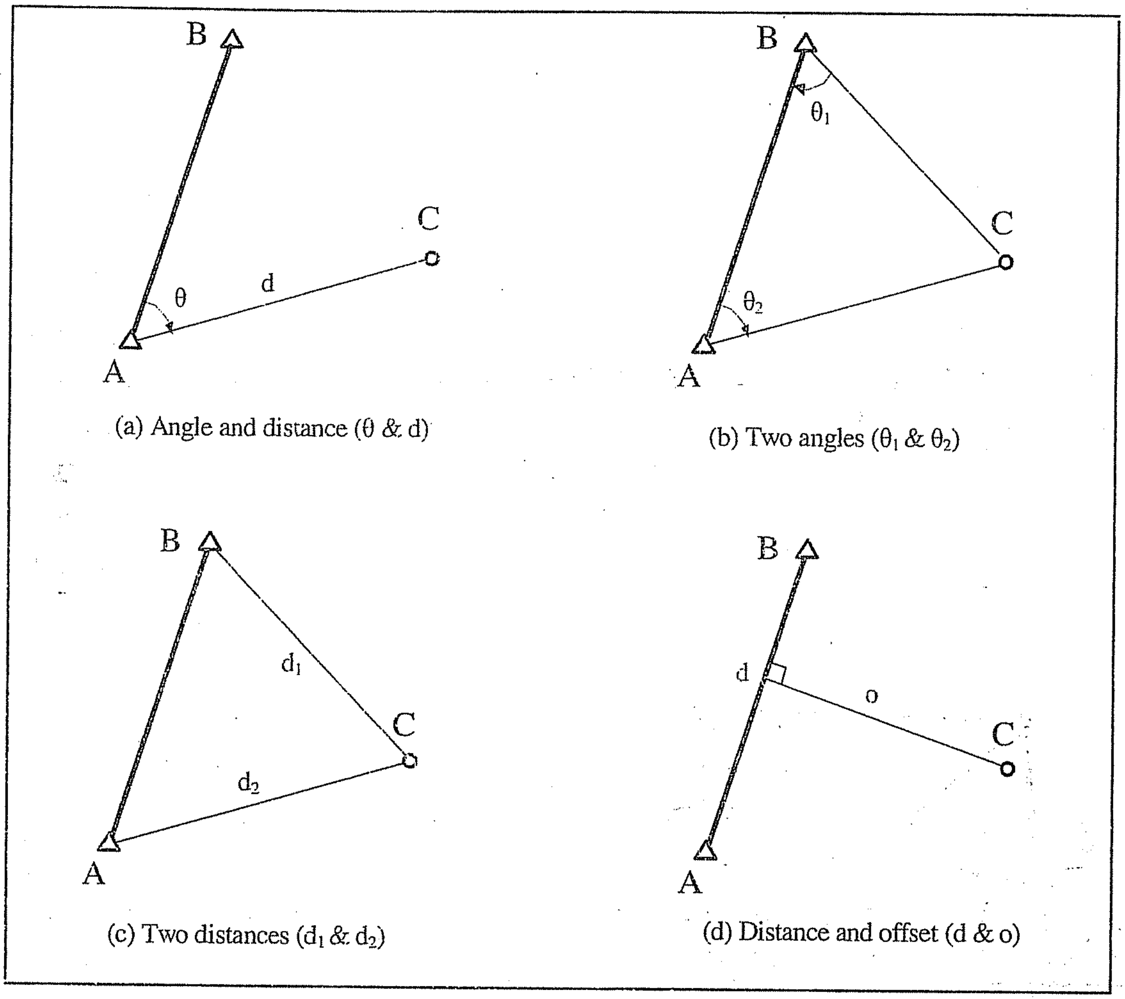


FIGURE 1.5: Methods of locating the position of an unknown point relative to a known line.

Note: The above two principles are no longer of high importance when using Global Positioning Systems – GPS (see Chapter 12) for survey measurements. GPS gives uniform accuracy and does not require inter-visibility between survey points.

PROBLEMS

1.1 What is the relationship between the earth's surface, the Geoid and the Ellipsoid?

1.2 What is the difference between topographic and hydrographic surveying?

1.3 Describe how Eratosthenes managed to measure the perimeter of the earth.

1.4 What is one radian equivalent to in both the decimal and sexagesimal systems?

1.5 Transform the angle 118° 33' 57" into its equivalent values in both the decimal and radian systems.

1.6 How many donums are there in an acre?

1.7 The area of a land parcel is $2780 m^2$. How much is this area equivalent to in ft^2 and acres?

1.8 What are the basic principles of surveying? Why are these principles not important or valid when using the latest GPS technology?

2 ERRORS IN SURVEYING

2.1 INTRODUCTION

To understand the subject of errors in surveying, let us assume that we need to measure a certain distance AB using a measuring tape. To do so, we gave a team of two people a tape and asked them to measure the distance under the following circumstances:

- Perform the measurement on a beautiful day (no wind, comfortable temperature and so on) and write down the result.
- Repeat the measurement immediately on the same day with the same tape.
- Repeat the measurement on a windy day.
- Repeat the measurement on a hot day.
- Repeat the measurement on a cold day.
- Repeat the measurement under the previous conditions but with a different tape.
- Repeat the measurement under the previous conditions, but two different people are asked this time to do the job.

If we compare the various measurements obtained in the different previous scenarios, we will notice that they are not equal. The reason is that all surveying operations are subject to three sources of error, which could lead to harmful and unexpectable results. These sources are:

a) The imperfections of the instruments,
b) The fallibility of the human operator, and
c) The uncontrollable nature of the environment.

Actually, no surveying measurement is exact and free of error (unless by chance), and the true values of the measured parameters are never known. Therefore, a surveyor must thoroughly understand the sources of error in the various methods of surveying, as well as, the methodology for evaluating the achievable accuracy of a surveying program.

Due to the high importance of the subject of errors, and the need to know how to control, avoid and minimize them, I felt the need to introduce this subject to the reader before he/she learns about the techniques and equipment needed to perform the various surveying operations. This chapter will present the fundamental principles of measurement errors and the basic statistical techniques used for evaluating the accuracy of various methods of surveying and of survey results.

2.2 ERRORS IN SURVEYING MEASUREMENTS

The *true error* in a surveying measurement is defined as the difference between the measured value of a parameter and its true value.

Let e_i = true error
x_i = measured value
x = true value

$$\Rightarrow e_i = x_i - x \qquad \text{.........................(2.1)}$$

But since, as mentioned earlier, the true value (x) can never be determined, the true error (e_i) too can never be exactly determined. Therefore, the error in a measurement must be estimated or calculated by comparing it with another more accurately determined value of the same parameter, such as the mean of several measurements. Let $\hat{x}$ represent such a value. Then, an estimate (v_i) of the true error (e_i) is:

$$v_i = x_i - \hat{x} \qquad (2.2)$$

This estimate error is sometimes called the residual error. Once this error is calculated, it should be removed from the measured value (x_i) by subtracting it from x_i. An alternative way is to add a correction (C_i) to the measured value. This correction is equal to the error in magnitude but with opposite sign, that is: correction = - error.

In general, errors in surveying measurements can be divided into three different types:

1. Blunders (also referred to as mistakes),
2. Systematic errors, and
3. Random errors (also referred to as compensating or accidental errors).

2.2.1 BLUNDERS (MISTAKES)

These are simply mistakes caused by human carelessness, fatigue and haste. Blunders can be positive or negative, large or small and their occurrence is unpredictable. Some examples of blunders are the transposition of digits in recording a measurement (such as recording 43.18 instead of 34.18) and sighting a wrong target when measuring an angle.

Blunders are disastrous if left in the surveying measurements, and therefore, must be eliminated by careful work and by using field procedures that provide checks for blunders as will be explained later in several places in this book.

2.2.2 SYSTEMATIC ERRORS

These are mostly caused by the maladjustment of the surveying instruments and by the uncontrollable nature of the environment. Both the signs and magnitudes of systematic errors behave according to a particular system or physical law of nature, which may or may not be known. When the law of occurrence is known, systematic errors can be calculated and eliminated from the measurements.

One example of systematic errors is the tape length error when, for various reasons, the actual length of the tape will be different from its nominal length under calibration conditions.

EXAMPLE 2.1:

A line was found to be 376.4 m when measured with a tape, which is believed to be 20 m long (nominal length). On checking, the actual tape length was found to be 20.04 m. What is the correct length of the line?

SOLUTION:

$$\text{Correct length of the line} = \text{measured length} \cdot \frac{\text{actual tape length}}{\text{nominal tape length}}$$

$$\Rightarrow \text{Correct length of the line} = 376.4 \text{ x } \frac{20.04}{20} = 377.2 \text{ m}$$

For areas (see Chapter 3: section 3.7):

$$\text{Correct Area} = \text{measured area x } \left(\frac{\text{actual length of the tape}}{\text{nominal length of the tape}}\right)^2$$

A special type of systematic error is an error that always occurs with the same sign and magnitude and is therefore often referred to as a *constant error*. The most common source of constant error is the measuring instruments. For example a 30-m tape may in fact be missing the first 0.10 m (i.e., 10 cm) due to the deterioration of the tape after the repeated use. Then, if not noticed, every time the tape is used would contain a constant error of +0.10 m. Constant errors of this type can be detected by careful attention and calibration of the instruments. More examples of systematic errors will be given in the next chapters.

2.2.3 RANDOM ERRORS (COMPENSATING OR ACCIDENTAL ERRORS)

These are caused by imperfections of the measuring instruments, imperfections of the surveyor to make an exact measurement, and the uncontrollable variations in the environment. These errors can be minimized by using better instruments and properly designed field procedures and by making repeated measurements.

Random errors have the following characteristics:

1. Positive and negative errors of the same magnitude occur with the same frequency.
2. Small errors occur more frequently than large ones.
3. Very large errors seldom occur.
4. The mean of an infinite number of observations is the true value.

For example, let us assume that a distance is measured using the same instrument and the same degree of care, a large number of times, say 1000 times. Then, the mean or average of the 1000 repeated measurements is computed, and the estimated (residual) error in each individual length measurement is calculated using Equation (2.2). The estimated error computed in this manner is called the deviation from the mean because it is a measure of how far is the measurement from the mean. Now, calculate the range of these errors (range = maximum error – minimum error), divide this range into a suitable number (5 to 8 intervals) of equal intervals and count the number of occurrences in each interval. Plotting the frequency of occurrence against the interval limits of the estimated error may result in a histogram similar to that shown in Figure 2.1.

For an infinite number of repetitions of the measurements, this histogram approximates to a continuous normal curve with the following probability density function (p.d.f):

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{1}{2}\left(\frac{v}{\sigma}\right)^2} \qquad \text{.............................. (2.3)}$$

Where v = random error, and

σ = Standard error or deviation of the measurements (see next section).

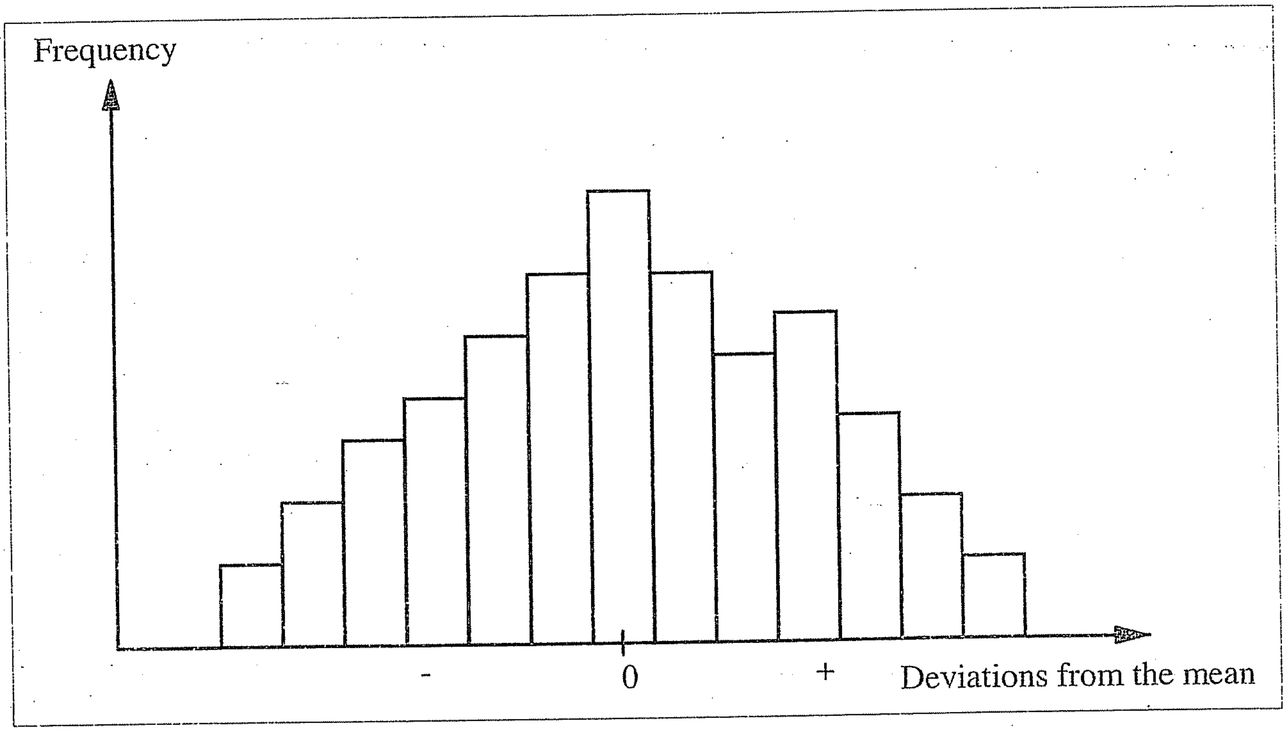


FIGURE 2.1: A histogram which shows the distribution of random errors.

This continuous curve is shown in Figure 2.2.

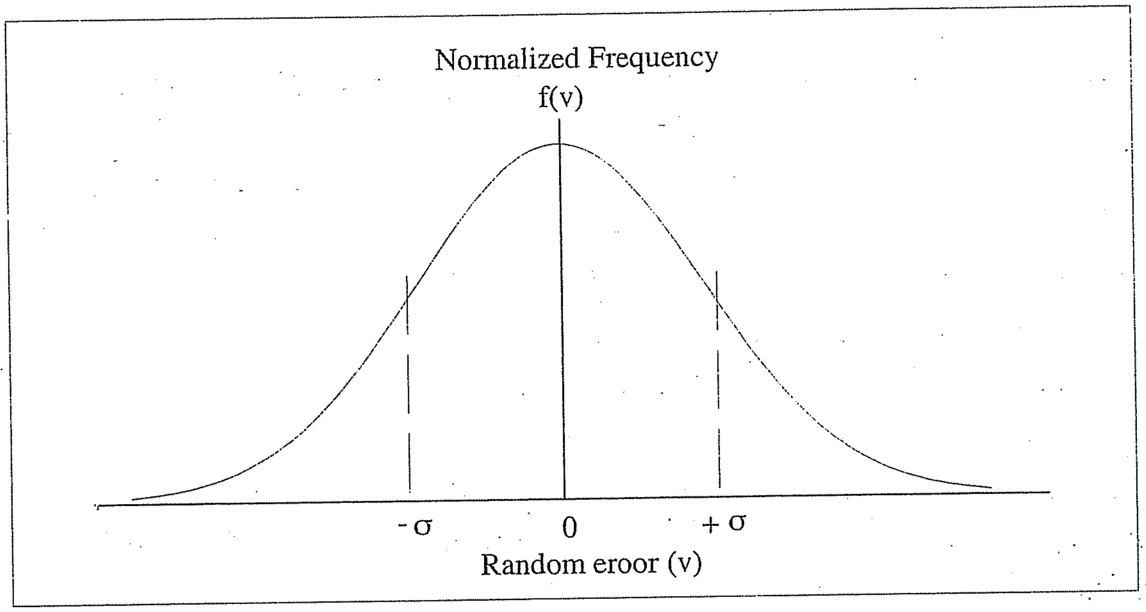


FIGURE 2.2: Normal curve of error.

The normal curve is symmetrical about v = 0. The probability that the random error in a measurement takes on a value between a and b, is equal to the area under the curve and bounded by the values of a and b as shown in Figure 2.3

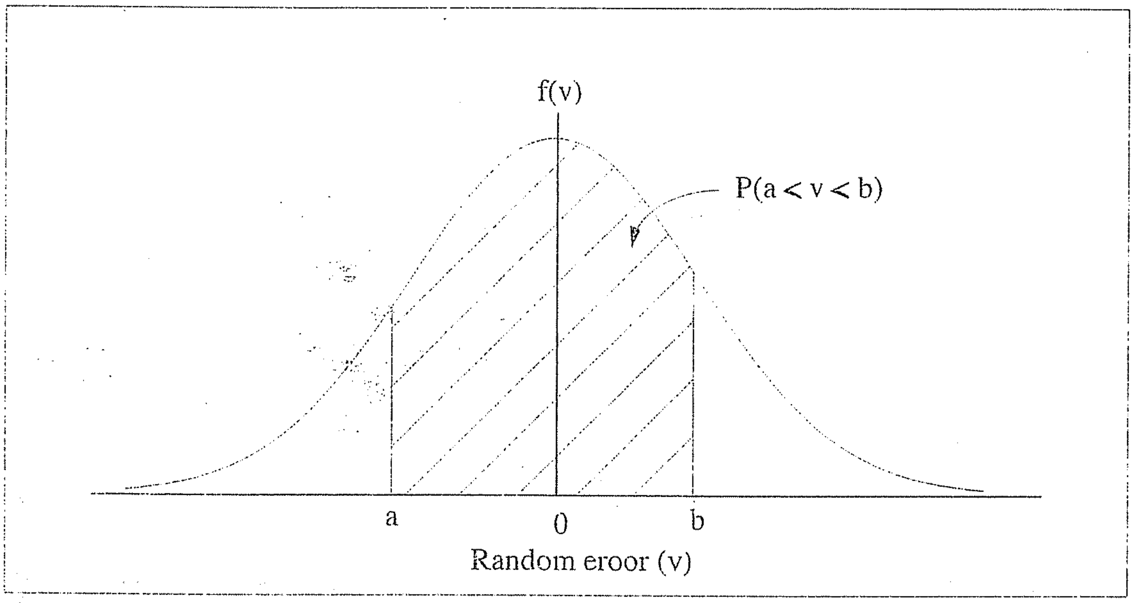


FIGURE 2.3: Probability of random errors.

In mathematical terms, if $P(a \leq v \leq b)$ represents that probability, then

$$P(a \leq v \leq b) = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{1}{2}\left(\frac{v}{\sigma}\right)^2} . dv \qquad \text{............(2.4)}$$

The curve is normalized so that the area under the entire curve is equal to 1. Since this integral is so complicated, probability values can be taken from already prepared tables, which can be found in most statistics book.

Some representative probabilities for selected error ranges are:

Error Range	Probability (%)
± 0.6745 σ	50.0
± 1.00 σ	68.3
± 1.6449 σ	90.0
± 2.00 σ	95.4
± 3.00 σ	99.7

2.3 MEAN, STANDARD DEVIATION AND STANDARD ERROR OF THE MEAN

Let $x_1, x_2, x_3 \ldots x_n$ be n repeated measurements of the same quantity, and let us assume that all these measurements were made with the same instrument and same degree of care. Then:

1) The mean denoted by $\bar{x}$, of the n measurements is computed as follows:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \ldots\ldots (2.5)$$

2) An estimate of the standard error $\hat{\sigma}_x$ of *one measurement* of the quantity is:

$$\hat{\sigma}_x = \pm\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \qquad \ldots\ldots (2.6)$$

The standard error is sometimes called the standard deviation or root-mean-square (RMS) error of a single measurement. It is a measure for the error in a single measurement as compared to the calculated mean.

3) An estimate of the standard error of the *mean of the n measurements*, to be denoted by $\hat{\sigma}_{\bar{x}}$, can be computed as follows:

$$\hat{\sigma}_{\bar{x}} = \pm\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n(n-1)}} \qquad \ldots\ldots (2.7)$$

Or

$$\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}_x}{\sqrt{n}} \qquad \ldots\ldots (2.8)$$

$\hat{\sigma}_{\bar{x}}$ is also called the RMS error of the mean. It is a measure for the error in the mean itself as compared to the true value or an acceptable estimate of it.

2.4 PROBABLE AND MAXIMUM ERRORS

The *probable error* of a measurement is defined to be equal to 0.6745σ. There is a 50% probability that the actual error exceeds the probable error, as well as, a 50% probability that it is less than the probable error. The probable error was widely used in surveying in the past as a measure of precision, now it is replaced by the standard error.

The *maximum error* in a measurement is defined as being equal to 3σ. There is a 99.7% probability that the actual error falls within 3σ, and only a 0.3% probability that the actual error exceeds 3σ.

<u>**Example**</u>: If the standard error of an angle measurement is ± 3.0 seconds, then,
The probable error = ± (0.6745 x 3.0) = ± 2.0 seconds
The maximum error = ± (3 x 3.0) = ± 9.0 seconds

The maximum error is usually used as a measure for detecting and isolating blunders from the surveying measurements. For example, after the mean and standard deviation of *n* repeated measurements have been computed, the deviation (v_i) of each measurement from the mean can be computed ($v_i = x_i - \bar{x}$). If any measurement deviates from the mean by more than 3σ, the measurement is considered to have a blunder. It is rejected, and a new mean and standard deviation are computed without this particular measurement.

EXAMPLE 2.2:

A distance was repeatedly measured 12 times, and the following results (in meters) were recorded:
58.78, 58.83, 58.80, 58.85, 58.18, 58.77, 58.79, 58.80, 58.81, 58.82, 58.79 & 58.82

Check these measurements for the existence of any blunders, reject them (if any), and compute the mean, the standard deviation, and estimated standard error of the mean.

SOLUTION:

Measurement (m)	First Iteration $v_i = d_i - \bar{d}$ (m)	Second Iteration $v_i = d_i - \bar{d}$ (m)
58.78	0.03	-0.03
58.83	0.08	0.02
58.80	0.05	-0.01
58.85	0.10	0.04
58.18	-0.57	blunder ⇒ rejected
58.77	0.02	-0.04
58.79	0.04	-0.02
58.80	0.05	-0.01
58.81	0.06	0.00
58.82	0.07	0.01
58.79	0.04	-0.02
58.82	0.07	0.01

<u>First Iteration</u>: (n = 12)

Mean = 58.75 m

Standard deviation = ± 0.18 m

Estimated standard error of the mean = $\frac{\pm 0.18}{\sqrt{12}}$ = ± 0.05 m

Maximum error of a single measurement = ± 3 x 0.18 = ± 0.54 m

⇒ Reject measurement 58.18 m (possibly the surveyor recorded 58.18 m instead of 58.81 m)

<u>Second Iteration</u>: (n = 11)

Mean = 58.81 m

Standard deviation = ± 0.02 m

Estimated standard error of the mean = $\frac{\pm 0.02}{\sqrt{11}}$ = ± 0.007 m ≈ ± 0.01 m

Maximum error of a single measurement = ± 3 x 0.02 = ± 0.06 m

⇒ No more measurements are rejected.

2.5 PRECISION AND ACCURACY

Precision: A measurement is considered to have high precision if it has a small standard deviation. For example, assume that team A measured a distance with $\hat{\sigma}_A = \pm 0.05$ m, and team B measured the same distance with $\hat{\sigma}_B = \pm 0.10$ m. The measurement of team A is said to be more precise than that of team B. Figure 2.4 shows that a large standard deviation means a flatter distribution curve for the random errors.

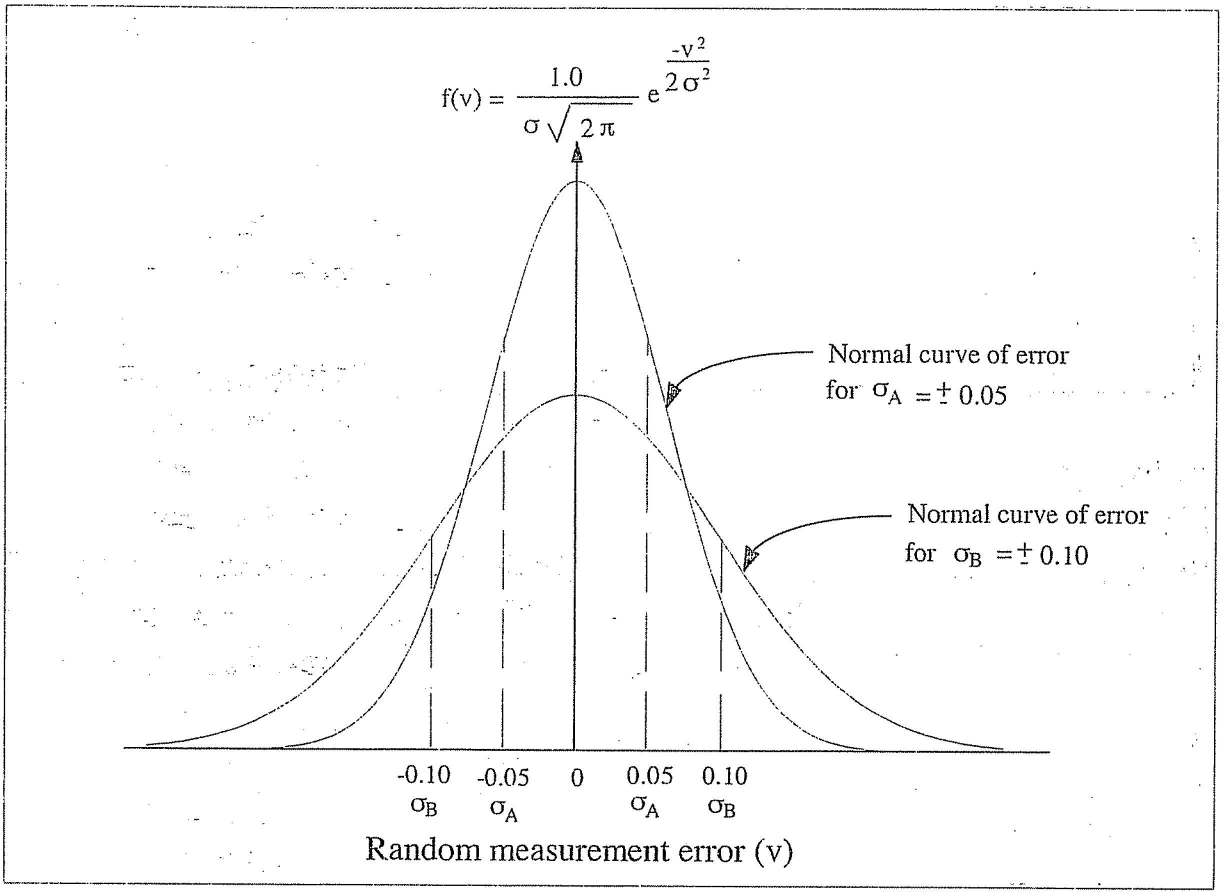


FIGURE 2.4: Standard error and the distribution of random errors.

Accuracy: A measurement is considered to have high accuracy if it is close to the true value. High precision does not necessarily mean high accuracy. A measurement that is highly precise is also highly accurate if it contains little or no systematic errors with all blunders removed. Figure 2.5 shows the four possible combinations of precision and accuracy.

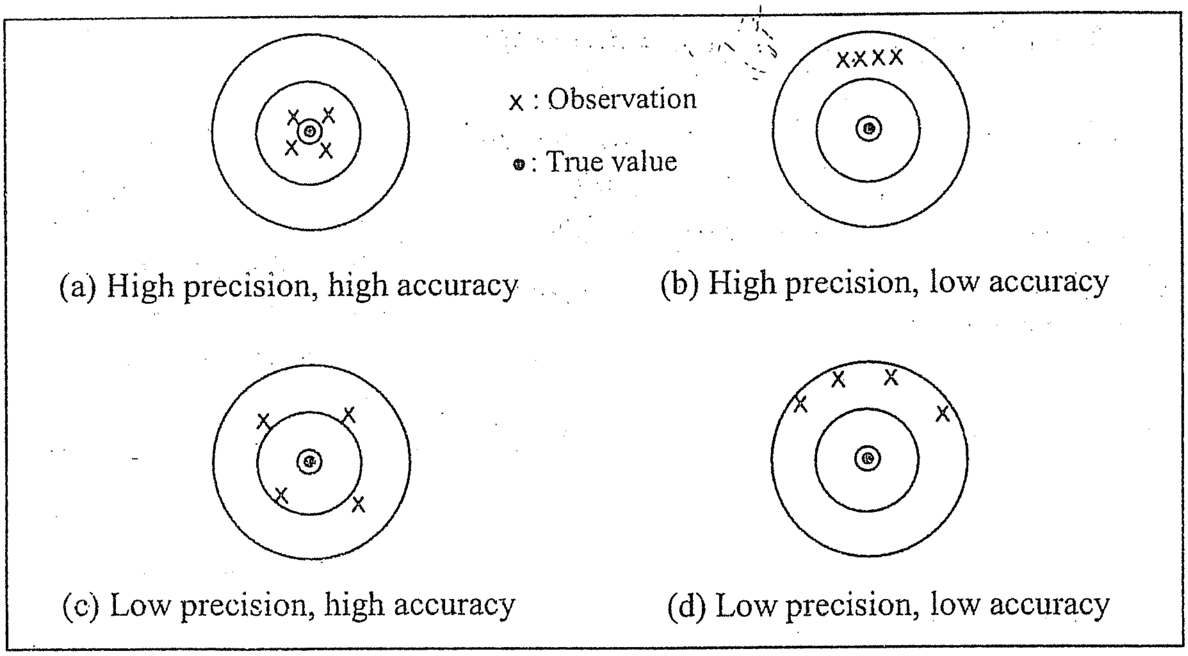


FIGURE 2.5: Possible combinations of precision and accuracy.

EXAMPLE 2.3:

A distance was measured by two independent parties, with the following results:

Party A: $D_A = 257.361 \pm 0.032$ m

Party B: $D_B = 257.538 \pm 0.011$ m

This distance was later measured by highly calibrated EDM (see Chapter 6) and found to be 257.407 m (with all blunders and systematic errors removed).

Compare between the two teams in terms of precision and accuracy.

SOLUTION:

$$\hat{\sigma}_B = \pm\, 0.011 \text{ m} < \hat{\sigma}_A = \pm\, 0.032 \text{ m}$$

$\Rightarrow$ Party B measurement is more precise than Party A.

True value of the distance can be assumed to be D = 257.407 m

Absolute error in party A measurement = $|257.361 - 257.407| = 0.046$ m

Absolute error in party B measurement = $|257.538 - 257.407| = 0.131$ m

$0.131 > 0.046 \Rightarrow$ Party B measurement is less accurate than Party A.

Result: Party B measurement is more precise but less accurate than Party A measurement.

In general, to obtain high precision and high accuracy in surveying, the following strategies must be followed:

1. Follow techniques that will help detect and eliminate all the blunders.
2. Eliminate or correct all systematic errors by frequent calibration and adjustment of the instruments, and
3. Minimize the random errors by using good instruments and field procedures.

2.6 RELATIVE PRECISION

Relative precision is a term that is commonly used to describe the precision of *distance measurement* in surveying. Suppose that a distance D is measured with a standard error σ_D, then:

$$\text{Relative precision of the measured distance at } 1\,\sigma = \frac{1}{D/\sigma_D} \qquad \ldots\ldots\ldots\ldots (2.9)$$

Usually, it is adequate enough to round off the denominator in the relative precision fraction to one or two non-zero digits. For example: a distance (D) was measured and found to be 4576.2 ± 0.3m, then:

$$\text{Relative precision of the measured distance at } 1\,\sigma = \frac{1}{\frac{4576.2}{0.3}} = \frac{1}{15{,}254} \approx \frac{1}{15{,}000}$$

This means that if we measure a distance of length 15,000 units (could be m or ft), then there is a 68.3% chance that the error is within 1 unit. Some agencies choose to represent the relative precision at 2 or 3σ level accuracy. For the previous distance:

Relative precision of measured distance at 3 $\sigma = \dfrac{1}{\dfrac{4576.2}{3 \times 0.3}} = \dfrac{1}{5{,}085} \approx \dfrac{1}{5{,}000}$

This means that if we measure a distance of length 5,000 units, then there is a 99.7% chance that the error is within 1 unit.

2.7 REPEATED MEASUREMENTS

Equation (2.8) [$\hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}_x}{\sqrt{n}}$] shows that $\hat{\sigma}_{\bar{x}} \propto \dfrac{1}{\sqrt{n}}$. This means that as the number of repeated observations (n) of a parameter increases, the standard error of the mean of these measurements ($\hat{\sigma}_{\bar{x}}$) decreases; leading to a high precision in the measured value of the mean. Equation (2.8) can be modified to look like this:

$$\sqrt{n} = \frac{\hat{\sigma}_x}{\hat{\sigma}_{\bar{x}}}$$

$$\Rightarrow \quad n = \left(\frac{\hat{\sigma}_x}{\hat{\sigma}_{\bar{x}}}\right)^2 \quad \text{.............................} \quad (2.10)$$

which means that if the standard deviation of a single measurement is ($\hat{\sigma}_x$), then n measurements are needed to achieve a certain value of ($\hat{\sigma}_{\bar{x}}$) for the standard error of the mean.

For example: Suppose that an angle can be measured with $\hat{\sigma}_x = \pm 3''$ in one repetition by using a certain instrument, then the number of repetitions required to determine the angle with $\hat{\sigma}_{\bar{x}} = \pm 0.8''$ is:

$$n = \left(\frac{3}{0.8}\right)^2 \approx 14$$

2.8 PROPAGATION OF RANDOM ERRORS

So far, the precision and accuracy discussed earlier have been about parameters (such as angles and distances) that are directly measured using surveying equipment. However, it often happens that a quantity is derived from the measured values of other parameters that could be statistically correlated or uncorrelated. For example, a long distance D (Figure 2.6) is obtained by adding the two individually measured sections d_1 and d_2. Now, assuming that each of these two sections has its own standard deviation (i.e. σ_{d_1} & σ_{d_2}), what would be the standard deviation of the derived quantity D?

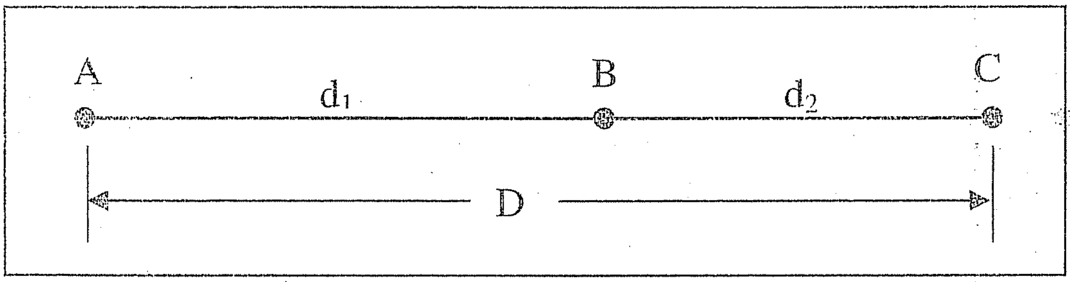


FIGURE 2.6: Measuring a long distance in two sections.

The answer for the previous question will be given here for the simplest case when the measured values are statistically uncorrelated. In general, assume that the value of parameter y can be derived from the measured values of *n* other uncorrelated parameters: $x_1, x_2, \ldots x_n$. Let y be related to the *n* parameters by a continuous function:

$$y = F(x_1, x_2, \ldots x_n) \quad \ldots (2.11)$$

Furthermore, let $\hat{\sigma}_{x_i}$ be the estimated standard error of parameter x_i and $\hat{\sigma}_y$ be the estimated standard error of y. Then:

$$\hat{\sigma}_y^2 = \left(\frac{\partial F}{\partial x_1}\right)^2 \hat{\sigma}_{x_1}^2 + \left(\frac{\partial F}{\partial x_2}\right)^2 \hat{\sigma}_{x_2}^2 + \ldots + \left(\frac{\partial F}{\partial x_n}\right)^2 \hat{\sigma}_{x_n}^2 \quad \ldots (2.12)$$

This law is called the ***law of propagation of random errors***. It is beyond the scope of this book to give the mathematical derivation for this law, but interested readers can refer to some of the references listed at the end of the book.

From the law of propagation of random errors, it follows that:

1) Error of a sum:

If $y = x_1 + x_2 + \ldots x_n$,

$$\Rightarrow \hat{\sigma}_y = \pm\sqrt{\hat{\sigma}_{x_1}^{\ 2} + \hat{\sigma}_{x_2}^{\ 2} + \ldots + \hat{\sigma}_{x_n}^{\ 2}}$$

2) Error of a product:

If $y = x_1 \cdot x_2$,

$$\Rightarrow \hat{\sigma}_y = \pm\sqrt{x_2^{\ 2} \cdot \hat{\sigma}_{x_1}^{\ 2} + x_1^{\ 2} \cdot \hat{\sigma}_{x_2}^{\ 2}}$$

3) Let $y = Ax$, where A is a constant and x is a measured quantity

Then,

$$\hat{\sigma}_y = A \,.\, \hat{\sigma}_x$$

Problem: Prove the above results.

EXAMPLE 2.4:

The radius (r) of a circular tract of land is measured to be 40.25 m with an estimated standard error ($\hat{\sigma}_r$) of ±0.01 m. Compute the area (A) of the tract of land and its estimated standard error ($\hat{\sigma}_A$).

SOLUTION:

$$A = \pi r^2 = \pi(40.25)^2 = 5089.58 \text{ m}^2$$

By the law of propagation of random errors:

$$\hat{\sigma}_A^{\ 2} = \left(\frac{\partial A}{\partial r}\right)^2 \hat{\sigma}_r^{\ 2} \Rightarrow \hat{\sigma}_A = \left(\frac{\partial A}{\partial r}\right)\hat{\sigma}_r$$

$$\hat{\sigma}_A = (2\pi r)\,\hat{\sigma}_r = \pm(2\pi \times 40.25)(0.01) = \pm\, 2.53 \text{ m}^2$$

$$\Rightarrow A = 5089.58 \pm 2.53 \text{ m}^2$$

EXAMPLE 2.5:

The radius (R) of the base of a cone is measured as 14.000 ± 0.002 cm. The height (h) of the cone is measured as 35.000 ± 0.018 cm. What is the standard error of the volume?

SOLUTION:

$$V = \frac{\pi R^2 h}{3} = \frac{\pi \times 14^2 \times 35}{3} = 7183.775 \text{ cm}^3$$

$$\hat{\sigma}_R = \pm 0.002 \text{ cm}, \qquad \hat{\sigma}_h = \pm 0.018 \text{ cm}$$

$$\frac{\partial V}{\partial R} = \frac{2\pi R h}{3} = \frac{2\pi \times 14 \times 35}{3} = 1026.254 \text{ cm}^3/\text{cm}$$

$$\frac{\partial V}{\partial h} = \frac{\pi R^2}{3} = \frac{\pi \times 14^2}{3} = 205.251 \text{ cm}^3/\text{cm}$$

$$\hat{\sigma}_V = \pm\sqrt{\left(\frac{\partial V}{\partial R}\right)^2 \hat{\sigma}_R^{\ 2} + \left(\frac{\partial V}{\partial h}\right)^2 \hat{\sigma}_h^{\ 2}}$$

$$= \pm\sqrt{(1026.254)^2(0.002)^2 + (205.251)^2(0.018)^2} = \pm 4.226 \text{ cm}^3$$

$$\Rightarrow V = 7183.775 \pm 4.226 \text{ cm}^3$$

EXAMPLE 2.6:

Two sides and the included angle of a triangular land parcel were measured with the following results: a = 45.12 ± 0.05 m, b = 38.64 ± 0.03 m, and $\theta = 52° \, 15' \pm 30''$. Calculate the area of the land parcel and its standard error.

SOLUTION:

The area of the triangle is given by the following relationship:

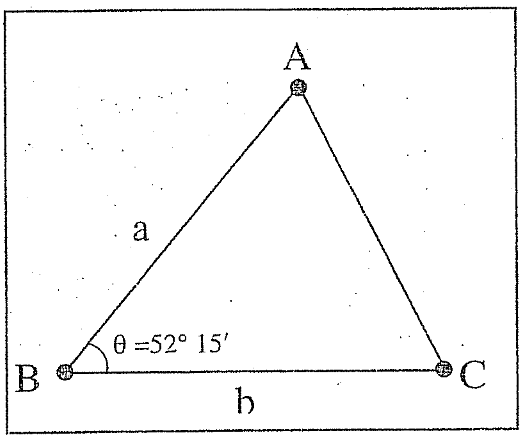


FIGURE 2.7: A triangular land parcel.

$$A = \frac{1}{2} ab \sin\theta$$
$$= \frac{1}{2} \times 45.12 \times 38.64 \sin(52^\circ\ 15')$$
$$= 689.26\ \mathrm{m}^2$$

The standard error of the area is (from Equation 2.12):

$$\hat{\sigma}_A = \pm\sqrt{\left(\frac{\partial A}{\partial a}\right)^2 \hat{\sigma}_a{}^2 + \left(\frac{\partial A}{\partial b}\right)^2 \hat{\sigma}_b{}^2 + \left(\frac{\partial A}{\partial \theta}\right)^2 \hat{\sigma}_\theta{}^2}$$

$$\hat{\sigma}_a = \pm\, 0.05\ \mathrm{m},\quad \hat{\sigma}_b = \pm\, 0.03\ \mathrm{m},\quad \hat{\sigma}_\theta = \pm\frac{30}{3600} \times \frac{\pi}{180} = 1.454 \times 10^{-4}\ \text{radian}$$

$$\frac{\partial A}{\partial a} = \frac{1}{2} b \sin\theta = \frac{1}{2} \times 38.64 \sin(52^\circ\ 15') = 15.28\ \mathrm{m}$$

$$\frac{\partial A}{\partial b} = \frac{1}{2} a \sin\theta = \frac{1}{2} \times 45.12 \sin(52^\circ\ 15') = 17.84\ \mathrm{m}$$

$$\frac{\partial A}{\partial \theta} = \frac{1}{2} ab \cos\theta = \frac{1}{2} \times 45.12 \times 38.64 \cos(52^\circ\ 15') = 533.68\ \mathrm{m}^2$$

$$\Rightarrow \hat{\sigma}_A = \pm\sqrt{(15.28)^2(0.05)^2 + (17.84)^2(0.03)^2 + (533.68)^2(1.454 \times 10^{-4})^2}$$
$$= \pm\, 0.94\ \mathrm{m}^2$$

2.9 WEIGHTS AND WEIGHTED MEAN

Sometimes, one measurement (observation) of a series may be more reliable than another. Such an observation should exert greater influence upon the calculation of the results. The degree of reliability is commonly termed the *weight* of the measurement. This is merely the relative value of that observation to the others of the series.

When calculating the mean value of some quantity from two or more sets of observations, it is logical to give consideration to the calculated precision of each of the sets. The weights are taken to be inversely proportional to the square of the standard error, that is.

$$\frac{w_1}{w_2} = \frac{\sigma_2^{\,2}}{\sigma_1^{\,2}}$$

Or $$w_i \propto \frac{1}{\sigma_i^{\,2}} \Rightarrow w_i = \frac{k}{\sigma_i^{\,2}}$$

Let $k = \sigma_0^{\,2}$

$$\Rightarrow \quad w_i = \frac{\sigma_0^{\,2}}{\sigma_i^{\,2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.13)$$

σ_0 is called the standard error of unit weight because if the standard error σ_i of a measurement is equal to σ_0, then it has a weight of 1.

Since the weight is inversely proportional to the square of the standard error σ_i , then, the more precise the measurement is, the smaller will be its standard error and the larger will be its weight.

Now, let $x_1, x_2, x_3 \ldots x_n$ be *n* independent measurements of a quantity, and let $\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3 \ldots \hat{\sigma}_n$ be the corresponding standard errors of these measurements. This means that the measurements are assumed to be made with different precision. It can be shown that the most probable or accepted value $(\hat{x})$ of the quantity is given by the weighted mean of these *n* measurements; that is:

$$\hat{x} = \frac{w_1x_1 + w_2x_2 + \ldots w_nx_n}{w_1 + w_2 + \ldots w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \qquad \ldots\ldots\ldots\ldots (2.14)$$

Moreover, an estimate of the standard error of the weighted mean $(\hat{\sigma}_{\hat{x}})$ can be computed as follows:

$$\hat{\sigma}_{\bar{x}} = \pm \frac{\sigma_0}{\sqrt{\sum_{i=1}^{n} w_i}} \qquad \text{.......(2.15)}$$

Problem: Use the law of propagation of random errors to prove that the previous equation is correct.

EXAMPLE 2.7:

Compute the weighted mean ($\hat{\ell}$) and the estimated standard error of the weighted mean ($\sigma_{\hat{\ell}}$) for the following four independent measurements of a distance:

$\ell_1 = 2746.34 \pm 0.02$ ft $\qquad \ell_2 = 2746.38 \pm 0.06$ ft

$\ell_3 = 2746.26 \pm 0.05$ ft $\qquad \ell_4 = 2746.31 \pm 0.04$ ft

SOLUTION:

Let $\sigma_0 = \pm 0.06$ ft, then:

$$w_1 = \left(\frac{0.06}{0.02}\right)^2 = 9, \qquad w_3 = \left(\frac{0.06}{0.05}\right)^2 = 1.44$$

$$w_2 = \left(\frac{0.06}{0.06}\right)^2 = 1, \qquad w_4 = \left(\frac{0.06}{0.04}\right)^2 = 2.25$$

$$\hat{\ell} = \frac{2746.34 \times 9 + 2746.38 \times 1 + 2746.26 \times 1.44 + 2746.31 \times 2.25}{9 + 1 + 1.44 + 2.25}$$

$$= 2746.33 \text{ ft}$$

$$\sigma_{\hat{\ell}} = \pm \frac{0.06}{\sqrt{13.69}} = \pm 0.02 \text{ ft}$$

Problem: Prove that the weighted mean and its standard error will not change regardless of the chosen value for σ_0. Support your proof by choosing a different value for σ_0 in the previous example and solve it again.

2.10 SIGNIFICANT FIGURES

The significant figures in a number are those digits with known values. They are identified by proceeding from left to right, beginning with the first non-zero digit and ending with the last digit of the number. The following rules may be helpful:

1 - All non-zero digits are significant
2 - Zeros at the beginning of a number merely indicate the position of the decimal point. They are not significant.
3 - Zeros between digits are significant
4 - Zeros at the end of a decimal number are significant.

Examples:

a - 456.300 has six significant figures
b - 0.0036 has two significant figures
c - 6.000350 has seven significant figures
d - 54.0 has three significant figures

The subject of significant figures is important in both fieldwork and office computations. Since neither the measurements nor the quantities mathematically deduced from them could be exact, it is essential to use the appropriate number of significant figures to express a final meaningful result.

When there are more significant figures in a quantity than are required, the number is rounded off to the number of places needed. The following points should be taken into consideration when deciding upon the number of significant figures in surveying operations:

1) Any calculated value should correspond with its standard error. For example, if the standard error of a distance is ±0.002 m, the value of the distance should be reported to the third decimal place.

2) The number of decimal places in a measurement should not exceed the accuracy of the fieldwork. For example, if a distance can be measured with a tape which can read up to 0.001 m, then it is not reasonable to report a distance with more than three decimal places.

3) When performing addition, subtraction, multiplication or division, the answer can not be more precise than the least precise number included in the mathematical operation.

For example:

$$
\begin{array}{r}
24.217 \\
+\,468.46 \\
+1563.1 \\
\hline
2055.777
\end{array}
$$

The sum must be rounded off to 2055.8 because 1563.1 has only one decimal place.

PROBLEMS

2.1 For the following set of repeated measurements of a distance: 576.39, 576.29, 576.31, 576.34, 576.35, 576.30, 576.33, 576.27, 576.34 and 576.30 m.

a. Compute the mean, standard deviation and estimated standard error of the mean.

b. Check for the presence of any blunders. Reject blunders if any and repeat calculations in part (a).

c. Choose an appropriate interval and plot a histogram for the errors. (for simplicity use 5 intervals)

2.2 A distance is measured to be 456.31 m with an estimated standard error of ± 0.05 m. Compute for this measured distance:

a. The probable error.
b. The maximum error.
c. The relative precision at 1σ.
d. The relative precision at 3σ.

2.3 The length and width of a rectangular field are 5420 ft and 1510 ft respectively. If the area of the field must be determined with a standard error of ± 0.1 acre, determine the relative precision at 1σ with which the length and width of the field must be measured.

2.4 A tract of land is trapezoidal in shape with the following dimensions:

$\ell_1 = 472.3 \pm 0.1$ m
$\ell_2 = 583.7 \pm 0.3$ m
$h = 241.8 \pm 0.2$ m

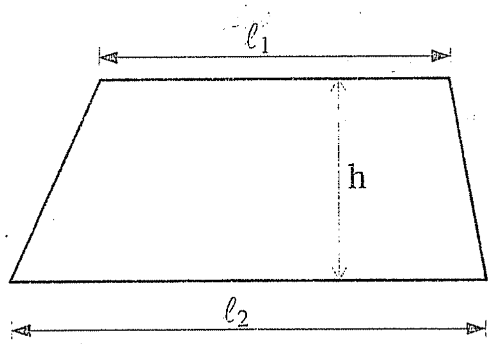


FIGURE 2.8: A trapezoidal land parcel

Compute the area of the tract and its estimated standard error.

2.5 A line was carefully measured 10 times on 3 different days. The mean and the estimated standard error of each day's measurement were computed to be as follows:

Day	Mean	Estimated Standard Error
1	2815.46 m	± 0.05 m
2	2816.72 m	±0.03 m
3	2816.38 m	±0.02 m

Compute:

a. The weighted mean of the three measurements.

b. The estimated standard error of the weighted mean.

2.6 Given below are the elevations and the RMS errors measured from two surveys for two subsidence-monitoring points:

POINT #	ELEVATIONS JUNE 1974	JUNE 1984
101	563.14 ± 0.03 m	563.01 ± 0.06 m
102	579.26 ± 0.04 m	579.05 ± 0.05 m

a. Compute for each point the change in elevation, the RMS error of the change, and the maximum expected survey error in the change. Make a table.

b. Which point has an elevation change exceeding the maximum expected survey error?

2.7 Derive equation (2.8).

3 CHAIN SURVEYING (TAPE MEASUREMENTS)

3.1 INTRODUCTION

As mentioned in chapter 1, measurements in surveying include both distances and angles performed in both the horizontal, as well as, the vertical direction. Often, distances measured in the horizontal direction (termed horizontal distances) between points on the ground surface are needed for several purposes, among which is preparing a plan for the area under consideration. The horizontal distance between two points is defined as the distance between the projections of those points on a reference horizontal plane. A horizontal plane at a point is defined, in turn, as the plane perpendicular to the direction of gravity at that point. Some of the most common methods for measuring distances are pacing, taping, tacheometry and electronic distance measurement.

Pacing is a method that is used for the approximate measurement of a relatively short distance between two points. The length of a stride is usually quite regular for each person. Thus by counting the number of strides a person takes to walk from one point to another, and then multiplying this number by the average length of that person's stride, the distance between the two points can be approximated. An experienced person can obtain an accuracy at 1σ of 1 part in 100 (that is, a relative precision of 1/100) of the distance by using pacing.

Distances can also be measured by a method called tacheometry, which requires an angle-measuring instrument (named theodolite) and a graduated rod (named staff). This method, in most cases, combines distance measurement with the measurement of elevation difference. It is best suited for topographic mapping and will be discussed in Chapter 5.

However, distances ranging from a few meters to several tens of kilometers can be measured in a short time with very high accuracy by using electronic distance measuring (EDM) equipment. Such instruments measure distances using electromagnetic waves and will be discussed in Chapter 6.

This chapter deals with the discussion of distance measurement using tapes in a simple process called *taping* or *chaining*. The term *chaining* has its origin from the use in the past of the link chain that was devised by the Englishman Edmond Gunter in the seventeenth century. Because of the wide availability and use of measuring tapes nowadays, the term *taping* is becoming more frequently used. Distances up to 100 m long can be easily measured with a tape to an accuracy (at 1σ level) of 1/3,000. By using proper care and field procedures, small areas can be mapped using a measuring tape alone with an accuracy that is adequate for many engineering projects. As in all surveying operations, the successful use of tapes for distance measurement requires a thorough understanding of measurement principles, equipment, field procedures and sources of error.

3.2 EQUIPMENT USED IN CHAIN SURVEYING

The equipment used in chain surveying falls under three broad categories: those used for linear measurement, those used for establishing right angles, as well as, other equipment.

a) Equipment used for the measurement of lines:

1) The Chain. This is made of links of tempered steel wire, each link being 0.20 m long from center to center of each middle connecting ring (Figure 3.1). Chains are 20 m or 30 m long with markers attached at every whole meter position and different color markers giving 5 m positions. They are not commonly used because of their heavy weight, especially when they are suspended, which leads to sagging errors. Also, their length easily changes with temperature because of their high coefficient of thermal expansion.

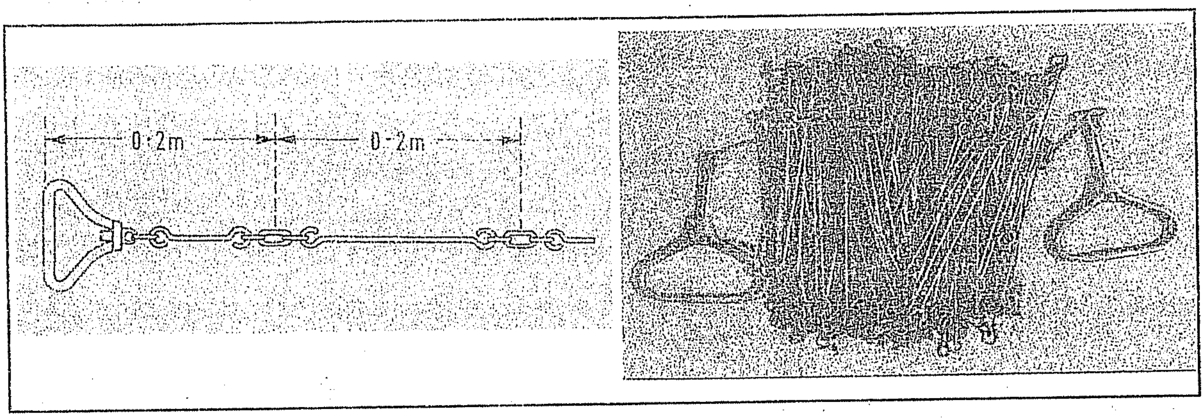


FIGURE 3.1: An example of chain links.

2) Tapes. These may be made of synthetic material, glass fiber being typical, or coated or plain steel. Tapes of lengths ranging from one meter to 50 meters are available, with 20-m, 30-m and 50-m lengths being most commonly used in surveying (Figure 3.2).

In general, most available tapes in the market nowadays are graduated and figured in a way that distances can be read and recorded to the nearest millimeter. Tapes are more accurate than chains, but their main disadvantages are their lack of robustness and the difficulty in doing field repairs.

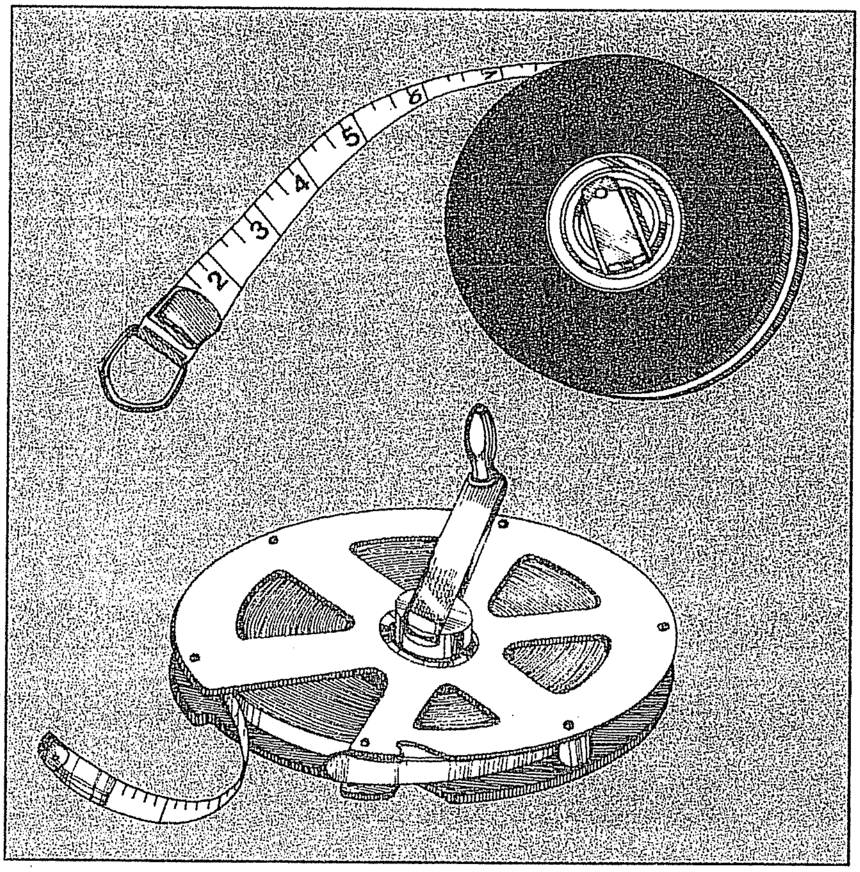


FIGURE 3.2: Typical measuring tapes.

3) Invar tapes. The word ***invar*** is an abbreviation of the English word ***invar***iable. These tapes are made of a mixture of steel (65%) and Nickel (35%), which results in a low coefficient of thermal expansion (about one-thirtieth that of steel tapes). As a result, their length is least affected by temperature, and this makes them more accurate than other types of tapes. Because they are expensive, invar tapes are primarily used for measuring lines that require a high degree of accuracy.

b) Equipment used for making right angles:

1) The cross staff. This tool consists essentially of an octagonal brass box with slits cut in each face so that opposite pairs form sight lines (Figure 3.3a). This enables sights to be taken through any two pairs of slits whose axes are perpendicular. The other two pairs enable angles of 45° and 135° to be set out. An alternate type of cross staff is shown in Figure 3.3b.

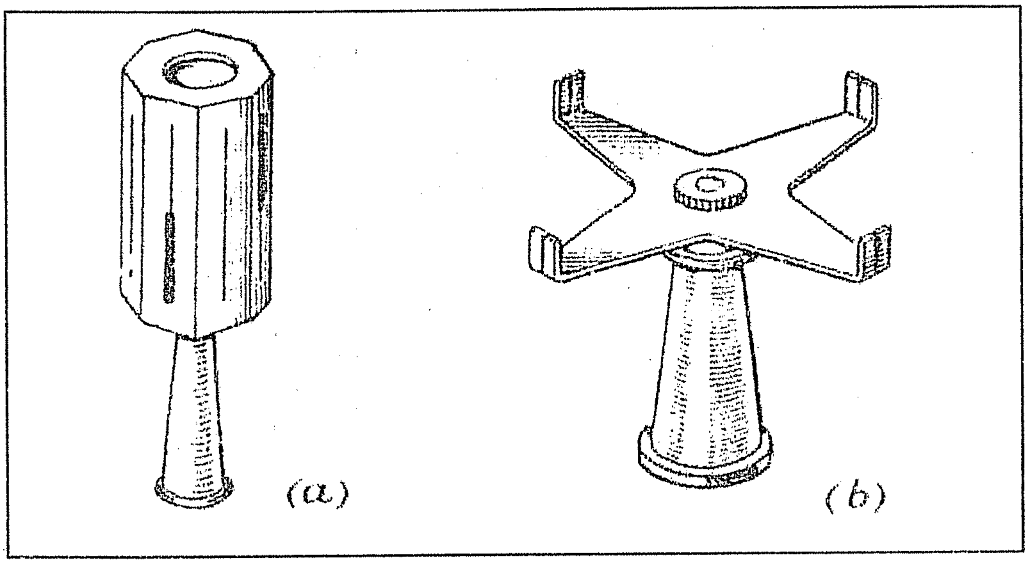


FIGURE 3.3: Cross staves.

2) The site-square. This consists basically of two telescopes, one mounted on top of the other, with their lines of sight at 90° to each other. The lower telescope is directed to a site mark positioned on one arm of the right angle to be established. The line of sight of the second telescope will lie along the other arm of the right angle and a further site mark can be positioned as required.

3) The optical square. This is a simple and compact instrument, the most widely available kind having a cylindrical shape of about 35-mm diameter and 40-mm thickness (Figure 3.5b). There are two types of optical squares, one using two mirrors and the other using a prism

The mirror type makes use of the fact that a ray of light reflected from two mirrors is turned through twice the angle between the mirrors (Figure 3.4). As can be seen from this figure, mirror A is completely silvered. Mirror B, on the other hand, is silvered to half its depth while the other half is left plain. Thus, the eye looking through the small eyehole will be able to see half an object at O_1. An object at O_2 is visible in the upper (silvered) half of mirror B, and when $O_1\hat{X}O_2$ is a right angle; the image of O_2 is in line with the bottom half of O_1 seen directly through the plain glass.

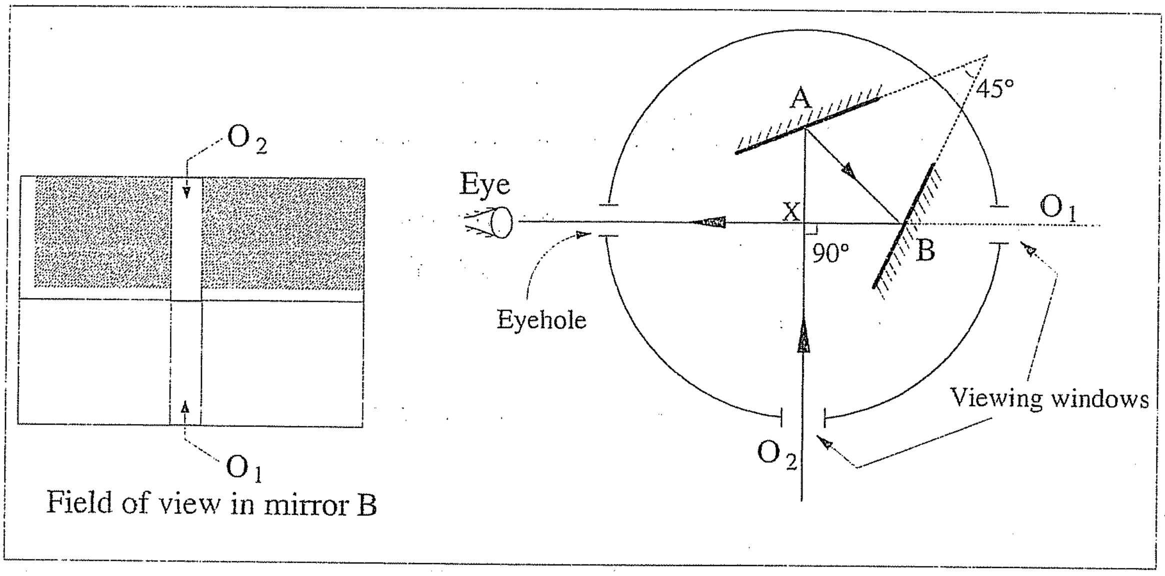


FIGURE 3.4: Mirror type optical square.

The prismatic type of optical square employs a pentagonal-shaped prism, cut so that two faces contain an angle of equal to 45° (Figure 3.5a), and is used in the same way as the mirror optical square. Figure 3.5b shows an example of the prismatic type that could contain

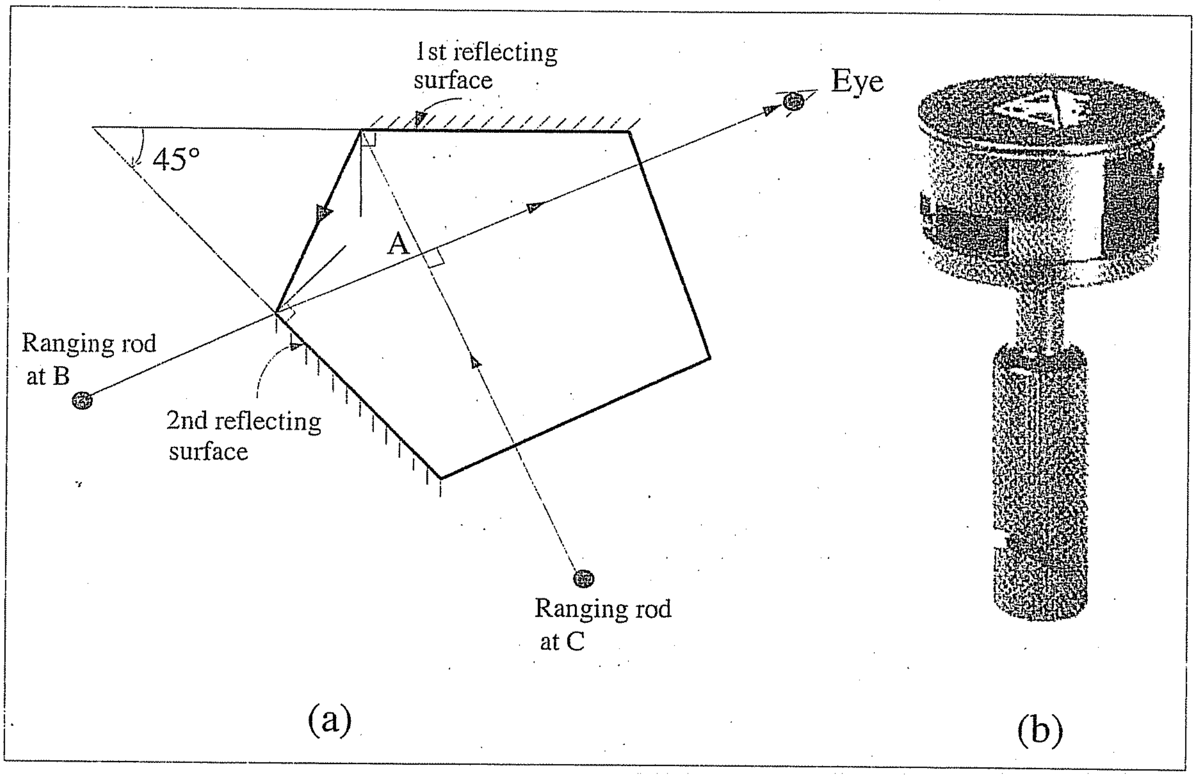


FIGURE 3.5: Prism type optical square.

only one prism (single prism) or two prisms (double prisms), with an opening on the right side of the optical square, and another opening on the left side.

c) Other equipment:

The following equipment is also used in chain surveying:

1) Ranging rods. These are poles of circular cross-section that is about 1-inch in diameter and that are 1 or 2 meters long. They are painted with alternate bands of red and white that are usually 0.5 m long, and are tipped with a pointed steel shoe to enable them to be driven into the ground (Figure 3.6). Rods that are 1-m long steel pipes are most common because they are easier to handle, and can be connected with each other to form longer rods. In general, ranging rods are used to help in the measurement of lines, and for marking any points which require to be seen. On hard surfaces (such as rocks or paved ground), a special tripod is used to support the rods.

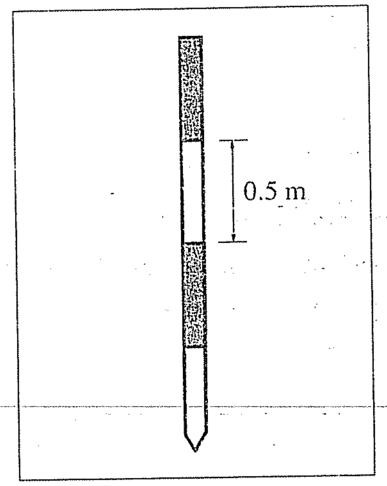


FIGURE 3.6: Ranging rod.

2) Arrows (also known as pins). These are steel skewers about 40 cm long and 3 to 4 mm in diameter (Figure 3.7). They are used to mark intermediate points when measuring a long line.

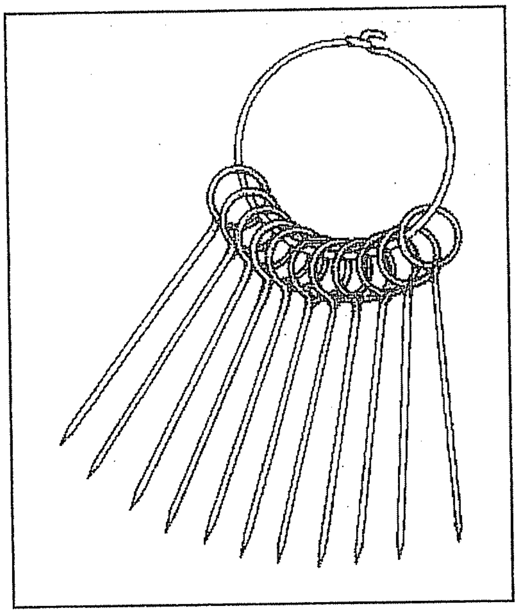
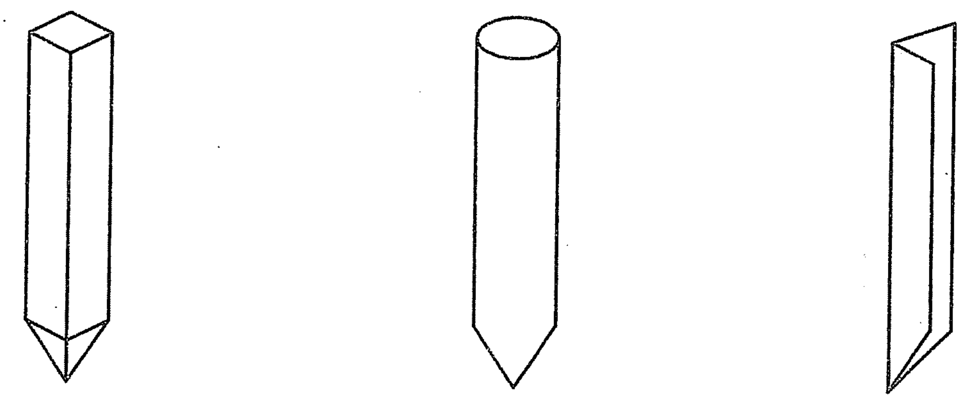


FIGURE 3.7: Arrows.

3) Pegs. Points which require to be more permanently marked, such as the intersection points of chain lines, are marked by pegs driven into the ground. These pegs can have a 4 cm x 4 cm square cross-section (Figure 3.8a), or a circular cross-section of 3 to 5 cm diameter (Figure 3.8b), both about 40 cm long. In very hard or frozen ground, steel angles are used instead (Figure 3.8c), while in asphalt roads, small 5 or 6 mm square brads are used.



(a) Square cross-section (b) Circular cross-section (c) Steel angle

FIGURE 3.8: Types of pegs.

4) Plumb bob. The plumb bob is a metallic object in the shape of a cone. When hung freely by a strong string from the center of its base, the tip of the cone points towards the direction of gravity. It is used to project a point on the ground up to the tape, to project a point on the tape down to the ground, and to center surveying equipment above stations (Figure 3.9).

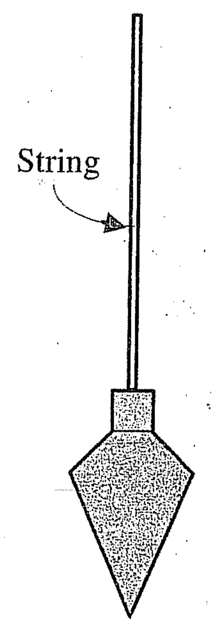


FIGURE 3.9: The plumb bob.

5) Clinometer. A small device used to measure the angle of inclination (slope) of a uniformly sloping ground. A simple clinometer can be made using a protractor and a plumb bob as shown in Figure 3.10.

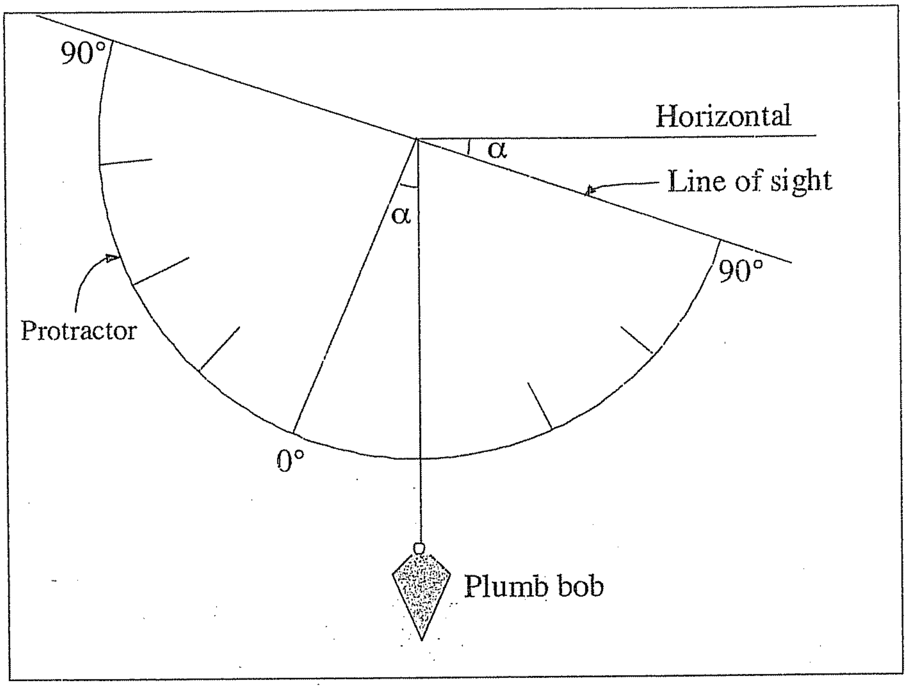


FIGURE 3.10: The clinometer.

6) Abney Level. This is an alternative device used to measure the inclination angle of uniformly sloping lines. Figure 3.11 shows a typical abney level.

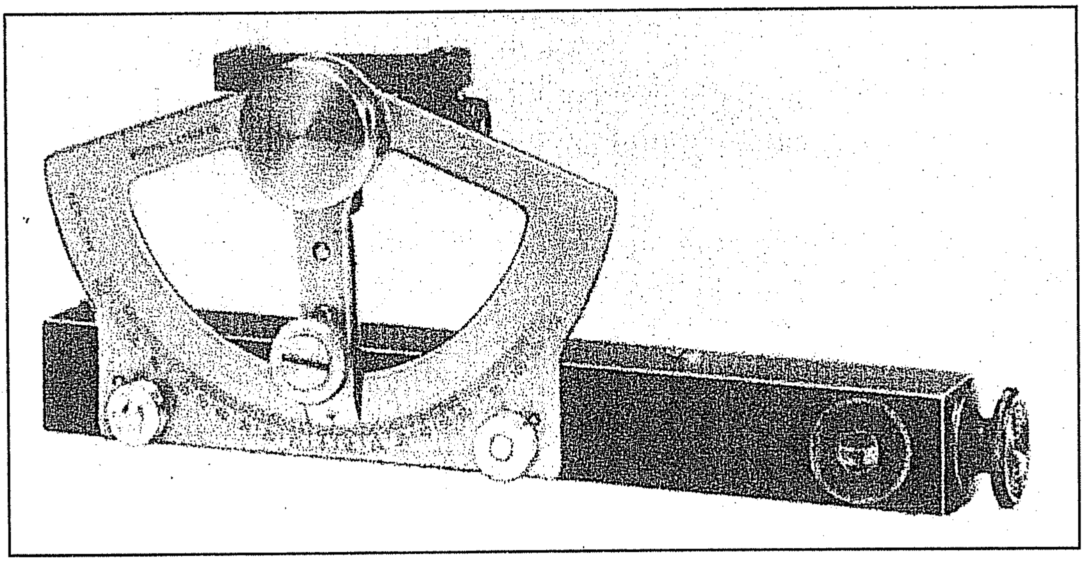


FIGURE 3.11: The Abney level.

3.3 PROCESSES IN CHAIN SURVEYING

Two types of measurement are performed in chain surveying. These are the ranging and measurement of lines, and the setting out of right angles.

3.3.1 RANGING AND MEASUREMENT OF LINES

Measuring a distance is one of the basic processes done in chain surveying. This operation is usually performed by two people, one acting as a leader and the other as a follower. If the line to be measured is shorter than one tape length, it is directly measured by extending the tape between the end points of the line. However, if the line is longer than one tape length, it is required to add intermediate points (which could be at equal or random distances) between the end points of the line by a process called forward ranging. In general, the following important points should be kept in mind when doing the measurement:

1) The measurement should be in a straight line, especially when measuring long lines. This will reduce the error caused by the individual sections of the line not lying in a straight line with each other.
2) The tape should be reasonably pulled to minimize sagging, and to avoid over-stretching the tape material at the same time.
3) A systematic way should be followed to count the number of times the tape is used between the end points of the line.

Now, depending on the topography of the ground where the line to be measured is located, the following procedure is followed:

A) ***Level or nearly level ground.*** When the ground is level or nearly so, the line is measured as follows:

1. Position two ranging rods at both ends of the line (A & B in Figure 3.12). The rods should be vertical.

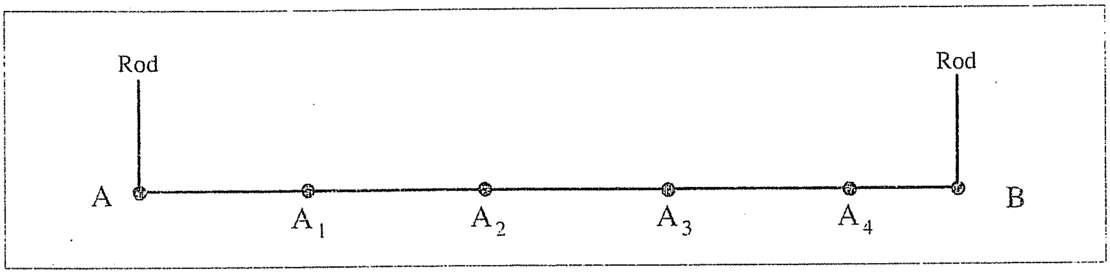


FIGURE 3.12: Line ranging.

2. The leader, holding a ranging rod and the end of the tape and several arrows (10 arrows for example), extends the tape horizontally in the direction of point B.
3. The follower, holding the zero of the tape and standing behind the rod at A, looks in the direction of B and begins giving right and left signals to the leader until the rods at A, A_1 and B lie in a straight line. The leader, then, drives an arrow at A_1.
4. The follower moves with the zero of the tape and a ranging rod to A_1. He then pulls out the arrow and drives the ranging rod in its place. At this time, the leader will extend the tape in the direction of B. The ranging process will be repeated until point A_2 is located by driving an arrow into it.
5. Step (4) is repeated to locate the next point A_3. At this moment the follower will be standing at A_2 with two arrows in his hand.
6. The previous steps are repeated until reaching a point (A_4 in Figure 3.12) so that the distance between this point and point B will be less than a tape length. When the follower stands at this point, he will have four arrows with him, which means that the distance measured so far will be four multiples of the tape length. If the last segment between A_4 & B is measured and added to the four multiples, the total horizontal distance AB will be known.

> **Note 1:** An alternative procedure to the one explained above will be to divide the distance between the end points A and B randomly into a number of sections which are not necessarily equal. These individual sections are then measured and their lengths are added to get the total length of the line.

Note 2: The process used to divide the line above is called forward ranging whereby additional points are added between the end points of the long line. Alternatively, a short line, say line CD, might need to be extended to form a longer line behind point C. In this way, with ranging rods driven vertically at points C and D, the surveyor holds a ranging rod in a vertical direction behind C, moves backwards and aligns himself with the rods at C & D until he gets to a point such as E and drives an arrow. He could move backwards again, align himself with C & D and add another point like F, and so on. Notice that this process of backward ranging is done with only one person as compared to forward ranging, which needs two people to do it.

B) ***Uniformly Sloping Ground***. When the ground between points A and B has a uniform slope (Figure 3.13), the slope distance (S) is measured by the tape, while the slope angle (α) is measured by a clinometer or an abney level. The horizontal distance (D) and elevation difference (Δh) between points A & B will be:

$$D = S \cdot \cos\alpha \qquad \text{.............................. (3.1)}$$

$$\Delta h = S \cdot \sin\alpha \qquad \text{.............................. (3.2)}$$

If the elevation difference (Δh) between points A and B is known, then there is no need to measure the slope angle (α). The horizontal distance (D) will be calculated, instead, from the following expression:

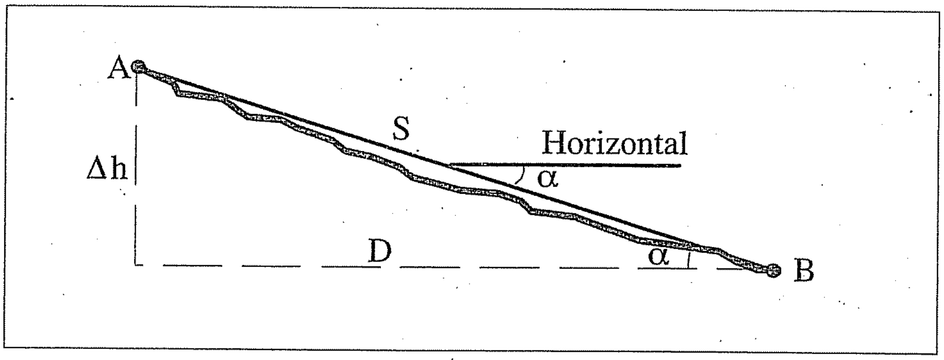


FIGURE 3.13: Distance measurement on a uniformly sloping ground.

$$D = \sqrt{S^2 - (\Delta h)^2} \quad \text{.............................. (3.3)}$$

C) ***Uneven Ground.*** On uneven or non-uniformly sloping ground, line measurement is done through a process called stepping. The measurement is done in short horizontal increments of 10 to 15 m long with the help of a plumb bob. The total length in this case will be the sum of all the short increments. Figure 3.14 illustrates this process.

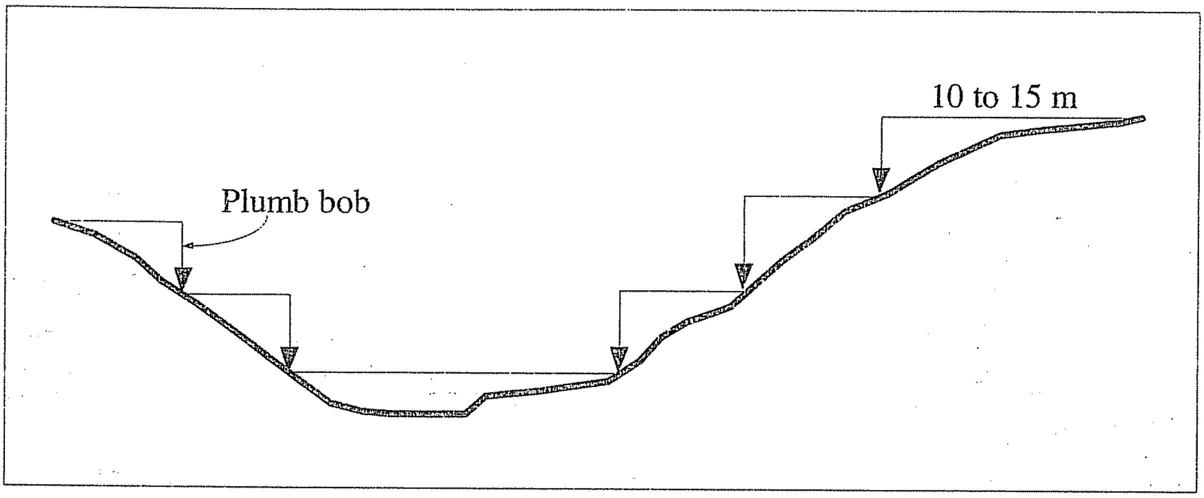


FIGURE 3.14: Stepping.

3.3.2 SETTING OUT RIGHT ANGLES

Two cases are to be considered here:

A) Dropping a perpendicular from a point C to a line AB. This can be done by any of the following methods:

1. With the free end of the tape at point C, extend it horizontally towards line AB and swing it left and right and observe the minimum reading at which it crosses the line AB (Figure 3.15a). This occurs when the tape is perpendicular to the line. This method is primarily used on smooth ground where a free swing of the tape is possible.

2. With the free end of the tape at point C (Figure 3.15b), strike an arc to cross the line AB at points D & E. Bisect DE at F. Then $C\hat{F}E = 90°$.

3. Run the tape from C to any point D on the line AB (Figure 3.15c). Bisect CD at E, and with E as center and a radius equal to ED, strike an arc to cross line AB at F. Then, $D\hat{F}C = 90°$, being the angle in a semi-circle.

4. Using the optical square with double prism. Three rods are placed at A, B & C. The surveyor carries the optical square in a vertical direction and starts moving left and right on the line AB while facing point C. When he sees the three rods at A, B & C along one line, then he will be standing at point D where $C\hat{D}B = 90°$ (Figure 3.15d).

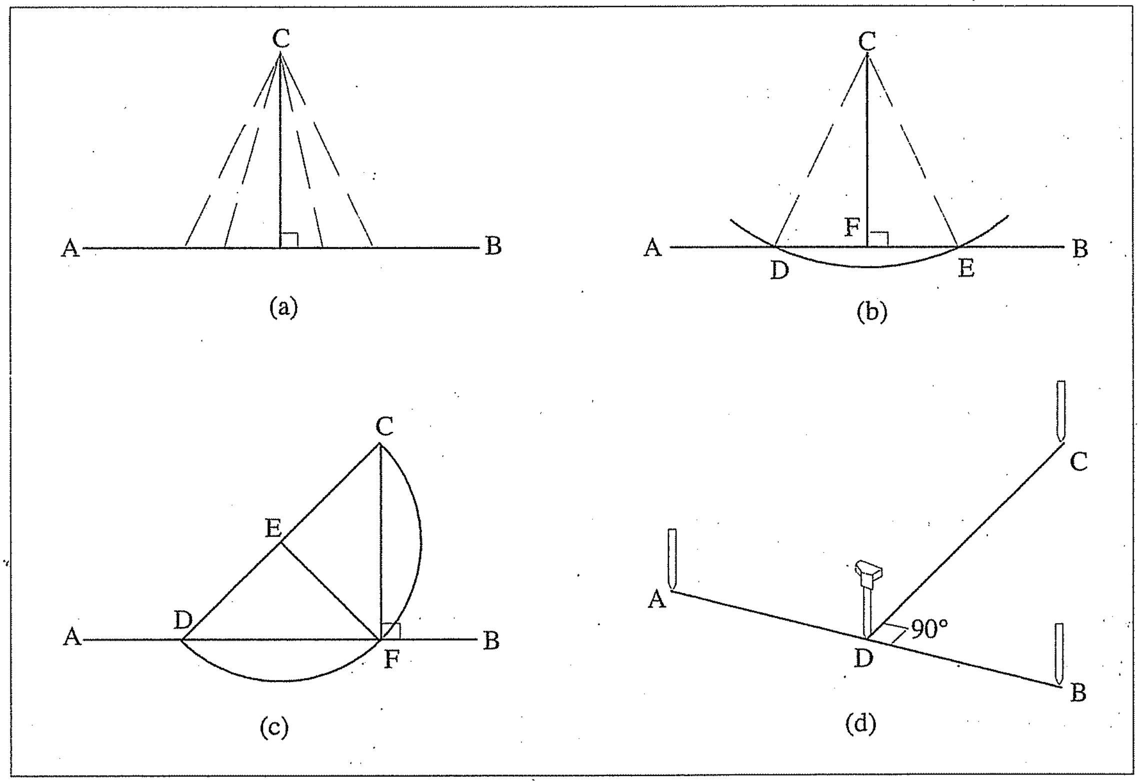


FIGURE 3.15: Methods of dropping a perpendicular from a point (C) to a chain line (AB).

B) Setting out a line at right angles to another line such as AB from a given point C on this line. This can be done by any of the following methods:

1. Using the cross staff or site-square as explained in section 3.2.

2. Using the optical square with double prism. In this method, the surveyor stands at point C, which is located on the line AB (Figure 3.16), and looks through his vertically held optical square until he sees the two rods at A and B as one continuous rod. Another person with a rod starts moving left and right in the field of view until this rod is seen by the surveyor to be in line with the rods at A & B. This rod will define the location of the end point of the perpendicular line.

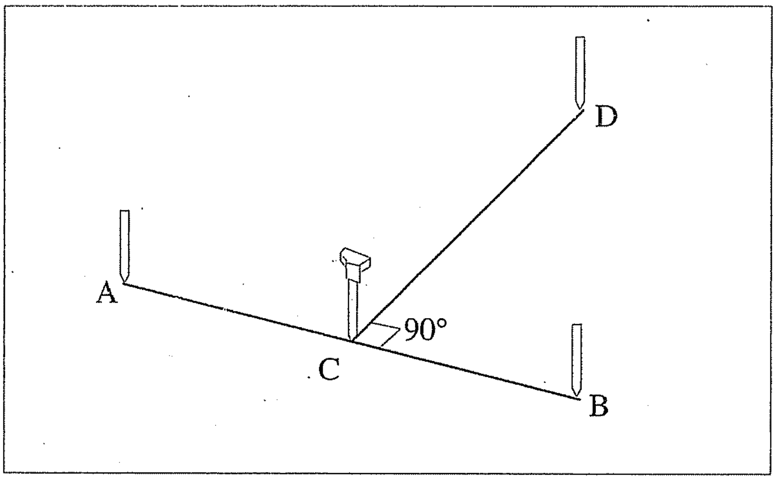


FIGURE 3.16: Erecting a perpendicular using the optical square with double prism.

3. The equilateral triangle method. Choose two points D & E on line AB such that CD is equal to CE (Figure 3.17). Then, with a radius bigger than CD or CE, strike arcs from D & E to intersect at F. Then $F\hat{C}E = 90°$.

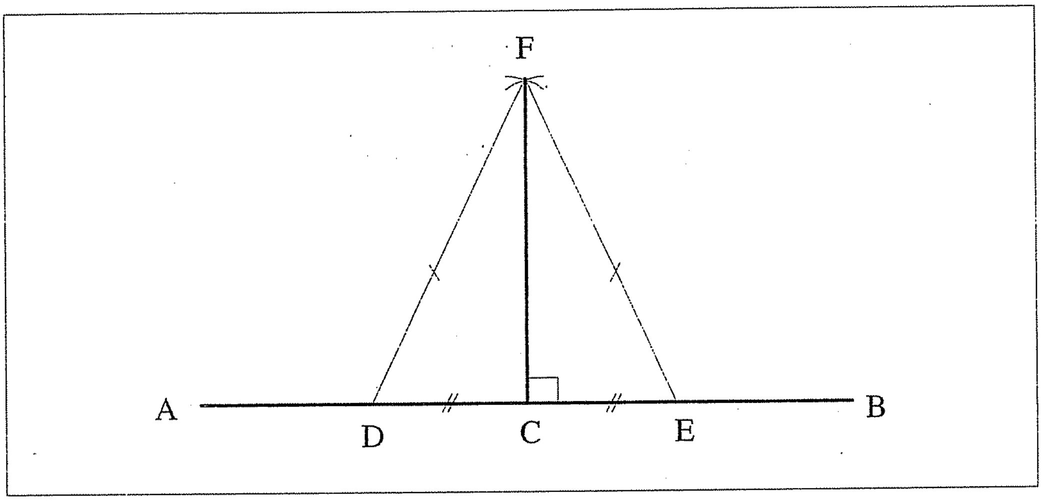


FIGURE 3.17: Setting out a perprndicular using the equilateral triangle.

4. Pythagoras' theorem method with the ratio between the right triangle sides being 3:4:5 or any multiple such as 6:8:10. To erect a perpendicular from point C at the line AB, measure a distance equal to 4 m on line AB and locate point D (Figure 3.18). Now, put the zero of the tape at C, and the 8 m mark (3 m + 5 m) at D, and with a pin or an arrow, pull the tape at the 3 m mark. The resulting point E will be the end point of the line perpendicular to AB at C.

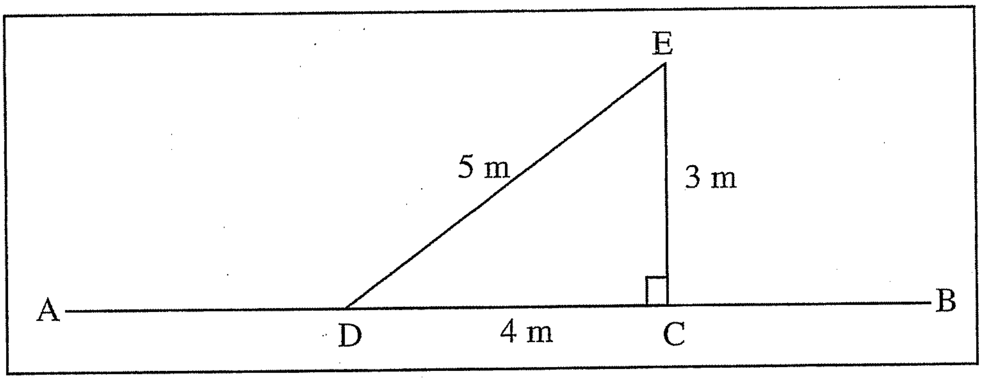


FIGURE 3.18: Pythagoras theorem method for setting out a perpendicular to a chain line.

3.4 MAPPING DETAILS USING CHAIN SURVEYING

In chain surveying, the topographical and man-made features are located and mapped by measuring with the tape the lengths of a series of selected reference straight lines, called chain lines, and then locating points on the ground relative to these lines. This is done by one of two ways:

1) The method of ties. In this method, a point (which could be an edge of a building) is located by measuring two reasonable distances, called ties, between this point and two selected points on the chain line. Figure 3.19a illustrates this process.

2) The method of offsets. In this method, a point is projected on the chain line, and then the distance between the point and its projection (called offset) as well as the distance from the beginning point of the line to the projected point are measured. Figure 3.19b illustrates this process.

Ground details may be located by the method of ties or the method of offsets or a combination of both.

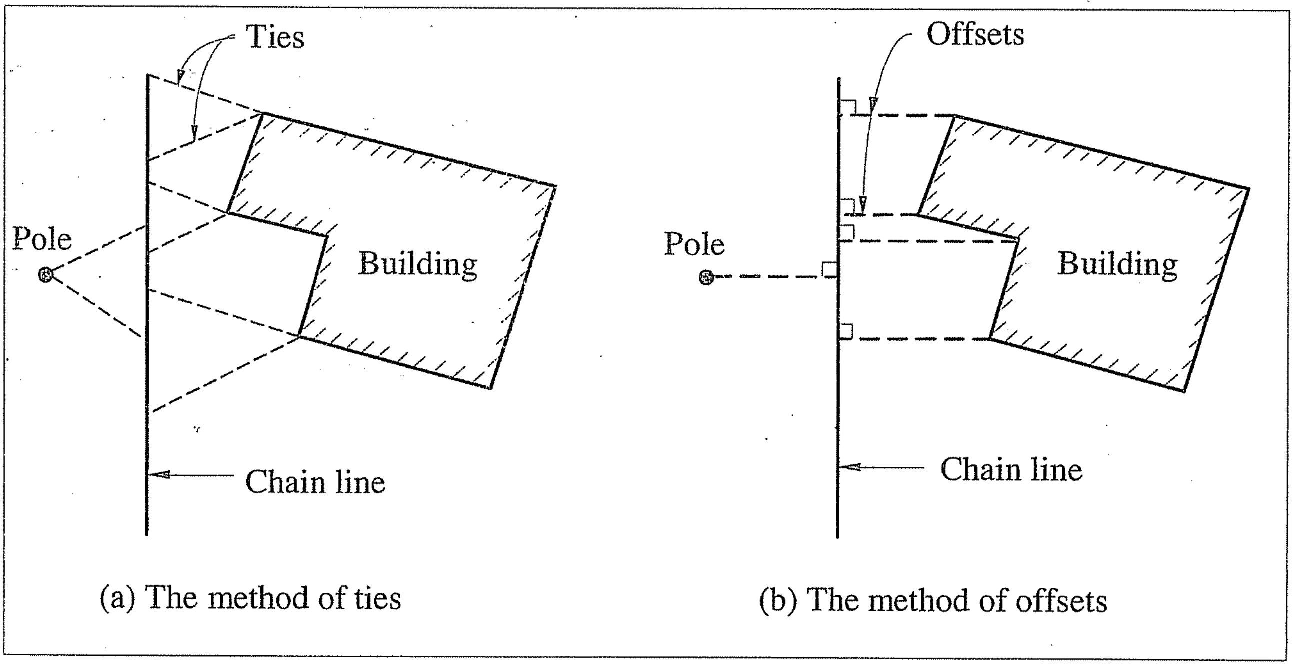


FIGURE 3.19: Methods of locating ground details.

Before performing any field measurements, the surveyor should do a reconnaissance process where he visits the area to be mapped. He notices the shape of the area, as well as, the existing details and draws a reasonable sketch of the area. The sketch is drawn by free hand using a pencil, but reasonably made so that the drawn distances will represent their counterparts on the ground. Sometimes a straight edge is used for greater clarity and time saving. All details such as roads, buildings, fences, electric poles, etc. should be included in the sketch, in addition to the approximate north direction. Figure 3.20 shows an example of a sketch.

The reconnaissance process will help the surveyor choose the most suitable location for the survey stations, which, in turn, form the chain lines. It also helps the surveyor plan his work including the time needed for the measurements, the number of people needed to do the work, the type of equipment necessary, and finally, the cost of performing the field work.

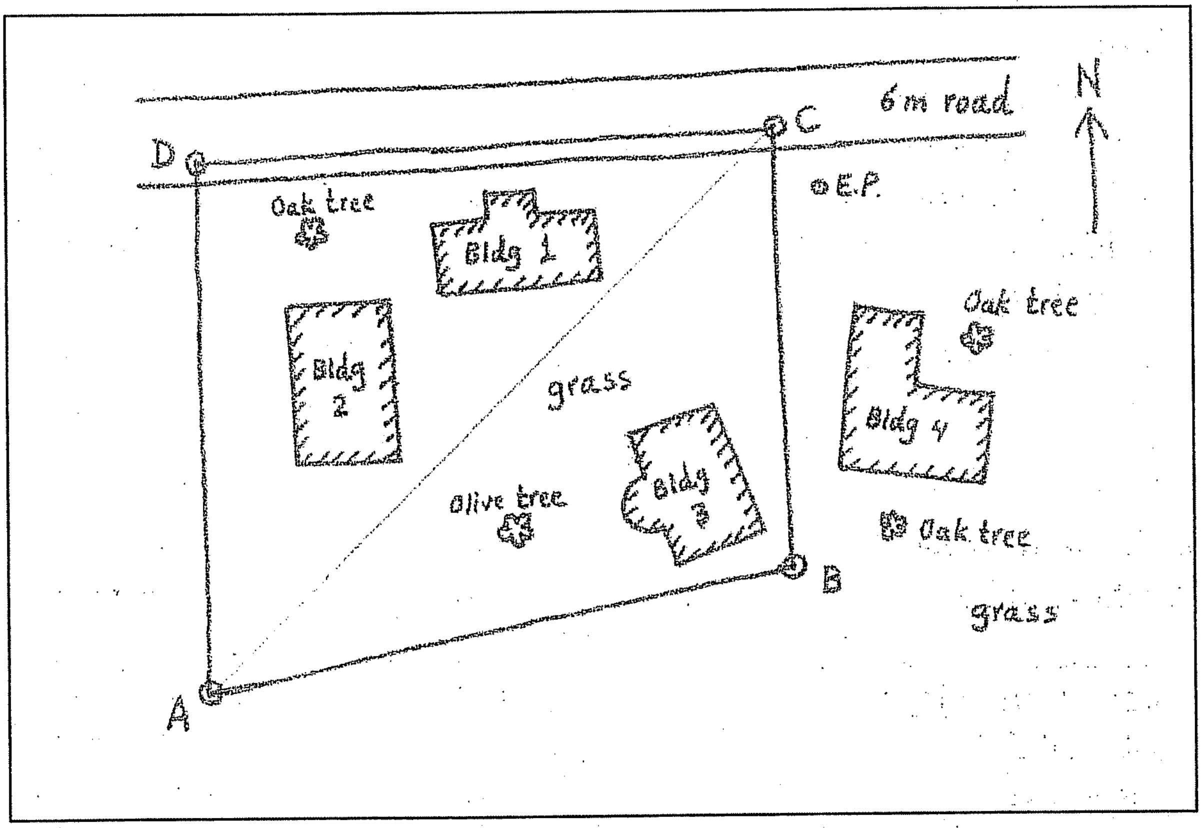


FIGURE 3.20: An example of a sketch for an area to be chained.

3.4.1 CHOICE OF CHAIN LINES

When choosing survey stations, and hence the chain lines, the following points should be taken into consideration:

1) Chain lines should form well-conditioned triangles where by the internal angles fall between 30° and 120°. This will help in forming a sharper determination of the point of intersection of the chain lines when plotting the details (Figure 3.21).

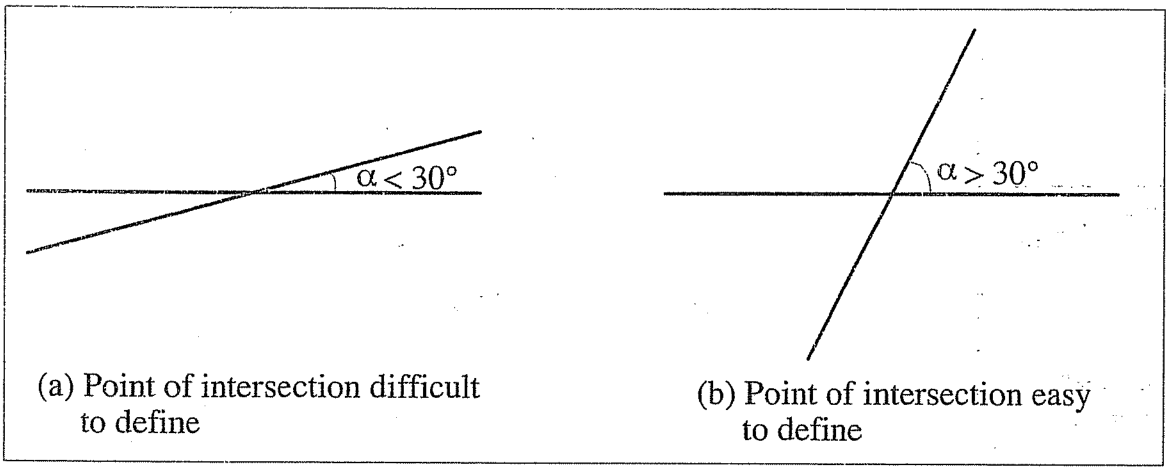


FIGURE 3.21: Relationship between the angle of intersection between chain lines and the location of the point of intersection.

2) Chain lines should be chosen as close as possible to the boundaries of the area or the details to be measured, so that the ties or offsets will be short.

3) It should be possible to see at least two other survey stations from each station.

4) The number of chain lines should be kept to a minimum, but at the meantime, enough to locate all the details. This will minimize the errors and the required amount of work.

5) Survey stations should be chosen in a way such that check lines will be provided (e.g. line AC in Figure 3.20).

6) Survey stations should be chosen on firm, easy to reach grounds. Obstacles along chain lines should be avoided as much as possible. It is an advantage to select stations close to stationary objects such as electric poles, trees, building corners, wall edges, etc. When a station is tied by at least two ties with such immovable objects, then it will be easy to recover such a station if it is lost for some reason.

3.4.2 BOOKING THE MEASUREMENTS

It is necessary to book the field measurements in an organized way so that they can be easily understood when plotting the details. Moreover, the person preparing the map could be different from those who do the fieldwork, and as a result the systematic organization of the field measurements forms the communication language between the field people and the draftsman. The booking is usually carried out in a field book that consists of good quality paper, the pages of which are approximately 20 cm long by 15 cm wide. Each page is divided in the middle by two lines, which run from the top to the bottom of the page, with a separation of about 1.5 cm between these two lines. These two lines represent a split chain line and the space between them is used for booking distances along the chain line. The spaces to either side of the chain line are used to show measurements to points of detail, whether these measurements being offsets or ties. Figure 3.22 shows the booking of details across and on both sides of chain line BC in Figure 3.20.

The following points should be taken into consideration when booking survey measurements:

1) Begin each line at the bottom of a fresh page. If one page is not enough for a particular long line, other pages can be used with special page numbering and continuation marks indicated.
2) All measurements made in the field must be recorded in the field at the time of measurement. Do not rely on memory!
3) Proceed with the booking from the bottom of the page to the top. The measurement should be recorded in the direction of chaining (i.e., left or right of the chain line).

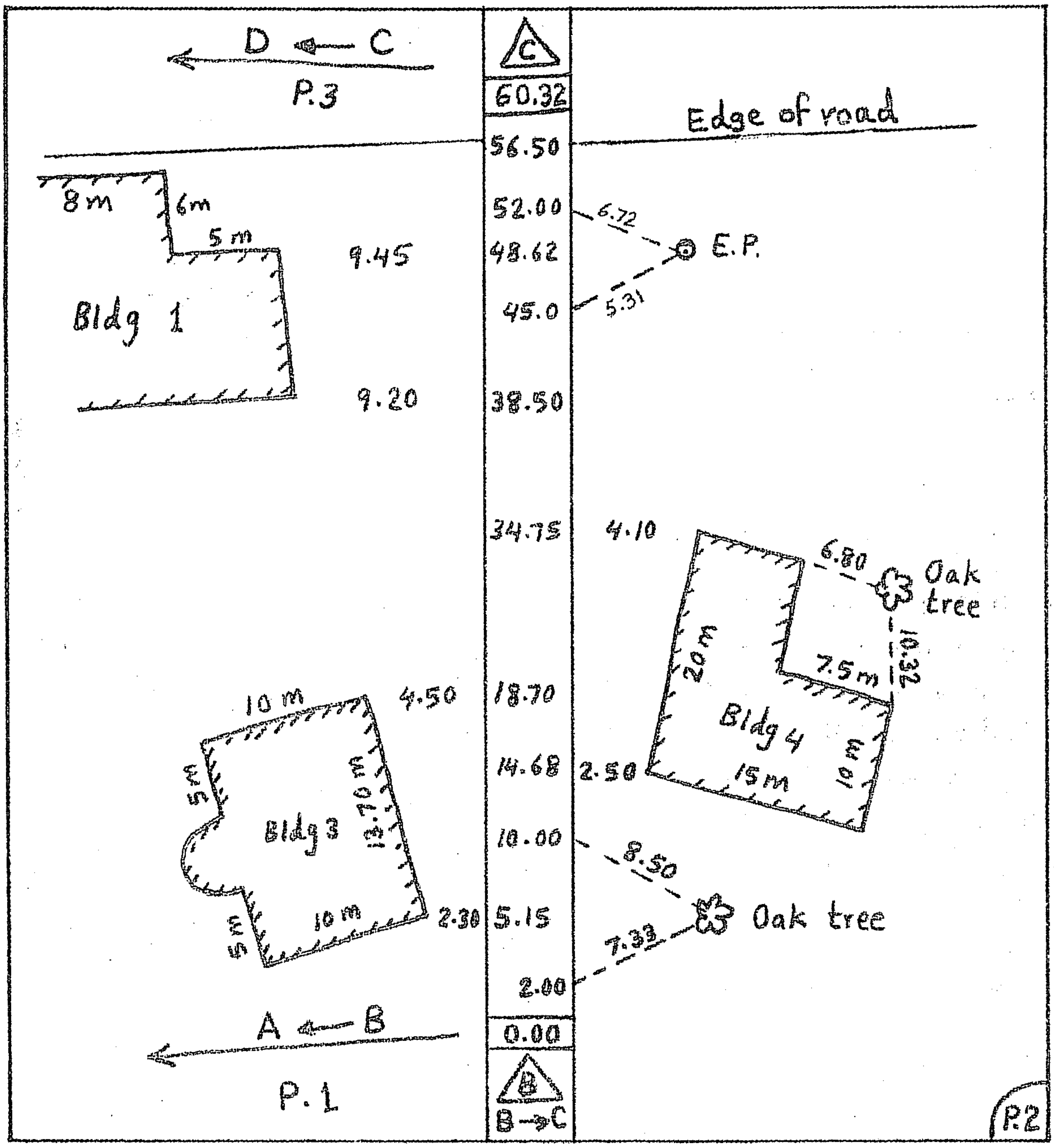


FIGURE 3.22: Booking of field measurements.

4) All details surveyed must be sketched neatly and roughly to scale in the appropriate place in the field book so that all measurements can be easily read and understood when plotting.

5) All other chain lines, which meet the chain line being measured, should be recorded in the appropriate place and a reference made to the page number of the field book on which its measurement can be found (see Figure 3.22).

6) To avoid confusion and crowding of the field book pages, offsets from the points of detail to the chain line are usually not drawn. However, offsets are booked by writing their values near the detail with the accompanying distance written in the space between the two middle lines. Ties, on the other hand, are drawn by a dashed line with their values indicated on these lines.

7) Names of houses, roads, rivers, etc. should be recorded plus any additional information about the surveyed detail that might be helpful when plotting. The name of the surveyed area, the name of the surveyor, the units of measurement, and the date of the survey should also be indicated.

3.4.3 PLOTTING THE DETAILS

After finishing the fieldwork, the plotting of the details proceeds according to the following steps:

1) Choose the appropriate scale, and hence the size of drawing paper to be used.
2) Using a pencil and necessary drawing tools, draw the chain lines on a normal (not tracing) paper. Begin plotting offsets and ties systematically in the same order in which they were measured and booked. Do not forget to indicate the north direction on your initial plan.
3) After all the details have been drawn by pencil, the plan is taken to the site and visually checked (if possible). If any mistakes or missing details are found, they are fixed in the field. Otherwise, the details are then inked on a final tracing paper. When doing so, and in order to obtain a neat looking map, try to do the following:

   a. Whenever possible, it is preferable to make the north arrow direction pointing towards the top of the sheet.
   b. Center the drawing in the middle of the sheet. Leave margins of 2 - 3 cm from the four sides. An extra space is left on the right and bottom sides of the sheet to write the relevant information which

includes the legend, scale, north arrow, office name, surveyor name, site and owner name, etc.

The plotting process has been facilitated lately by the vast developments in software and computer technology. Computer programs, such as AUTOCAD, are now available which make the drawing process much easier and more efficient than manual methods. Drawings can be revised, corrected and updated faster than ever before, and can be easily produced on plotters at any desired scale.

3.5 ACCURACY OF MEASUREMENT

To a certain extent, the accuracy of measurement depends on the plotting scale, and taking into consideration that this scale might be altered (increased or decreased), it is better to be more accurate than may appear to be strictly necessary. It is suggested that measurements be made to the nearest 10 mm (i.e. 1 cm).

A good draughtsman can plot a length to within 0.2 mm. Hence, if the plotting scale is 1/500, then 0.2 mm on the paper represents 10 cm on the ground. If the scale is 1/100, then 0.2 mm represents 2 cm on the ground. As a result, measuring a line to 1 cm accuracy is practically sufficient for most scales, especially when using the computer for drawing where lengths can be drawn with a very high accuracy.

3.6 CHAINING OBSTACLES

In general, chain lines should not be broken or obstructed by obstacles, but sometimes it is not possible to avoid the existence of such obstacles, especially when working in a field with a pond, building, standing crops, a small wood or a hilly area. The types of obstacles that arise may be grouped under the following three types:

(a) Those which obscure vision but do not prevent chaining such as a hill.

(b) Those which prevent chaining but not vision such as a pond or a river.

(c) Those which prevent both chaining and vision such as a building.

a) Vision obscured, chaining possible:

The usual obstacle in this category is a small hill, and is dealt with by the method of repeated alignment (repeated forward ranging) (Figure 3.23).

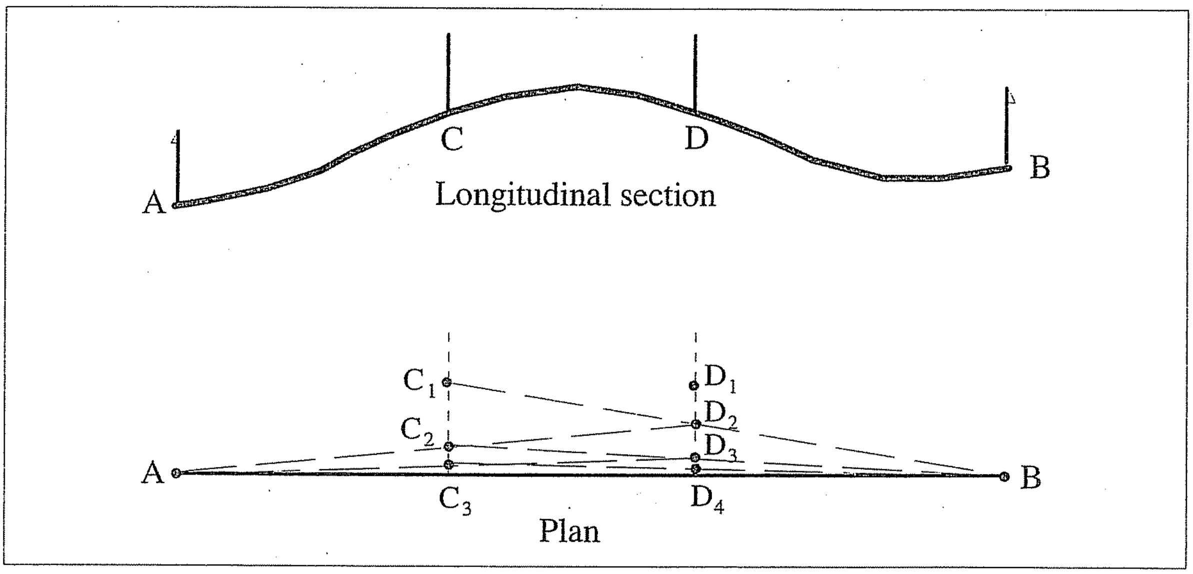


FIGURE 3.23: A hill which obscures vision but does not prevent chaining.

The surveyor and his assistant place themselves on both sides of the hill so that the person standing at C_1 can see the poles at D_1 and B, and the person standing at D_1 can see the poles at C_1 and A. Now assuming the surveyor is standing at C_1 and looking towards B, he directs his assistant to position D_2 on the line C_1B. The assistant who is now at D_2 and looking towards A ranges the surveyor to move to position C_2 on the line D_2A. This procedure is repeated alternatively between the surveyor and his assistant until the two poles C and D lie on the line AB.

b) Vision possible, chaining prevented:

Two types of obstacles can be recognized under this category. These are:

1. Closed obstacles such as a pond or standing crops. Measurements here are made around the obstacle in one of two ways:

a. The parallel line method. A distance that is parallel and equal in length to the missing one is made on the ground as explained in Figure 3.24. Two equal offsets CE and DF are set out perpendicular to AB, and then EF is chained to supply the missing length CD. As a check, GK and HL may be set out on the other side, if possible, and KL is measured. Then,

$$\begin{aligned} AB &= AC + EF + DB \\ &= AG + KL + HB \end{aligned} \qquad \text{(3.4)}$$

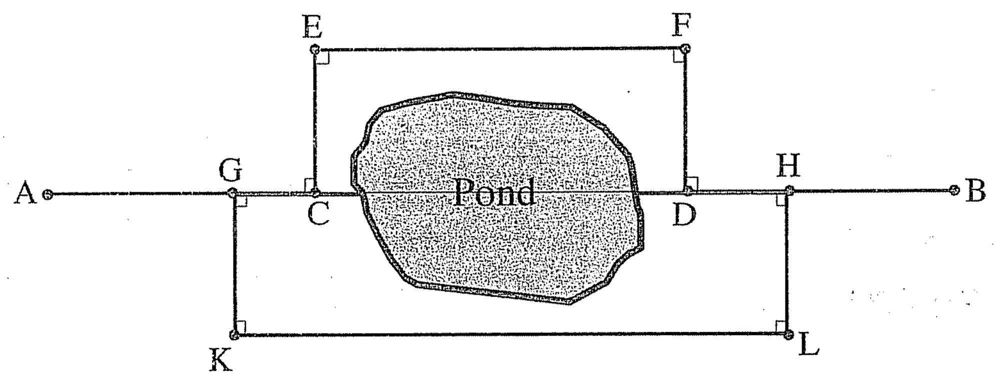


FIGURE 3.24: An obstacle which prevents chaining but not vision, such as a pond.

b. The capital letter A method. For example, assume that the distance AB (Figure 3.25) needs is be known. We choose an arbitrary point K from which points A & B can both be seen. Now, bisect KA at M and KB at N and then measure the distance MN. From the similar triangles KMN & KAB, distance $\overline{AB} = 2 \cdot \overline{MN}$. If M is chosen so that $\overline{KM} = \overline{KA}/3$ and N is chosen so that $\overline{KN} = \overline{KB}/3$, then $\overline{AB} = 3 \cdot \overline{MN}$. Any ratio other than 2 or 3 can also be used.

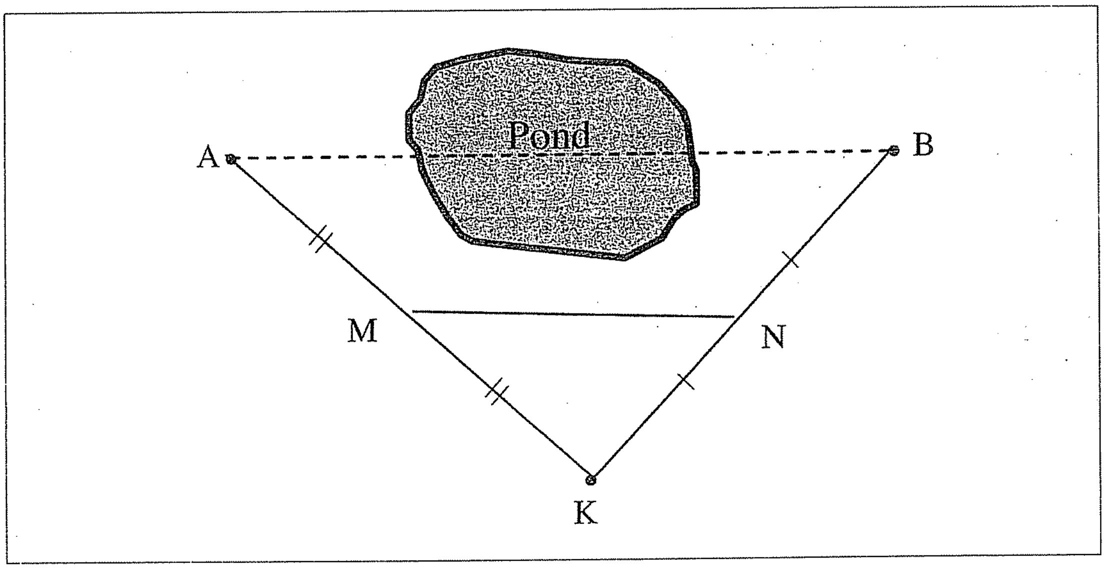


FIGURE 3.25: An obstacle which prevents chaining but not vision, such as a pond.

2. Linear obstacles such as a river. If the obstacle here is a river or a stream of greater width than a tape length, a geometrical construction is necessary. This can be done in one of three ways:

(a) In the first method shown in Figure 3.26a, a ranging rod is placed at H on the far bank of the river. CE is constructed on the near bank perpendicular to AB, and a rod is ranged in to point F between E and H. A perpendicular is then dropped from F on to AB, meeting it at G. CE, FG, AC, CG and HB are then measured. Now, from the similar triangles EDF & FGH:

$$\frac{HG}{FD} = \frac{FG}{ED}$$

$$\Rightarrow \quad HG = FD \cdot \frac{FG}{ED} = \frac{CG \times FG}{EC - FG} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

(b) The second method gives the answer directly, and does not involve setting out any angles (Figure 3.26b). A line DE is set out on the near bank and bisected at C. The line FCG is now constructed such that FC = CG. With a rod at H on the far bank and on the line AB, a rod can be set at J on the intersection of lines EG and HC with a double backwards ranging process. The unknown distance FH is then equal to JG that can be easily measured.

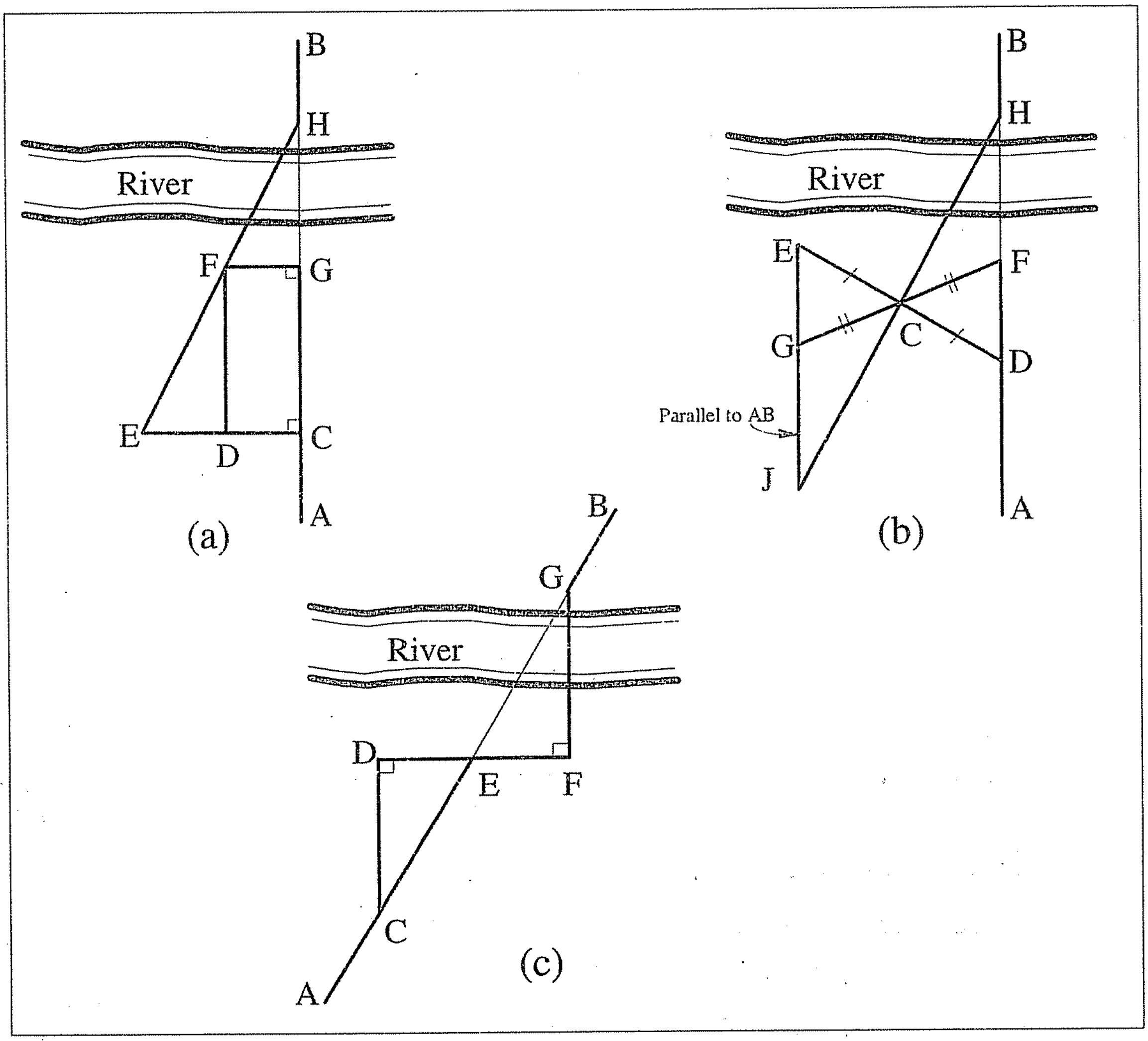


FIGURE 3.26: A river obstacle.

(c) In the third method, where the line crosses the river on the skew (Figure 3.26c), poles are placed on line AB at E and G on the near and far banks respectively. A line DF is set out along the bank, so that GF is perpendicular to DF (with the help of an optical square). A perpendicular from D is constructed to meet AB at C. Lines CE, EF & ED are measured. Then:

$$EG = \frac{CE \times EF}{ED} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

c) Both chaining and vision prevented:

This type of obstacle is encountered in two different situations. These are:

1) When prolonging a chain line (such as AC) past a fixed object (such as a building). It can be solved by the *Random line method* (Figure 3.27). A random line AH is constructed near the building and the lengths of AF, AG and AH are measured in addition to the length of the perpendicular CF. By similar triangles:

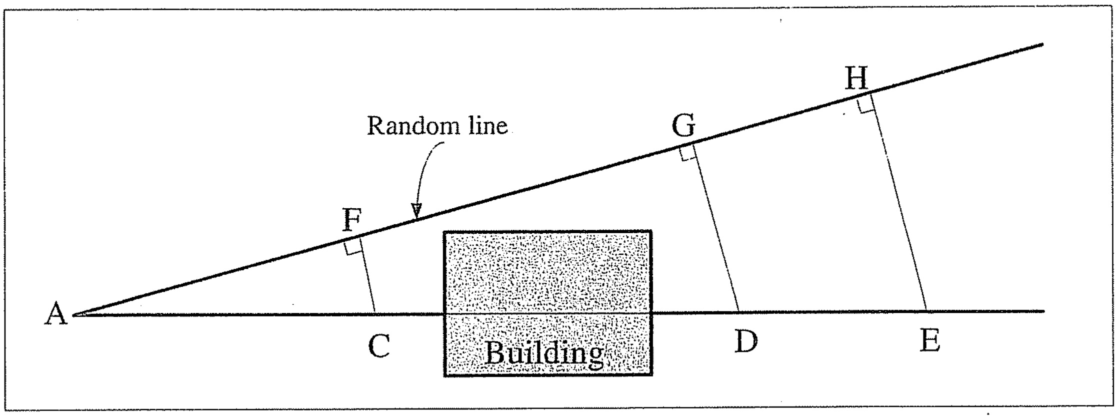


FIGURE 3.27: The random line method for prolonging a chain line past a building.

$$\frac{GD}{FC} = \frac{GA}{FA} \Rightarrow GD = FC \cdot \frac{GA}{FA}$$
$$\frac{HE}{FC} = \frac{HA}{FA} \Rightarrow HE = FC \cdot \frac{HA}{FA} \quad \ldots\ldots (3.7)$$

By erecting perpendiculars on AH from G and H with lengths GD and HE respectively, points D & E are located. They lie on the extension of line AC.

Another method to solve this problem is illustrated in Figure 3.28.

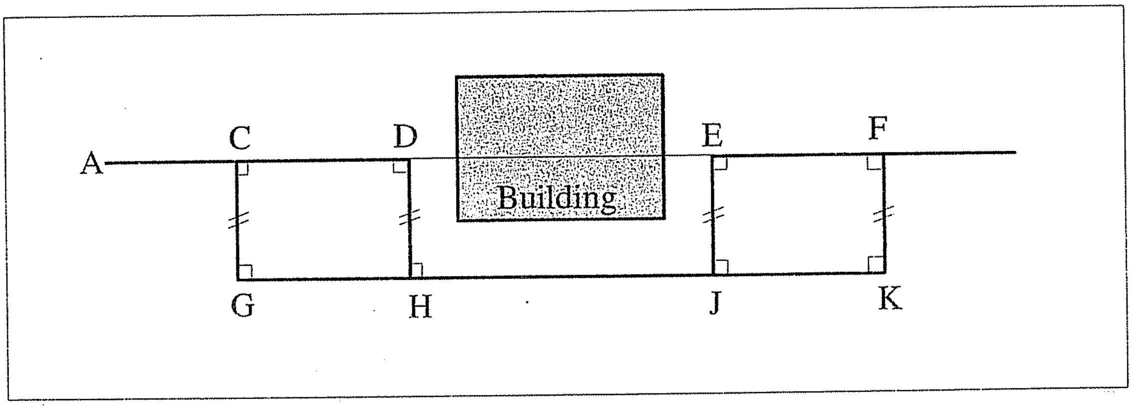


FIGURE 3.28: Prolonging a line past a building using equal offsets.

2) When trying to measure a distance between two points obstructed by a building (Figure 3.29). This problem is solved by the capital letter A method explained earlier.

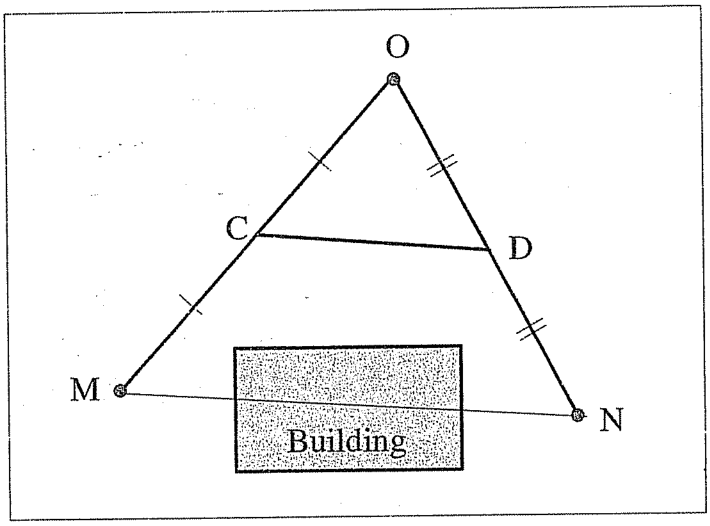


FIGURE 3.29: The letter (A) method.

3.7 ERRORS IN CHAINING AND THEIR CORRECTION

The same three types of errors explained in Chapter 2 occur in chain surveying (as all other types of surveying measurements). These are: blunders, systematic errors, and random errors.

a) Blunders (mistakes):

These are simply mistakes caused by human carelessness, fatigue and haste. They are random in both sign and magnitude. Typical mistakes in chaining a line are:

a - Omitting an entire chain length in booking.
b - Misreading the chainage by confusing the tallies - say the 14 m and 16 m tallies on a 30 m tape.
c - Erroneous booking such as writing 32.14 m instead of 23.14 m.

If allowed to pass unchecked, mistakes could lead to an unpredictable erroneous result. They can be eliminated by careful work and by using field procedures that provide checks for blunders.

b) Systematic errors:

These arise from sources that are known, and their effects, therefore, can be eliminated. Some of the systematic errors in chaining to which corrections can be applied are:

1) Temperature Correction: The correction C_t to be applied to the observed length of a survey line because of the effect of temperature on a *steel tape* can be computed as follows:

$$C_t = 0.0000116(T - T_0)L_m \qquad \ldots\ldots\ldots\ldots\ldots (3.8)$$

Where 0.0000116 is the coefficient of thermal expansion of steel per °C
T is the field temperature
T_0 is the temperature under which the tape is calibrated
L_m is the measured length of the line

2) Sag Correction: A tape supported only at the ends will sag in the center by an amount that is related to its weight and the amount of pull. Sag causes the recorded distance to be greater than the actual length being measured. When the tape is supported at its midpoint, the effect of sag in the two spans is considerably less than when it is supported at the

ends only. The sag correction C_s can be calculated by either one of the following two equations:

$$C_s = -\frac{W^2L}{24P^2} \quad \ldots\ldots (3.9)$$

Or

$$C_s = -\frac{w^2L^3}{24P^2} \quad \ldots\ldots (3.10)$$

Where W = the total weight of the section of tape located between supports
w = unit weight per meter of tape (or ft)
L = the interval (open length) between supports (in m or ft)
P = the tension on the tape (pull)

The units of weight and tension should be compatible (in kg or lb). The total sag correction for a tape resting on multiple supports would be the sum of the sag corrections for the separate intervals. Hence, the total sag effect for a 30 m tape supported at its midpoint and the ends would be twice the calculated sag for a 15 m span.

EXAMPLE 3.1:

Calculate the sag correction for:
(a) A 100 ft steel tape weighing 2 lb and supported at the ends only with a 12 lb pull.
(b) A 30 m steel tape weighing 0.0112 kg/m and supported at 0, 15 and 30 m points under a tension of 5 kg.

SOLUTION:

(a) $$C_s = -\frac{W^2L}{24P^2} = -\frac{2^2 \times 100}{24 \times 12^2} = -0.116 \text{ ft}$$

(b) $$C_s = -2\left(\frac{w^2L^3}{24P^2}\right) = -2\left(\frac{0.0112^2 \times 15^3}{24 \times 5^2}\right) = -0.001 \text{ m}$$

3) Tension Correction: Since the tape material is elastic to a small extent, its length is changed by variations in the tension applied. It is not related to the sag, but to the elastic deformation of the tape. It can be calculated from the following expression:

$$C_p = \frac{(P - P_0)L_m}{AE} \qquad \text{(3.11)}$$

Where C_p = the tension correction in ft or m

P = the applied tension

P_0 = the calibration tension

A = the cross-sectional area of the tape in in^2 or cm^2

E = the modulus of elasticity of the tape material (for steel: E = 29,000,000 lb/in^2)

L_m = the measured length of the line.

4) Length Correction: Checking the tape frequently is necessary for this correction to be effective, because the tape length changes due to wear and tear. The difference between the actual length used in measurement and the nominal length of a tape is known as the length correction. It can be calculated from the following equation:

$$C_\ell = (\ell_a - \ell_o)\frac{L_m}{\ell_o} \qquad \text{(3.12)}$$

Where C_ℓ = length correction in the line of length L

ℓ_a = actual length of the tape

ℓ_o = nominal length of the tape

L_m = the measured length of the line.

EXAMPLE 3.2:

A line is measured with a tape believed to be 50 m long that gives a length of 205.76 m. On checking, the tape is found to measure 50.03 m. What is the correct length of the line?

SOLUTION:

$\ell_a = 50.03$ m, $\ell_o = 50$ m, $L = 205.76$ m

$C_\ell = (50.03 - 50) \times \dfrac{205.76}{50} = +0.12$ m

Corrected length = 205.76 + 0.12 = 205.88 m

Now that the different types of systematic corrections are calculated, the correct line length (L_c) will be:

$$L_c = L_m + C_t + C_s + C_p + C_\ell \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.13)$$

Note: When the length correction is the only correction to be considered, an alternative direct way to correct for lines and areas is as follows:

$$L_c = L_m + C_\ell = L_m + (\ell_a - \ell_0)\frac{L_m}{\ell_0} = L_m \cdot \frac{\ell_a}{\ell_0} \qquad \ldots\ldots\ldots (3.14)$$

That is,

$$\text{Correct line length} = \text{measured length} \cdot \frac{\text{actual length of the tape}}{\text{nominal length of the tape}}$$

Similarly,

$$\text{Correct area} = \text{measured area} \cdot \left(\frac{\text{actual length of the tape}}{\text{nominal length of the tape}}\right)^2 \qquad \ldots (3.15)$$

c) Random errors (compensating or accidental errors):

As explained in Chapter 2, these errors arise from lack of perfection in the human eye and in the method of using the equipment. As there is as much chance of these errors being negative as being positive, they tend to cancel out (i.e. tend to compensate for each other). Usually, no attempt is made to correct for them in chaining. Repeating the measurement several times and taking the average value will reduce these errors to a minimum.

PROBLEMS

3.1 The slope distance between two points was measured to be 463.21 ft with an estimated standard error of ± 0.05 ft. The difference in elevation between the two ends of the line was found to be 35.6 ± 0.5 ft. Compute:

a. The horizontal distance.

b. The estimated standard error of the computed horizontal distance.

3.2 Two edges of an axis of a road deflect by an angle = 90° (Figure 3.30). Using a tape of length = 30 m, ranging rods, pegs, arrows and an optical square, show how to connect those lines by a circular curve of radius = 26 m. Give the solution for the two cases when the center of the curve can be reached, and when the center can not be reached due to the existence of an obstacle such as a building.

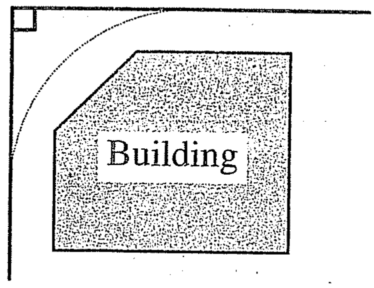


FIGURE 3.30

3.3 A survey line was measured by Mr. X and found to be 150.00 m. The same distance was measured by Mr. Y and found to be 150.08 m. The plotting scale was decided to be 1:1000 with plotting accuracy = 0.5 mm.

a. Do you think it is required to repeat the measurements?

b. If it doesn't need to be re-measured for such a scale, in what cases do you think it does need?

3.4 Using only tapes and arrows, describe how to measure the horizontal angle (α) between the two chain lines AB & AC (Figure 3.31).

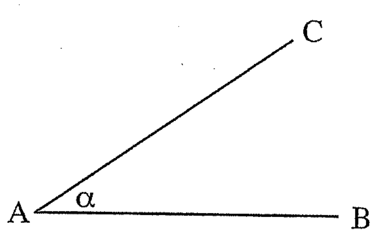


FIGURE 3.31

3.5 Using only an optical square and ranging rods, describe how to construct a circle on the ground around a line such as AB so that AB will be the diameter.

3.6 The piece of land of irregular boundaries (Figure 3.32) is to be mapped on a plan in order to calculate its area by the use of a planimeter (Chapter 8). Choose the number and location of the chain lines needed to chain this land. If this figure represented a lake instead, would the choice of chain lines be different?

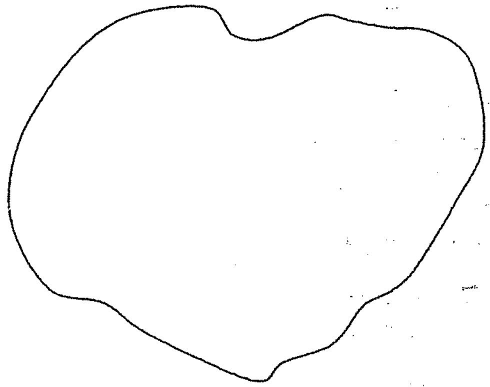


FIGURE 3.32

3.7 A four-sided land parcel ABCDA has a building inside it, which obscures vision along the diameters AC and BD. Describe a simple way to indirectly measure these diameters. Assume that the internal angles of the land parcel are different from 90°.

3.8 A line is measured along a long gentle slope using a 20 m tape. The gradient of the line is measured with an Abney level and found to be 5° 30'. The slope distance is recorded as 150.25 m. The length of the tape was found subsequently to be only 19.96 m. Calculate the correct horizontal length of the line.

3.9 A 50-m tape was calibrated under a tension of 7 kg and a temperature of 20° C while fully supported. When carefully checked, the tape was found to be 50.005 m long. In the field it was used under a tension of 7 kg, a temperature of 35° C, and supported at the two ends only. A line was measured in 4 sections with the following results: 50 m, 50 m, 50 m and 48.631 m. Determine the correct length of the line. The tape weighted 0.50 kg.

4 LEVELING

4.1 INTRODUCTION

The elevation of a point is defined as the vertical distance between the point and a reference level surface called *datum*. The most commonly used datum is the mean sea level (MSL). The elevation of a point can be considered as its vertical coordinate, and is considered to be positive if the point is above the datum (e.g. Jerusalem), and negative if the point is below the datum (e.g. Jerico).

Leveling may be simply defined as the process by which the elevation of a point above a reference elevation datum, or the elevation difference between two or more points on the earth's surface is determined. Its purpose may be to provide spot heights or contour lines on a plan, to provide data for making longitudinal and cross-sections, or to provide a level or inclined surface in the setting out of construction works. Leveling can be done in several ways, which include:

1) *Chain Surveying*. By measuring the slope distance and angle of inclination for a uniformly sloping ground, the elevation difference between two points can be calculated (Section 3.3.1).

2) *Barometric Leveling*. This is the process of determining elevation by measurement of atmospheric pressure, and is based on the principle that atmospheric pressure decreases with increase in elevation. An altimeter or barometer is used for this purpose. This method is not highly accurate and is therefore restricted to situations where high accuracy is not required.

3) *Trigonometric Leveling*. The elevation difference between two points may be calculated by measuring the horizontal or slope distance between the two points, in addition to the vertical or zenith angle to the line of sight. This method is explained in chapters 5 and 6.

4) *Photogrammetric Leveling*. The elevation difference is measured from photographs taken for the area under consideration by a camera mounted in an airplane. This technique is discussed in Chapter 11.

5) *GPS Leveling*. The elevation of a point, or the elevation difference between two points is measured from special signals received from orbiting satellites using equipment known as GPS receivers. This subject is discussed in Chapter 12.

6) *Differential Leveling*. This is the most commonly used method in leveling for its high degree of accuracy. It is performed using an instrument called a level and a leveling staff, and will be the subject of this chapter.

4.2 BASIC DEFINITIONS

Various terms are used in the process of leveling, and it is useful at this stage to define them. These include:

1 - *Vertical line*. The vertical line at a point is the line formed by a freely falling body or by the string of a plumb bob when the tip is located directly over the point.

2 - *Horizontal line*. The horizontal line at a point is the straight line perpendicular to the vertical line at that point. There is an infinite number of horizontal lines at each point.

3 - *Horizontal plane*. This is the plane that passes through all the horizontal lines at a particular point. It is perpendicular to the vertical line at this point.

4 - *Level surface*. A level surface is a continuously curved surface that is perpendicular to the direction of gravity at all its points, such as the Geoid or any surface parallel to it (see Figure 4.1).

5 - *Level line*. This is a line that lies on the level surface, and is, therefore, perpendicular to the direction of gravity at all its points. The level line that passes through a particular point is tangent to the horizontal line at this point (Figure 4.1).

6 - *Difference in elevation between two points*. This is the vertical distance between the two level surfaces passing through these two points.

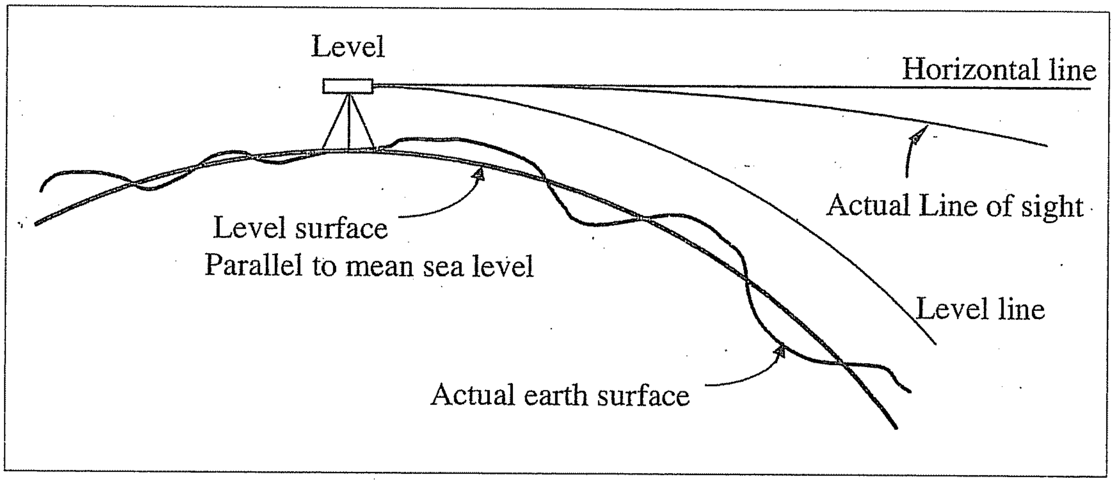


FIGURE 4.1: Relationship between the horizontal line, level line and the line of sight

7 - *Actual line of sight or collimation.* This is neither a horizontal nor level since it is affected by atmospheric refraction, which bends the line of sight downwards from the horizontal line (Figure 4.1).

8 - *Bench mark (BM).* This is a marked point whose elevation has been accurately measured. Benchmarks are established by the survey departments and made evenly distributed to enable surveyors to use them for measuring the elevations of nearby points without the need to start over from the elevation datum. They are well marked on the ground and have precise description to locate them easily.

9 - *Height of instrument.* This is the elevation of the line of collimation above the datum after setting up the level above a certain point.

4.3 BASIC PRINCIPLE OF A LEVEL

The basic instrument used in differential leveling to measure elevation or height differences is called a level. Although of many types and designs, a level consists essentially of:

a) A telescope for sighting, and
b) A leveling device for maintaining the line of sight in a horizontal position.

A level is set up so that the line of sight of the telescope is perpendicular to its vertical axis. If the vertical axis of the level is made to coincide with the direction of gravity, then the line of sight will be in a horizontal direction. Now, as the telescope is rotated around the vertical axis, the line of sight will move in a horizontal plane (Figure 4.2).

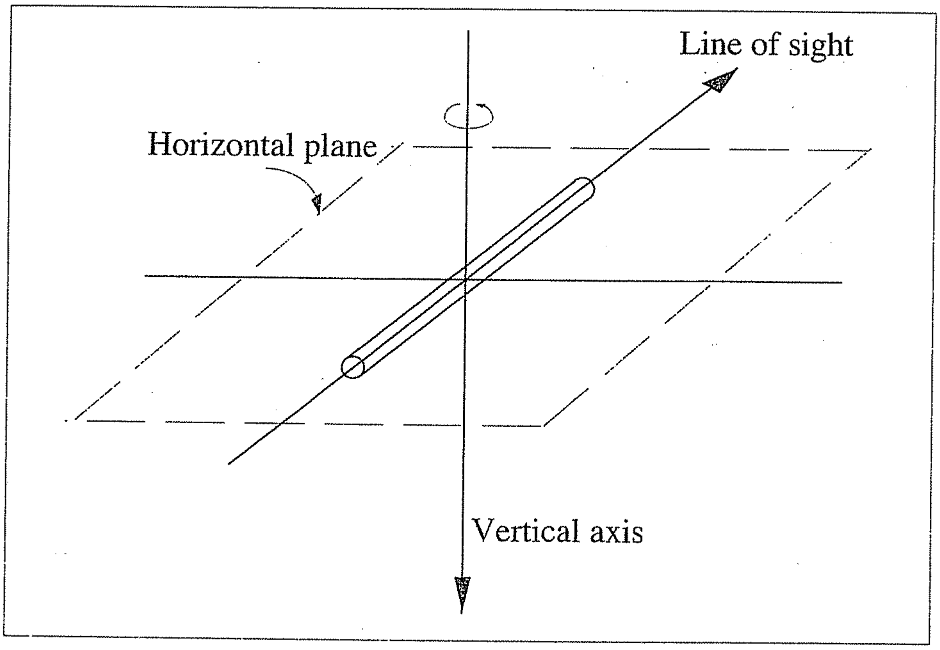


FIGURE 4.2: Basic principle of a level.

4.4 BUBBLE TUBE

A bubble tube, as shown in Figure 4.3 is used in most levels to establish a horizontal line. The bubble tube is a glass vial of uniform cross section. Its inside, upper surface is accurately ground in longitudinal section to the arc of a circle of specific radius. The tube is nearly filled with ether or some nonfreezing liquid, the remaining volume being a vapor space called the *bubble*. The buoyancy of the liquid lifts the bubble to a position symmetrical with the highest point in the tube. Since this highest point is on the arc of a circle that lies in a vertical plane, the tangent at that point will be truly horizontal.

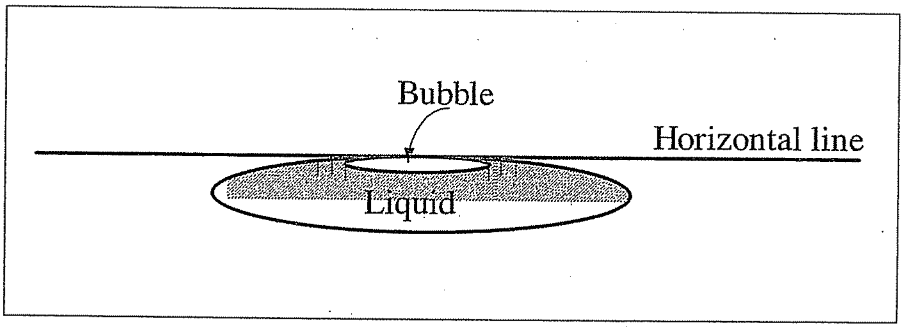


FIGURE 4.3: Cross section in a bubble tube.

A circular bubble vial, as shown in Figure 4.4, is used in many modern levels to approximately establish a horizontal plane.

(a) Bubble not centered (b) Bubble centered

FIGURE 4.4: Centering the bubble.

By centering the bubble within the circular mark, the surface is approximately leveled. In some levels like the Tilting Dumpy Level, the horizontality of the plane is accurately controlled by another bubble that can be seen through an eyepiece near the telescope. This bubble consists of two separate halves. When the bubble is off-centered, a split image of the two ends of the bubble is seen through the viewing microscope, as shown in Figure 4.5a. When the bubble is correctly centered, the two images coincide to form a continuous U-shaped curve, as shown in Figure 4.5b.

(a) Bubble off-centered (b) Bubble centered

FIGURE 4.5: Auxiliary bubble tube.

4.5 EQUIPMENT USED IN DIFFERENTIAL LEVELING

The following types of equipment are used for differential leveling:

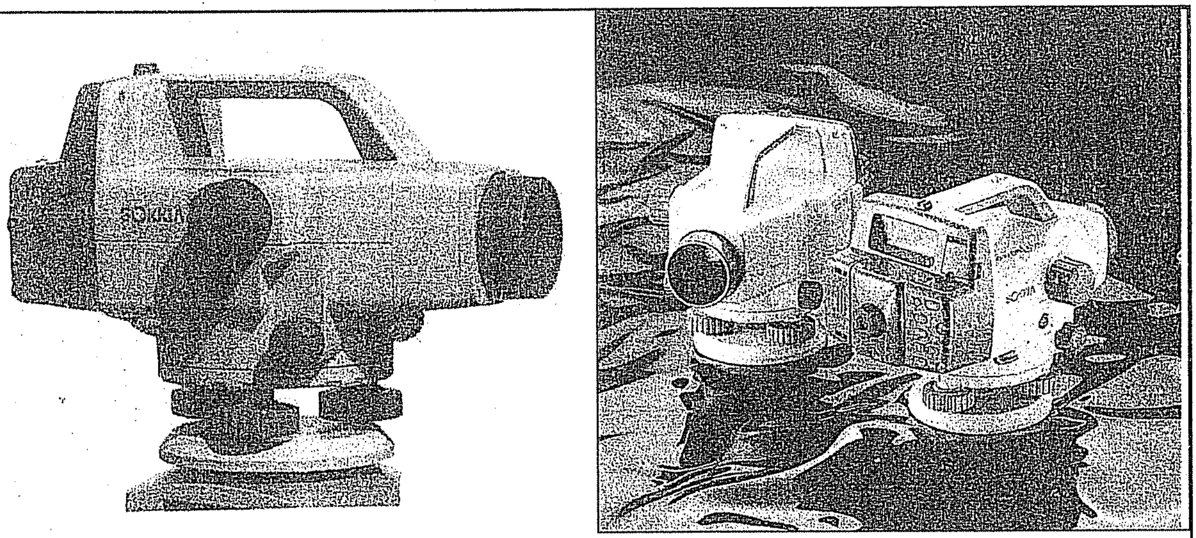
A) **The Level.** There are three basic types of level:

1) *The Dumpy Level.* In this type of instrument the line of sight defined by the center of the cross hairs and the optical center of the objective lens - the line of collimation - is fixed at right angles to the vertical axis of rotation of the instrument. The spirit level attached to the telescope should also be perpendicular to the vertical axis of the instrument and parallel to the line of collimation. The spirit level is brought to the horizontal plane using the three leveling screws on the tribrach. When level, the line of collimation should describe a true horizontal plane around the instrument. This type of level is mainly confined to construction sites or other cases where large numbers of level sights are required from a single instrument position.

2) *The Tilting Level.* In these instruments the telescope is hinged near the top of the vertical axis to allow a limited degree of movement with respect to the vertical axis. Like the dumpy level the spirit level is attached to the telescope. The vertical axis is made approximately vertical either by a ball and socket mounting or a three-screw tribrach. In both cases a small circular spirit level is used. The telescope is then leveled using the telescope spirit level by means of a tilting screw that tilts the telescope with respect to the vertical axis. It is important that this is done for each observation. The telescope bubble may be observed either by a mirror reflecting the bubble of the spirit level to the observer or by a split bubble prism. This allows the observer to see both ends of the bubble either in the telescope eyepiece or through a separate eyepiece. Leveling is achieved by making the two halves of the bubble coincide (Figure 4.5).

3) *The Automatic Level.* Many modern levels use a system of self-leveling compensators within the optical system of the telescope. This requires the observer to level the instrument within the working range of the compensator, which is usually about ±20' of the horizontal. The vertical axis may be supported by a three-screw or ball and socket mounting, the rough leveling being carried out by reference to a circular spirit level. The compensator, which consists of a system of fixed and suspended prisms, brings the line of sight to the horizontal plane even though the axis through the telescope is not truly horizontal. This type of level more normally gives an erect image as opposed to the inverted image of most other surveying telescopes. Figure 4.6a shows an example of an automatic level.

4) *The Electronic Digital Level.* This kind of level combines the merits of the automatic level with the fact that it is user friendly and easy to use (Figure 4.6b). All the user has to do is aim the staff, adjust the focus and then – with a single touch of a key – the level will accurately measure and record the staff reading on a display.



(a) Automatic level.

(b) Electronic digital level.

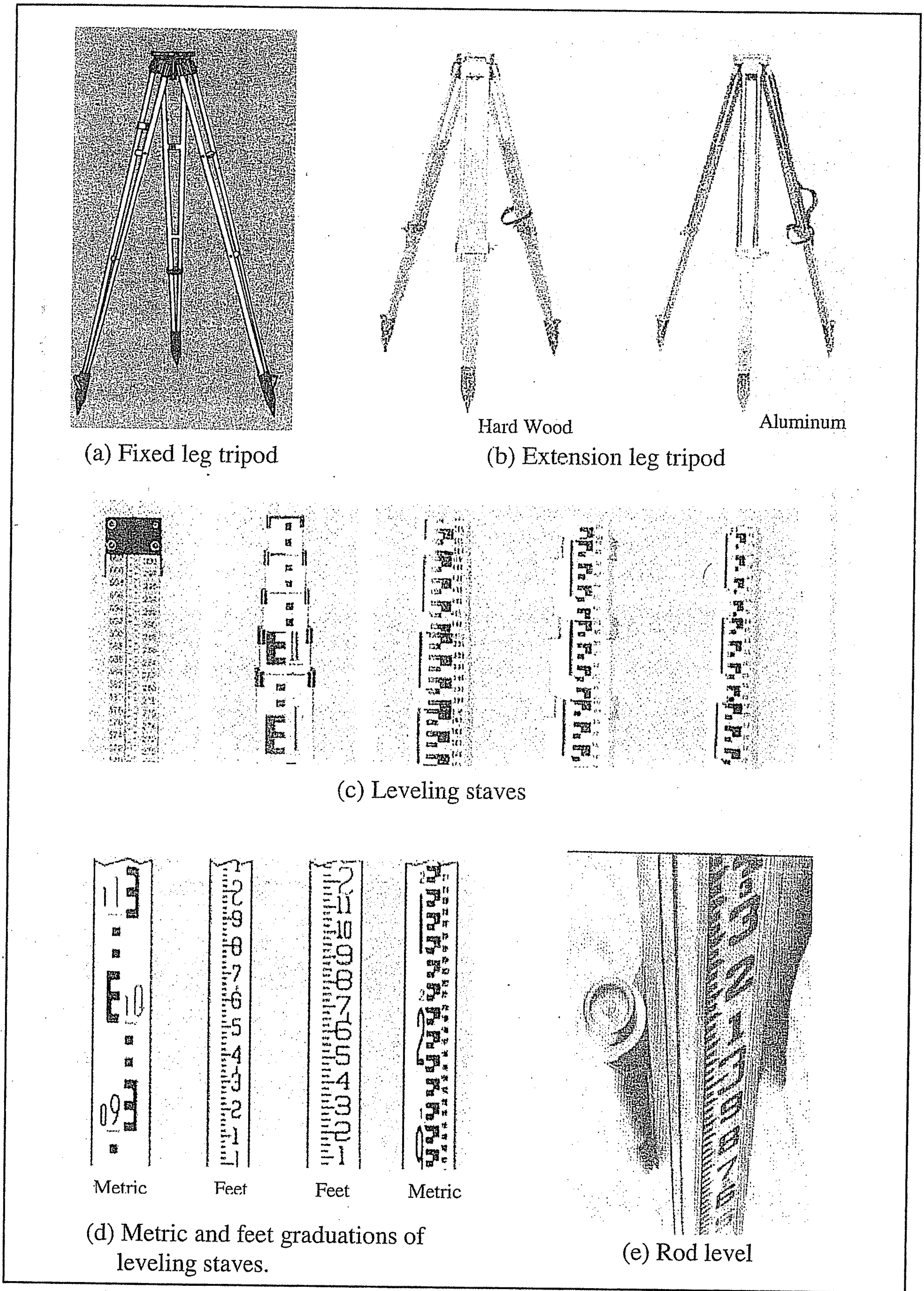
FIGURE 4.6: Automatic and electronic digital Levels.

B) **Tripod**. This is a three-legged stand used to support a level or other surveying instrument during field measurements. It consists of a head and three legs that are fitted with pointed metal shoes. The legs could be made of aluminum or hard wood, and are either with fixed length or adjustable length depending on the height of the user (Figure 4.7a & b). A survey instrument is usually secured to the tripod head by a threaded bolt.

C) **Level Rods (Staves)**. The level rod (staff) is used to measure the vertical distance between the point on which it is held and the line of collimation of the instrument, and is usually 3 - 5 m long. There are many different types of level staves (Figure 4.7c & d). These include:

1) The telescopic staff. This is composed of several sections that slide inside each other in a way similar to the radio antennae. It is made of either wood or metal. Currently, this is the most popular and widely used type of staves.
2) The folding staff. The most widely used type of this kind is the one composed of four folding sections into a 1-m long piece designed to fit easily into a car boot.
3) The one-piece staff. This is usually 1 - 3 m long and is difficult to transport in a car, and therefore it is suitable to be used in open rural areas.

D) **Rod Level**. This is a small device with a circular bubble tube mounted on a metal angle and attached to the edge of the level staff to ensure vertical standing over a point (Figure 4.7e).



(a) Fixed leg tripod

(b) Extension leg tripod

(c) Leveling staves

(d) Metric and feet graduations of leveling staves.

(e) Rod level

FIGURE 4.7: Tripods, leveling staves and rod levels.

4.6 SETTING UP THE LEVEL

The level should be mounted at a comfortable height on a rigid legged tripod making sure that it is firmly placed and the screws at the tripod head are finger tight. With practice, it should be possible to have the tripod head almost level before attaching the level. For a three-foot screws level, as in the case with most modern survey instruments, make sure that the three screws are in the middle of their paths. To level the instrument, bring the telescope parallel to one pair of foot screws (A and B in Figure 4.8), and by turning each screw either inwards or outwards, bring the small circular bubble to a position in line with the third foot screw C (Figure 4.8a). Now centralize the bubble with this third screw (Figure 4.8b).

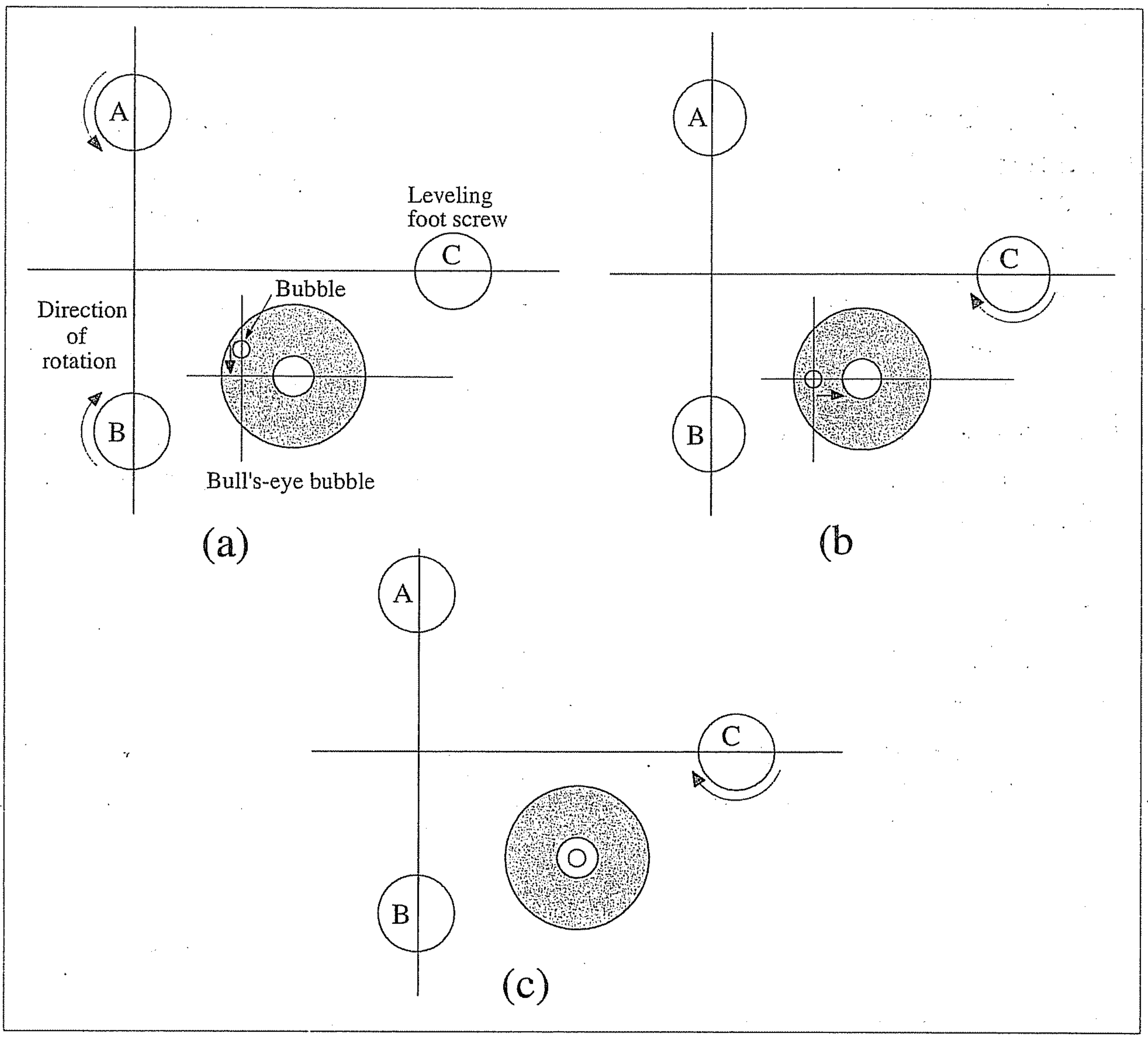


FIGURE 4.8: Leveling an instrument with three foot screws and a circular bubble tube.

Once the level has been roughly leveled using the circular bubble (Figure 4.8c), the cross-wires in the eyepiece should be focused preferably against a light background, until the lines are sharp and clear. The level should now be pointed at the staff and focused to make the staff graduations sharp. When using a dumpy or automatic level, the staff can now be read. If a tilting level is being used, the split level bubble must be brought into coincidence by turning the gradienter screw before the staff is read and this must be done before each reading. It is important to check the level bubble before every reading is taken, or in the case of an automatic level, that the compensator is operating correctly. This can be checked by tapping the instrument gently.

4.7 MEASURING ELEVATION DIFFERENCE USING A LEVEL

The basic operation in differential leveling is the determination of elevation difference between two points. Consider two points A & B as shown in Figure 4.9. Set up the level so that readings may be made on a staff held vertically at A, and then at B. Let point A be a benchmark whose reduced level (RL) or elevation = 520.43 m AMSL.

If the readings at A and B are 2.56 m and 0.93 m respectively (Figure 4.9a), then the difference in elevation between points A and B is equal to the vertical distance AC = 2.56 - 0.93 = 1.63 m. This positive value represents a rise of point B relative to A.

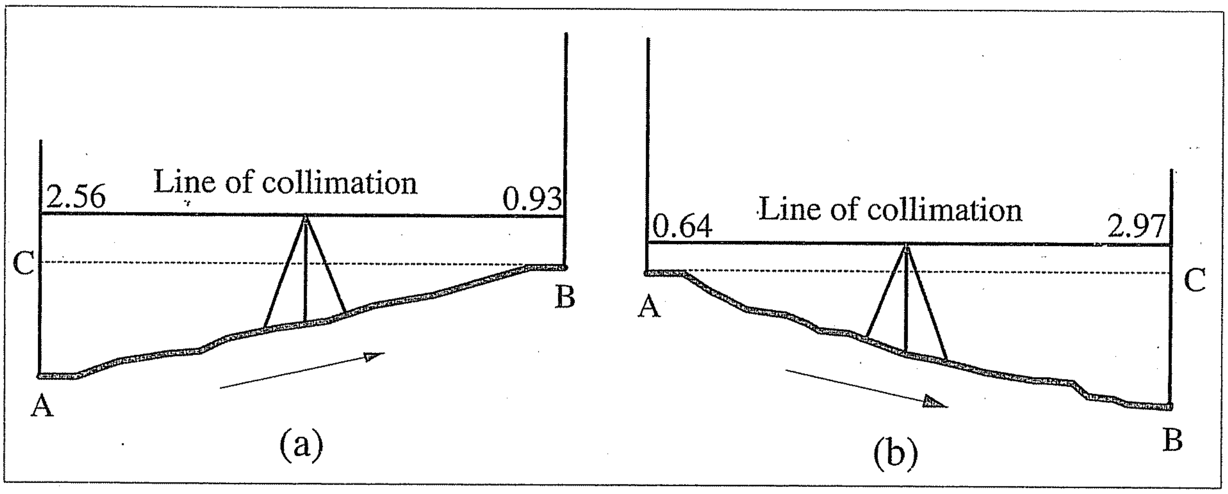


FIGURE 4.9: Principle of differential leveling.

If, on the other hand, the readings at A and B were 0.64 m and 2.97 m respectively (Figure 4.9b), then the difference in elevation between points A and B is equal to the vertical distance BC = 0.64 - 2.97 = -2.33 m. The negative sign indicates a fall of point B relative to A. Thus, we have for any two successive staff readings:

Second reading smaller than first reading represents a ***rise***.
Second reading greater than first reading represents a ***fall***.

If the actual elevation of one of the two points above an elevation datum is known, then the elevation of the second point can be calculated. For the cases of Figure 4.9, and given that the elevation of point A is 520.43 m AMSL, the elevation of point B can be calculated by one of the following two methods:

A) The Height of Instrument method:

1 - For Figure 4.9a:

Height of Instrument (HI)	= elevation of A + staff reading at A
	= 520.43 + 2.56 = 522.99 m AMSL
Elevation of B (or RL)	= HI - staff reading at B
	= 522.99 - 0.93 = 522.06 m AMSL

2 - For Figure 4.9b:

Height of Instrument (HI)	= elevation of A + staff reading at A
	= 520.43 + 0.64 = 521.07 m AMSL
Elevation of B (or RL)	= HI - staff reading at B
	= 521.07 - 2.97 = 518.10 m AMSL

B) The Rise and Fall method:

1 - For Figure 4.9a:

Elevation difference	= first reading at A - second reading at B
	= 2.56 - 0.93 = 1.63 m (positive ⇒ rise)
Elevation of B (or RL)	= elevation of A + rise
	= 520.43 + 1.63 = 522.06 m AMSL

2 - For Figure 4.9b:

Elevation difference = first reading at A - second reading at B
= 0.64 - 2.97 =-2.33 m (negative ⇒ fall)

Elevation of B (or RL) = elevation of A + fall
= 520.43 + (-2.33)
= 520.43 - 2.33 = 518.10 m AMSL

It is obvious that the results agree in both methods. Now, let ΔH_{AB} represent the elevation difference between points A and B, H_A and H_B represent the elevations or reduced levels of points A and B respectively, and r_A and r_B represent the staff readings at A and B respectively, then the following equation always holds:

$$\Delta H_{AB} = H_B - H_A = r_A - r_B \quad \text{.....................} (4.1)$$

4.8 PROCEDURE IN DIFFERENTIAL LEVELING

The main purpose here is to provide reduced levels for a large number of points. These points could be located along the centerline of a construction project such as a highway, or in an area to produce a contour plan. Before explaining the general procedure, the following definitions need to be known:

1 - *Backsight (BS)*. This is the first reading taken by the observer at every instrument station after setting up the level.

2- *Foresight (FS)*. This is the last reading taken at every instrument station before moving the level.

3- *Intermediate Sight (IS)*. This is any reading taken at an instrument station between the backsight and foresight readings.

4- *Turning Point (TP)*. This a point at which both a foresight and a backsight are taken before moving the staff.

4.8.1 GENERAL PROCEDURE

This will be illustrated through an example, which is going to be about the production of a longitudinal section (profile) of a road (Figure 4.10).

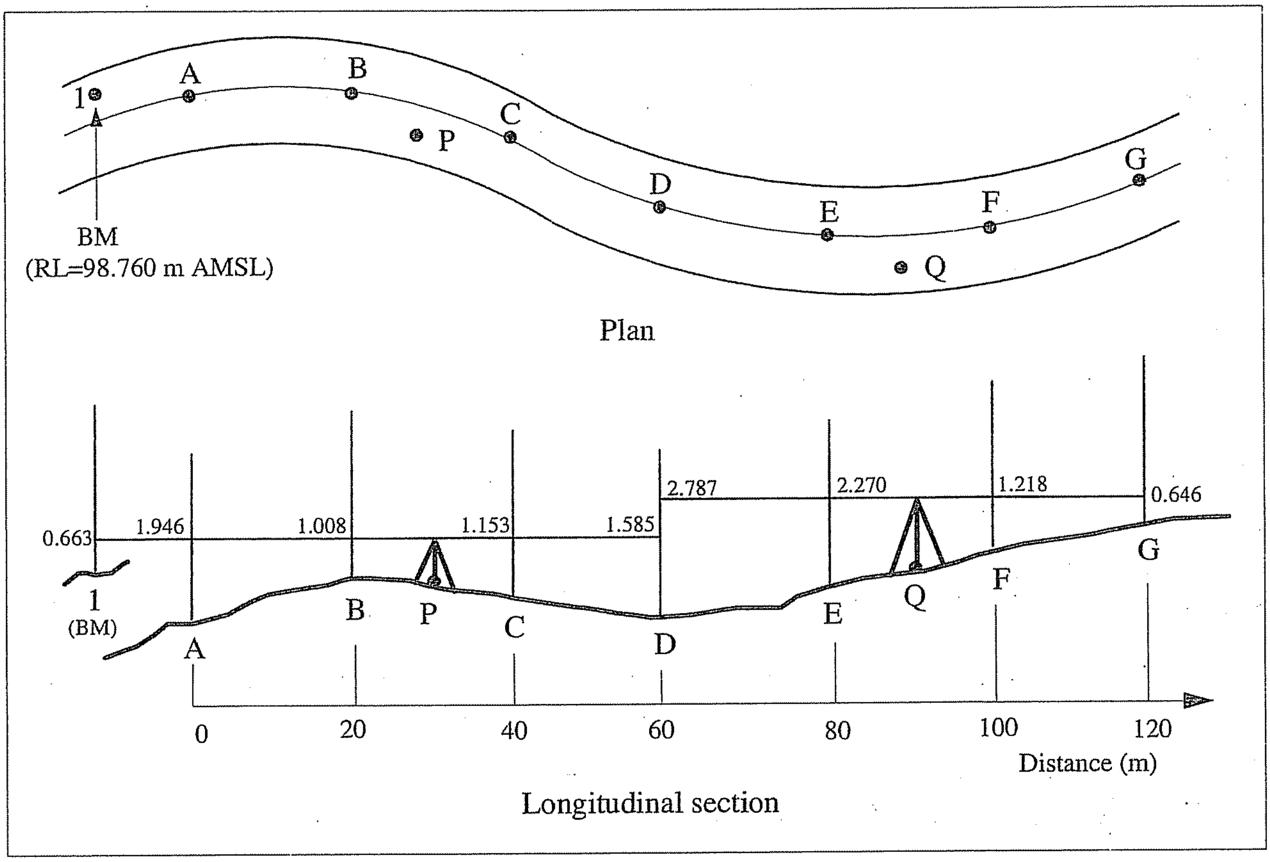


FIGURE 4.10: General procedure in differential leveling.

(1) After setting up the level at P, take the first (BS) reading on a nearby BM (point 1 in Figure 4.10).

(2) Take readings on the staff held vertically at points A, B and C respectively. These readings are called intermediate sights (IS).

(3) If the distance after point D, for example, becomes more than 100 m, or if the topography makes it difficult to take readings after this point from the same setup of the instrument, take the last reading at D. This reading is called foresight (FS). Point D is considered to be a turning or change point because the level position is changed at this stage.

(4) Move the level to point Q, and with the staff remaining at D, take a BS reading. Proceed taking intermediate sights at E & F, and a FS at G. If any further work is required, continue in accordance with the above procedure.

4.8.2 BOOKING AND CALCULATIONS

Staff readings are booked in a level field book that is mainly designed for this purpose. Table 4.1 shows a typical page from this book. The reduction of the readings is carried out in the same book. It can be done either by the height of instrument method or by the rise and fall method, whichever is more suitable. The following will explain the reduction of the readings using both of these methods on the data shown in Figure 4.10.

A) The Height of Instrument Method:

Table 4.2 shows the data of Figure 4.10. First of all, point numbers or names are booked in the left column (i.e. points 1, A, B through G). Each point is booked on a separate line. At point 1 (the bench mark), we took a BS of 0.663 m and therefore this value is written on the same line near point 1 but in the BS column. The elevation or reduced level of this point is known to be 98.760 m and is booked in the RL column. At points A, B & C we read IS of 1.946 m, 1.008 m & 1.153 m respectively. These values are booked in the IS column. The final reading taken from the first instrument setup is a FS of 1.585 m at point D and is therefore booked at the same line as point D in the FS column. Now, from the second setup of the instrument, a BS of 2.787 m was read at point D, and therefore is booked at the same line but in the BS column. The two IS readings of 2.270 m and 1.218 m taken at points E and F are then booked in the IS column. Finally, we book the last FS reading of 0.646 m taken at point G at the same line as point G, but in the FS column.

TABLE 4.2: Calculations using the height of instrument (HI) method.

Point	BS	IS	FS	HI	RL	Distance (m)	Notes
1	0.663			99.423	98.760	-	BM
A		1.946			97.477	0	Beginning of project
B		1.008			98.415	20	
C		1.153			98.270	40	
D	2.787		1.585	100.625	97.838	60	TP
E		2.270			98.355	80	
F		1.218			99.407	100	
G			0.646		99.979	120	
SUM	3.450	7.595	2.231				

TABLE 4.1: A typical page from a level field book

Project and Location: -

Surveyor: - Date: / /

Point #	BS	IS	FS	Rise	Fall	HI	RL (m)	Distance (m)	Remarks

The reduction of data is done in the following sequence:

1) Calculate the first height of instrument (HI). This is equal to the sum of the elevation of the BS point (point 1 or BM here) and the BS reading. In this case:
HI = 98.760 + 0.663 = 99.423 m

This value is entered in the same line as the BS reading, but in the HI column.

2) Calculate the RL of points A, B, C and D as follows:
RL of A = HI - IS at A = 99.423 - 1.946 = 97.477 m
RL of B = HI - IS at B = 99.423 - 1.008 = 98.415 m
RL of C = HI - IS at C = 99.423 - 1.153 = 98.270 m
RL of D = HI - FS at D = 99.423 - 1.585 = 97.838 m

These reduced levels are entered in the RL column in the corresponding lines.

3) Calculate the next height of instrument. This is equal to:
HI = RL of point D + BS reading at point D
= 97.838 + 2.787 = 100.625 m
This value is entered in the same line as point D, but in the HI column.

4) Calculate the RL of points E, F and G as follows:
RL of E = HI - IS at E = 100.625 - 2.270 = 98.355 m
RL of F = HI - IS at F = 100.625 - 1.218 = 99.407 m
RL of G = HI - FS at G = 100.625 - 0.646 = 99.979 m

These reduced levels are again entered in the RL column in the corresponding lines.

Note: The distance column in Table 4.2 indicates the cumulative distance between each point and the beginning point of the project.

Checks: The following checks on the booking and arithmetic calculations are performed:

1) Number of BS readings = number of FS readings = 2 $\Rightarrow$ OK

2) $\Sigma BS - \Sigma FS$ = RL of last point - RL of first point ...... (4.2)
$\Sigma BS - \Sigma FS$ = 3.450 - 2.231 = 1.219
RL of last point - RL of first point = 99.979 - 98.760 = 1.219 $\Rightarrow$ OK

3) Sum of all reduced levels excluding the first reduced level = sum of the HI for each setup multiplied by the number of IS and FS readings taken from that setup - sum of IS - sum of FS .... (4.3)

For the data of Table 4.2
- Sum of all reduced levels excluding the reduced level of point 1 = 689.741
- Sum of HI multiplied by the number of IS and FS readings = 99.423 x 4 + 100.625 x 3 = 699.567
- ΣIS = 7.595
- ΣFS = 2.231

The right part of Equation (4.3) = 699.567 - 7.595 - 2.231 = 689.741
Left side = right side $\Rightarrow$ OK

The arithmetic check is now complete. This indicates that there are no mistakes in the calculations.

B) The Rise and Fall Method:

The booking of the staff readings is done in the same way as explained earlier. However, the reduction of the data is done as follows:

1) Calculate the elevation difference (ΔH) between points 1 and A. This is calculated as follows.

ΔH_{1A} = staff reading at 1 - staff reading at A
= 0.663 - 1.946 = -1.283 m

This value is negative which indicates a fall from 1 to A, and is entered in the fall column without a sign in the line of point A.

The reduced level of A = RL of 1 - fall

= 98.760 - 1.283 = 97.477

This RL of A is entered in the table in the RL column.

2) Calculate the elevation difference (ΔH) between points A and B. This is calculated as follows:

ΔH_{AB} = staff reading at A - staff reading at B

= 1.946 - 1.008 = +0.938

This value is positive which indicates a rise from A to B, and is entered in the rise column in the line of point B.

The reduced level of B = RL of A + Rise

= 97.477 + 0.938 = 98.415 m

This RL of B is entered in the table in the RL column.

3) The computations proceed in a similar manner until calculating the RL of point D. Next, we proceed with the calculations of point E using the BS at D (and not the FS), and so on until all the reduced levels are calculated. Table 4.3 shows the results of these computations.

TABLE 4.3: Calculations using the rise and fall method.

Point	BS	IS	FS	Rise	Fall	RL	Distance (m)	Notes
1	0.663					98.760	-	BM
A		1.946			1.283	97.477	0	Beginning of project
B		1.008		0.938		98.415	20	
C		1.153			0.145	98.270	40	
D	2.787		1.585		0.432	97.838	60	TP
E		2.270		0.517		98.355	80	
F		1.218		1.052		99.407	100	
G			0.646	0.572		99.979	120	
SUM	3.450	7.595	2.231	3.079	1.860			

Checks: The following checks on the booking and arithmetic calculations are performed:

1) Number of BS readings = number of FS readings = 2 $\Rightarrow$ OK

2) $\Sigma BS - \Sigma FS = \Sigma Rise - \Sigma Fall$

$$= \text{RL of last point} - \text{RL of first point} \quad \ldots\ldots\ldots\ldots (4.4)$$

$\Sigma BS - \Sigma FS = 3.450 - 2.231 = 1.219$

$\Sigma Rise - \Sigma Fall = 3.079 - 1.860 = 1.219 \quad \Rightarrow$ OK

$RL_{last} - RL_{1st} = 99.979 - 98.760 = 1.219$

The arithmetic check indicates that there are no mistakes in the reduction of the data.

4.9 GENERAL NOTES

1) The accuracy of the reduced levels does not depend only on correct calculations, but also on correct measurements (staff readings) and the correct booking of these measurements. For example, a staff reading of 2.38 m can be booked as 2.83 m and still all the arithmetic checks will indicate correct calculations, but the reduced levels will be incorrect. Therefore, to ensure accurate elevations of the level points, the fieldwork should start at a benchmark and close at another benchmark of known elevation. If no other benchmark is available, the surveyor should go back and close at the starting point.

2) If the purpose of leveling is to find out the elevation difference between two points, which are unseen from each other, or if they are far apart, no intermediate sights are necessary in this case. Only backsight and foresight readings are made.

3) The backsight and foresight distances should be approximately equal to avoid errors, which will result if the line of sight was not completely horizontal.

4) Turning points should be chosen on firm ground. On soft ground, a special triangular base is used.

5) If possible, staff readings should be made to the nearest mm at turning points and to the nearest cm at other points.

6) If the fieldwork starts at a point of an unknown elevation, but passes through a benchmark somewhere in the leveling chain, then the calculations are carried out as follows:

   a - If the benchmark is the last point, then the elevation of the first point is calculated from Equation (4.2) as follows:
   RL of first point = RL of last point (BM) + ΣFS - ΣBS ......(4.5)
   Now, that the RL of the first point is known, the computations are performed as explained earlier.

   b - If the benchmark is not the first point, neither the last point, we add the staff reading at this bench mark to the known BM elevation to get the HI. This HI is recorded in the table on the same line as the corresponding BS reading. Now, the computations are started from this known HI and continued until calculating the RL of the last point. The RL of the first point is then calculated using Equation (4.5). With the RL of the first point known, the computations are continued until reaching the BM, and with this, the table will be complete.

7) If the point whose elevation is to be calculated is the bottom of a bridge or a ceiling or the top of a barrier such as a wall or a column, the staff is held at this point in an upside-down position so that the zero of the staff will be at the point. The reading at this point is booked in the table with a negative sign and the calculations are carried out in the normal way as explained in the previous section.

EXAMPLE 4.1:

Using the data shown in Figure 4.11, do the required booking, and calculate the reduced levels of points A, B, C & D by both the height of instrument and rise and fall methods. Make the required arithmetic checks.

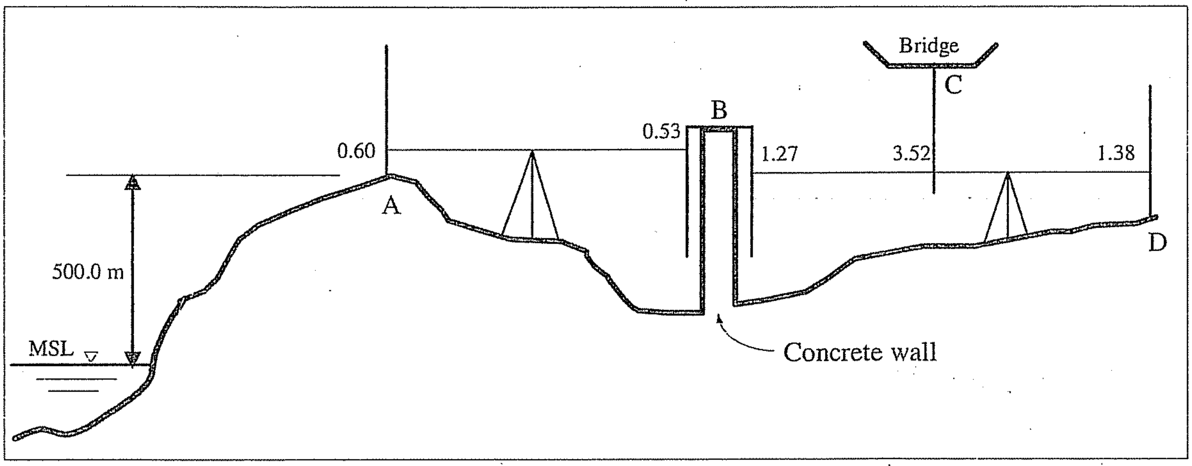


FIGURE 4.11: A leveling section with a bridge and a concrete wall obstacle.

SOLUTION:

Point	BS	IS	FS	Rise	Fall	HI	RL	Notes
A	0.60					500.60	500.00	BM
B	-1.27		-0.53	1.13		499.86	501.13	Top of concrete wall (TP)
C		-3.52		2.25			503.38	Bottom of bridge
D			1.38		4.90		498.48	
SUM	-0.67	-3.52	0.85	3.38	4.90			

Checks:

1) # of BS = # of FS = 2 $\Rightarrow$ OK

2) $\Sigma BS - \Sigma FS = -0.67 - 0.85 = -1.52$

$\Sigma Rise - \Sigma Fall = 3.38 - 4.90 = -1.52$ $\Rightarrow$ OK

$RL_{last} - RL_{1st} = 498.48 - 500.00 = -1.52$

3) ΣRL excluding first RL = 501.13 + 503.38 + 498.48 = 1502.99

Σ (HI x # of FS and IS) = 500.60 x 1 + 499.86 x 2

= 1500.32

ΣFS = 0.85

ΣIS = -3.52

1500.32 - 0.85 - (-3.52) = 1502.99 $\Rightarrow$ OK

4.10 ERRORS IN DIFFERENTIAL LEVELING

As in any other type of surveying work, errors in leveling can be divided into systematic errors, random errors and blunders.

a) Systematic errors:

There are two major sources of systematic errors in differential leveling. These are:

1) Inclination of the line of sight due to the earth's curvature and atmospheric refraction, and
2) Inclination of the line of sight due to maladjustment of the level.

1) Earth's curvature and atmospheric refraction:

As defined before, a level line is a curved line and is everywhere normal to the plumb line. However, a horizontal line of sight is perpendicular to the plumb line only at the point of observation. Hence, it should be carefully distinguished from a level line.

Because of atmospheric refraction, rays of light transmitted along the earth's surface are refracted or bent downward so that the actual line of sight is along a curve that is concave downward. This curve has a radius that is seven times the radius of the earth.

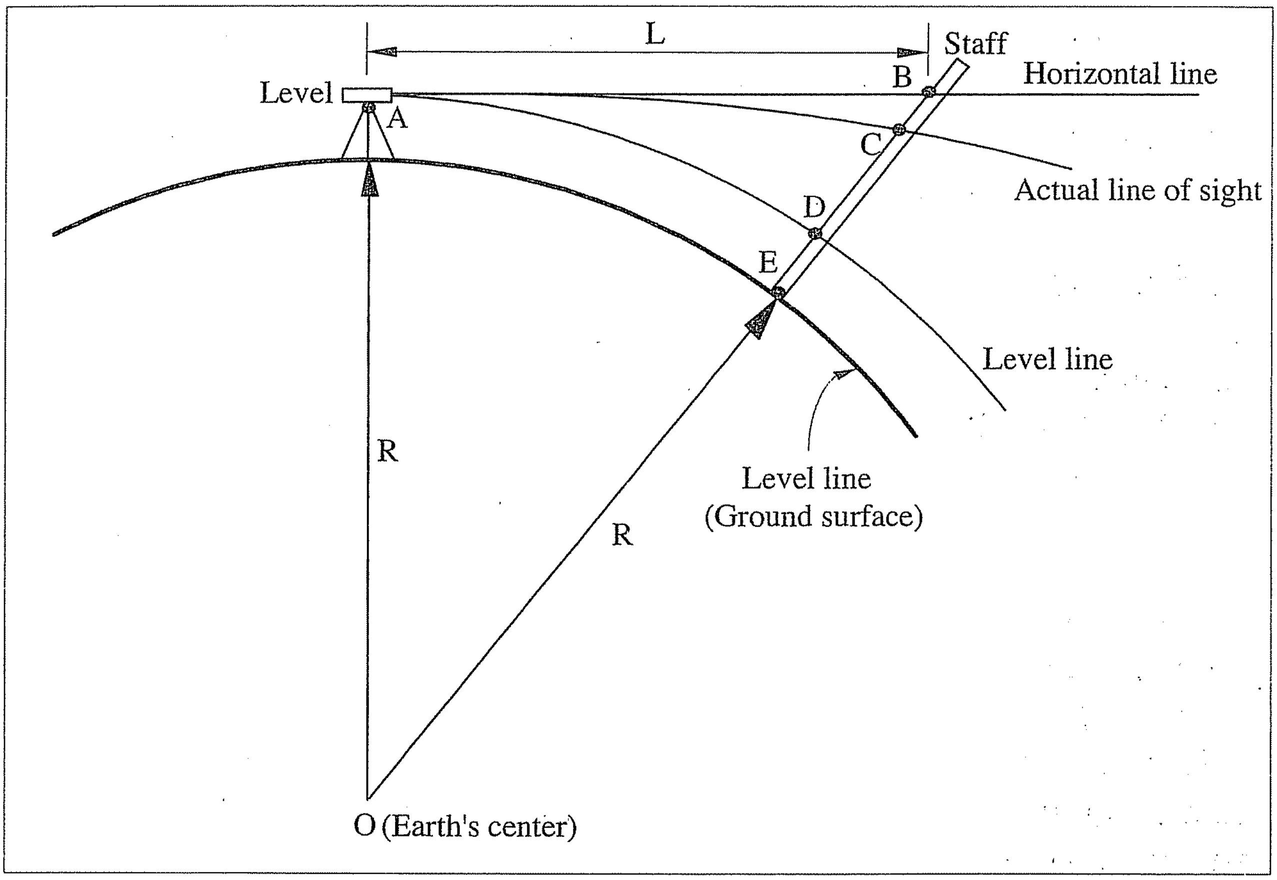


FIGURE 4.12: Effect of earth's curvature and atmospheric refraction on differential leveling.

In Figure 4.12,

BC = refraction of line of sight from horizontal
BD = Error due to earth's curvature
CD = Actual net error in the staff reading = BD - BC

In the triangle ABO,

$$(BD + DO)^2 = AB^2 + AO^2, \qquad AB = L, \qquad AO = DO \approx R$$

$$\Rightarrow \quad BD^2 + 2R \cdot BD + R^2 = AB^2 + R^2$$

$$BD^2 + 2R \cdot BD = L^2$$

Since BD is very small compared to R and L, BD^2 can be ignored.

$$\Rightarrow \quad 2R \cdot BD \approx L^2$$

$$\Rightarrow \quad BD \approx \frac{L^2}{2R}$$

Substituting R = 6365 km

$$\Rightarrow \quad BD \approx 0.0786\ L^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.6)$$

Where BD is in m

L is in km

$$\text{Refraction} = BC \approx \frac{BD}{7} \approx \frac{0.0786L^2}{7}$$

Actual error in staff reading $= CD = BD - BC$

$$\Rightarrow \quad CD = 0.0786L^2 - \frac{0.0786L^2}{7}$$

$$\Rightarrow \quad CD = 0.0673L^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.7)$$

Where CD is in m

L is in km

When $L = 1\ \text{km} \Rightarrow CD \approx 0.07\ \text{m} = 7\ \text{cm}$

$L = 100\ \text{m} = 0.1\ \text{km} \Rightarrow CD \approx 0.001\ \text{m} = 1\ \text{mm}$

Therefore, to keep the effect of the earth's curvature and atmospheric refraction to a minimum, it is advisable that the distance between the level and the staff should not exceed 100 m.

2) Maladjustment of the level (collimation error):

When the line of sight of a level is not perfectly parallel to the axis of the level bubble due to maladjustment of the level, the line of sight will be actually inclined from the horizontal even though the level bubble is perfectly centered. This kind of error can be completely eliminated by balancing the BS and FS distances. Figure 4.13 illustrates the geometry of this source of error.

Let a = Actual BS reading with staff held at point A,

m = Correct BS reading at A if the line of sight was perfectly horizontal,

ε_1 = Reading error at A due to maladjustment of the level,

b = Actual FS reading with staff held at B,

n = Correct FS reading at B if the line of sight was perfectly horizontal,

ε_2 = Reading error at B due to maladjustment of the level, and

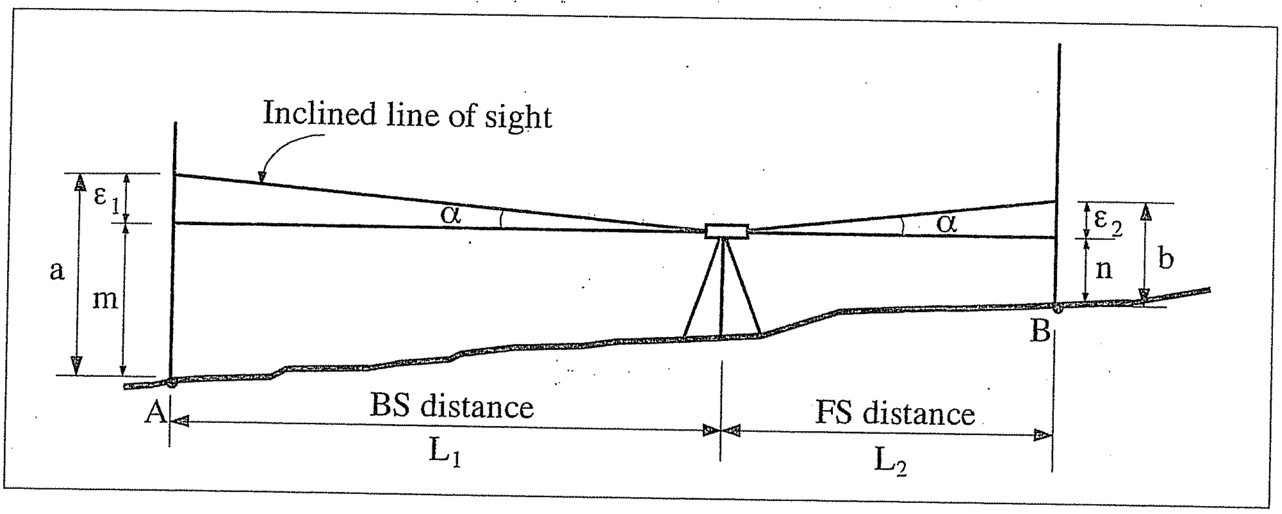


FIGURE 4.13: Error due to the maladjustment of the level.

α = Angle of inclination of the line of sight from the horizontal. It is positive if above the horizontal and negative if below the horizontal.

Then,

$$\begin{aligned} \varepsilon_1 &= L_1 \cdot \tan\alpha \\ \varepsilon_2 &= L_2 \cdot \tan\alpha \end{aligned} \qquad \text{..............(4.8)}$$

Correct elevation difference $(\Delta h) = m - n$

$$= \left(a - \varepsilon_1\right) - \left(b - \varepsilon_2\right)$$

$$= \left(a - b\right) - \left(\varepsilon_1 - \varepsilon_2\right) \qquad \text{..........(4.9)}$$

Substitute equations (4.8) into (4.9),

$$\Rightarrow \quad \Delta h = \left(a - b\right) - \tan\alpha\left(L_1 - L_2\right) \qquad \text{..........(4.10)}$$

It is seen for Equation (4.10) that if $L_1 = L_2$, then $\varepsilon_1 = \varepsilon_2$, and the correct elevation difference will be equal to the difference between the two actual readings a & b, and will be free of error caused by the maladjustment of the level.

EXAMPLE 4.2:

To check a level for the existence of collimation error, the level was set up mid-way between points A and B and the following two staff readings were taken: 1.92 m at A and 1.40 at B. The level was then moved to another position and the readings in Figure 4.14 were taken. Is there a collimation error? If the answer is yes, then calculate the angle of inclination of the line of sight from the horizontal, as well as the correct readings that should have been taken at A and B in the second setup if there was no collimation error.

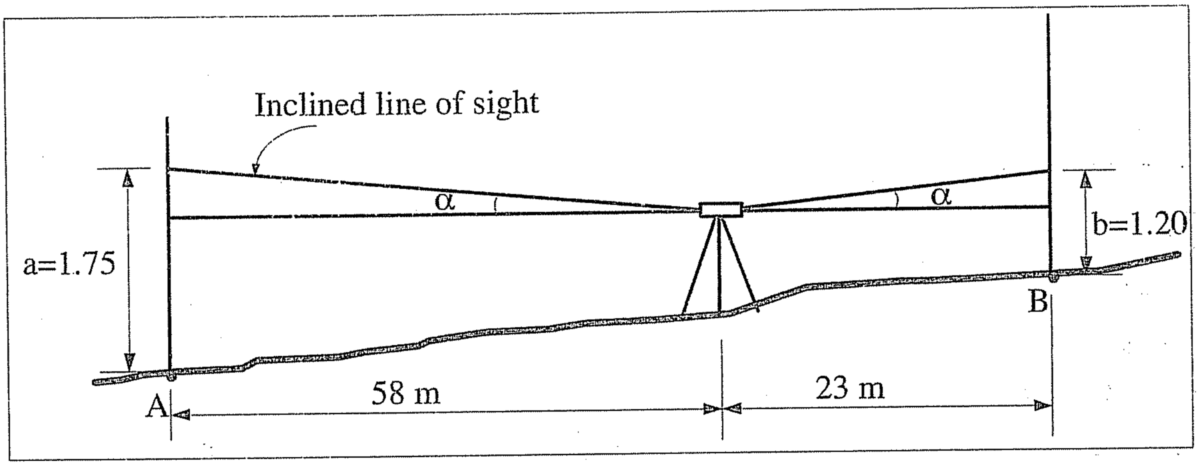


FIGURE 4.14: Checking a level for the existence of collimation error.

SOLUTION:

First setup: ΔH_1 (correct) $= 1.92 - 1.40 = 0.52$ m

Second setup: ΔH_2 $= 1.75 - 1.20 = 0.55$ m

$\Delta H_1 \neq \Delta H_2 \Rightarrow$ There is a collimation error

From Equation (4.10)

$0.52 = (1.75 - 1.20) - \tan\alpha\,(58 - 23)$

$\Rightarrow \alpha = 0° \, 2' \, 57''$

Correct reading at A (m) $= 1.75 - 58 \tan\alpha = 1.70$ m

Correct reading at B (m) $= 1.20 - 23 \tan\alpha = 1.18$ m

Check:

$\Delta H = 1.70 - 1.18 = 0.52 \text{ m} = \Delta H_1 \quad \Rightarrow$ OK

b) Random errors:

The principal sources of random errors that affect the accuracy of leveling results are:

1) The staff not held plumb
2) The bubble of the level not perfectly centered
3) The incorrect reading of the staff
4) The instability of turning points
5) Wind. Wind may vibrate the level and the staff and make it difficult to keep the bubble centered and to read the staff correctly.

c) Blunders or mistakes:

Blunders commonly made in leveling include the following:

1) Misreading the staff especially when the marks on the staff are obscured by a tree, fence and so on.
2) Not setting the staff on the same point for a FS and the subsequent BS readings.
3) Recording or booking of data. Examples of this kind of error will be reading 2.58 m as 2.85 m, or booking a FS in the BS column and vice versa. Page checks should always be made before leaving the field.

4.11 RECIPROCAL LEVELING

When a line of levels crosses a wide body of water (e.g., a river) or a ravine, it becomes impossible to balance the BS and FS distances, and it might be necessary to take sights at distances much longer than ordinarily permissible (greater than 100 m). Under such circumstances, errors due to earth's curvature, atmospheric refraction and the inclination of the line of sight become particularly significant. A special procedure called reciprocal leveling is used to overcome this problem and obtain the best results.

In Figure 4.15, the elevation difference between the two points A and B is to be determined. Using reciprocal leveling, the procedure is described as follows:

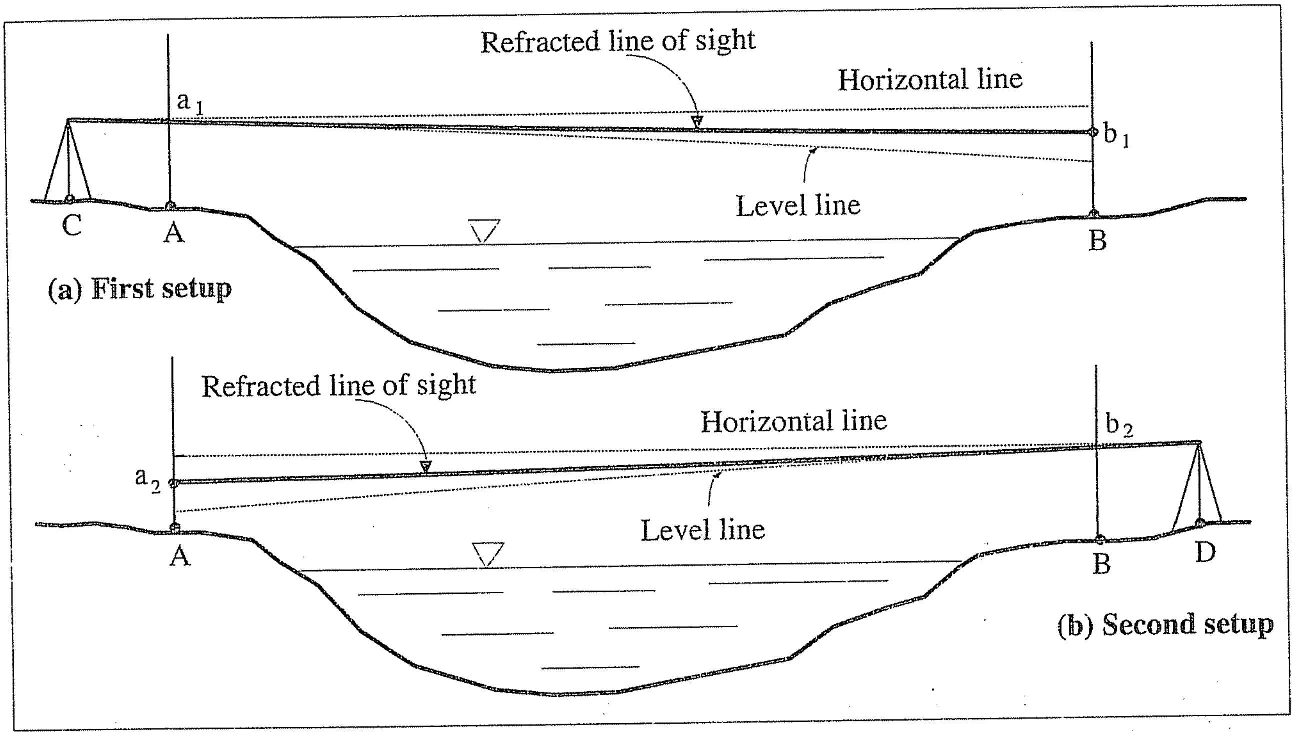


FIGURE 4.15: Reciprocal leveling.

1) Set up the level at point C (Figure 4.15a), 2 to 3 m from A and take the readings a_1 at A and b_1 at B. Calculate the first elevation difference:

$$\Delta H_1 = a_1 - b_1$$

2) Move the level to point D (Figure 4.15b) so that the distance AC = BD. Take the two readings a_2 at A and b_2 at B. Calculate the second elevation difference:

$$\Delta H_2 = a_2 - b_2$$

3) Calculate the correct elevation difference (ΔH) as follows:

$$\Delta H = \frac{\Delta H_1 + \Delta H_2}{2} = \frac{(a_1 - b_1) + (a_2 - b_2)}{2} \qquad \text{...........(4.11)}$$

EXAMPLE 4.3:

The elevation of point A in Figure 4.15 is 917.34 m. From a setup on the left bank, the BS reading at A was 1.44 m and the FS reading at B was 1.90 m. At the second setup (on the right bank) of the level, the BS reading at A was 1.80 m and the FS reading at B was 2.34 m. Find the elevation of point B.

SOLUTION:

Using Equation (4.11),

$$\Delta H_{AB} = \frac{(1.44 - 1.90) + (1.80 - 2.34)}{2} = -0.50 \text{ m}$$

The elevation of point B (H_B) $= H_A + \Delta H_{AB}$

$$= 917.34 + (-0.50) = 916.84 \text{ m}$$

4.12 CLOSURE ERROR

For the case where leveling starts at a known BM and ends at the same or at another BM, the calculated elevation for the end station must be equal to the elevation of the known station if the leveling is free of errors. However, this case rarely occurs and a closure error results.

In Figure 4.16, leveling starts at BM1 and ends at BM2 with n_i and Δh_i referring to the number of level setups and elevation difference between consecutive stations. Assume that the known elevation of BM2 is (h) and the calculated elevation from leveling is (h'), then the closure error (ε) is:

$$\varepsilon = h' - h \qquad \text{.................................(4.12)}$$

Corrections for closure error can be distributed to the line sections by proportion. Assuming that all the staff readings along a level line are made with the same care and accuracy, the number of setups needed for each of the line sections can be used for proportioning. Therefore:

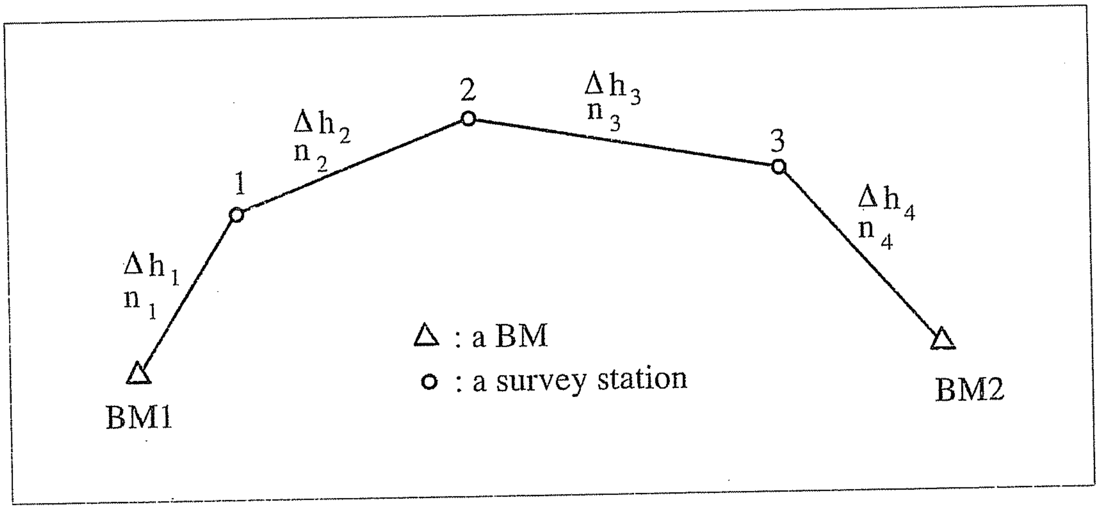


FIGURE 4.16: A level line which starts at a BM and closes at a different BM.

$$\text{Closure correction for } \Delta h_i = -\frac{n_i}{\Sigma n_j}(\varepsilon) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

$$\text{Corrected } \Delta h_i = \text{measured } \Delta h_i + \text{closure correction for } \Delta h_i \quad \ldots\ldots\ldots\ldots (4.14)$$

4.13 CLASSES AND ACCURACY OF LEVELING

Leveling might be divided primarily into two main classes:

1) Precise Leveling. This type of leveling is very accurate and is used mainly for the establishment of benchmarks, for extensive engineering projects, for regional crustal movement investigation, and for any other project, which requires high accuracy. For this purpose, special high precision levels and leveling rods, as well as, field procedures are used.

2) Ordinary Leveling. This type of leveling does not require very high accuracy, and is therefore, suitable for small engineering project, for topographic mapping, for making longitudinal and cross-sections, and for other projects for which medium accuracy is sufficient.

The allowable closure error in differential leveling is normally given in the form: $\varepsilon = \pm k\sqrt{D}$ mm where k is a constant determined by the type of leveling and D is the total leveled distance in kilometers. The value of k in ordinary leveling may vary from 10 to 30 mm depending on the type of ground being leveled, the accuracy required and the type of instrument being used. For precise leveling, the value of k may vary from 2 to 5 mm.

4.14 APPLICATIONS OF LEVELING

Apart from the general problem of determining elevation difference between two points, which has been fully dealt with in the previous sections, the main uses of leveling are:

1) The making of longitudinal sections (profiles)
2) The making of cross-sections
3) Contouring, and
4) Setting out levels.

4.14.1 LONGITUDINAL SECTIONS (PROFILES)

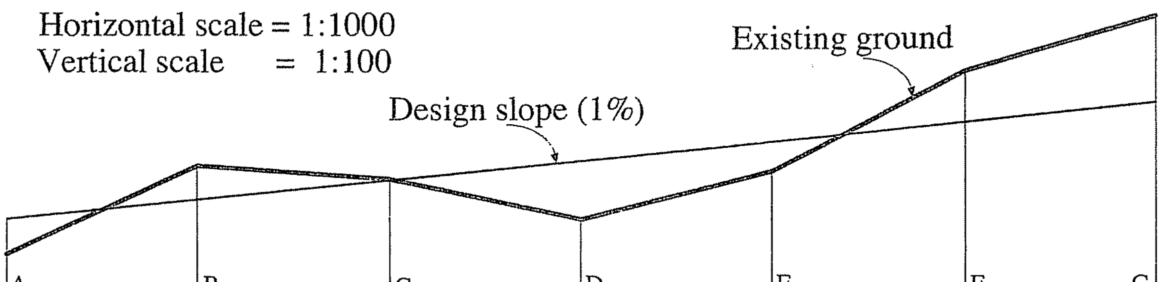
An example of such a section is given in Figure 4.10. The objective here is to reproduce on paper the existing ground profile along a particular line such as the center line of an existing or proposed work like the center line of a railway, road, canal, sewer or water main. As a general guide, levels are taken at:

1 - Every 20 m, 50 m or 100 m depending on the topography
2 - Points at which gradient changes, and
3 - Street intersections.

Staff readings to 0.01 m accuracy are generally adequate. The sections are usually plotted to a distorted scale, a common one for roadwork being 1/1000 horizontal and 1/100 vertical. This is due to the fact that the horizontal distance represented on the horizontal axis is much larger than the elevation variations of profile points. Figure 4.17 shows an example of a plotted profile with points connected by straight lines.

To avoid the build up of errors, the following points should be kept in mind when leveling to produce a longitudinal section:

1 - Start the work at a benchmark (BM) and make use of nearby benchmarks.
2 - Try to keep BS and FS distances approximately equal. This will minimize errors resulting from earth's curvature, atmospheric refraction and maladjustment of the level.



Point	A	B	C	D	E	F	G
Distance	0.0	20.0	40.0	60.0	80.0	100.0	120.0
Ground Elevation	97.48	98.42	98.27	97.84	98.36	99.41	99.98
Design Elevation	97.87	98.07	98.27	98.47	98.67	98.87	99.07
Cut		0.35	0.00			0.54	0.91
Fill	0.39		0.00	0.63	0.31		

FIGURE 4.17: A longitudinal section (profile) of the data of Table 4.2.

3 - Make all changes (TP) on firm ground to make checks if required later.
4 - Take the final FS on a BM or close back to the starting point.
5 - Do not work with the staff extended in high wind.

4.14.2 CROSS-SECTIONS

Some engineering works require that cross sections be taken at right angles to the centerline of a proposed or existing project such as a road. The width of these sections must be sufficient to cover the proposed works, e.g. 15 m either side of the centerline for a normal road. The longitudinal spacing of the sections depends on the nature of the ground, but should be constant if earthworks are to be computed. A spacing of 20 m is common.

The centerline is first set, and then perpendiculars are erected by the methods discussed earlier (see Chapter 3). If all the staff readings can be taken by a single setup of the level, a neat way to book the readings is as follows (Figure 4.18).

It is common to plot cross-sections to a normal, i.e. undistorted scale. A scale of 1/50 or 1/100 for both horizontal and vertical axes is good for this purpose.

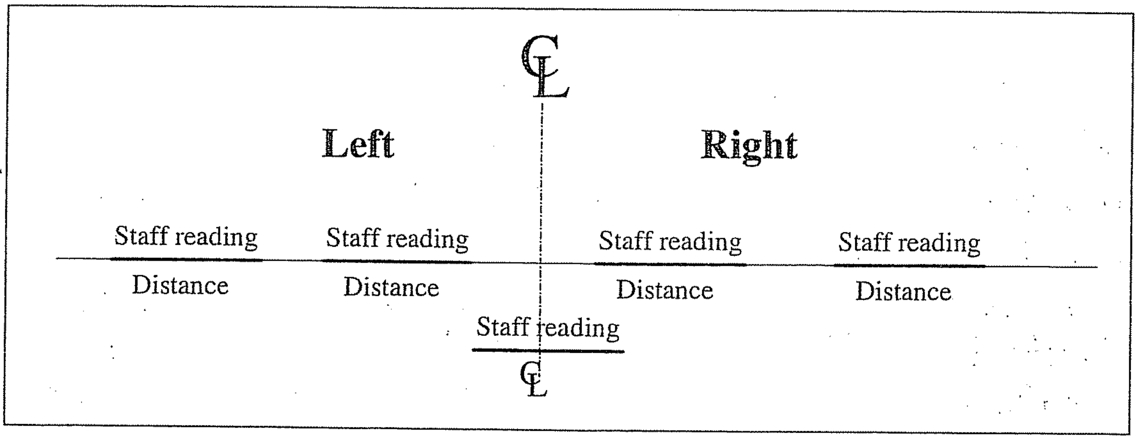


FIGURE 4.18: Booking leveling data for making cross-sections.

4.14.3 CONTOURING

A *contour* is an imaginary line connecting points on the ground that have the same elevation. It may be considered as the trace of the intersection of a level surface with the ground. The shoreline of a body of still water is a typical example of a contour. The vertical distance or elevation difference between two successive contours is called *contour interval*. Figure 4.19 shows an example of contours that have a contour interval of 20 m.

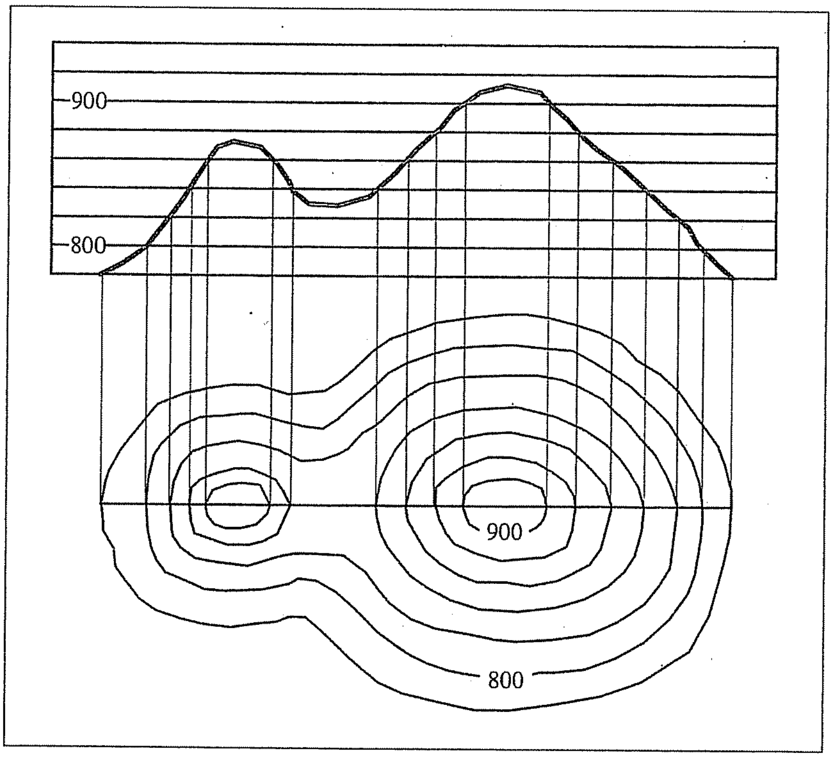


FIGURE 4.19: Contours and ground profile.

Characteristics of Contours:

The principal characteristics of contours are:

1) Contours spaced closely together represent a steep slope. When spaced far apart, they indicate a flat slope (Figure 4.20).

2) Contours are perpendicular to the direction of the steepest slope (Figure 4.20).

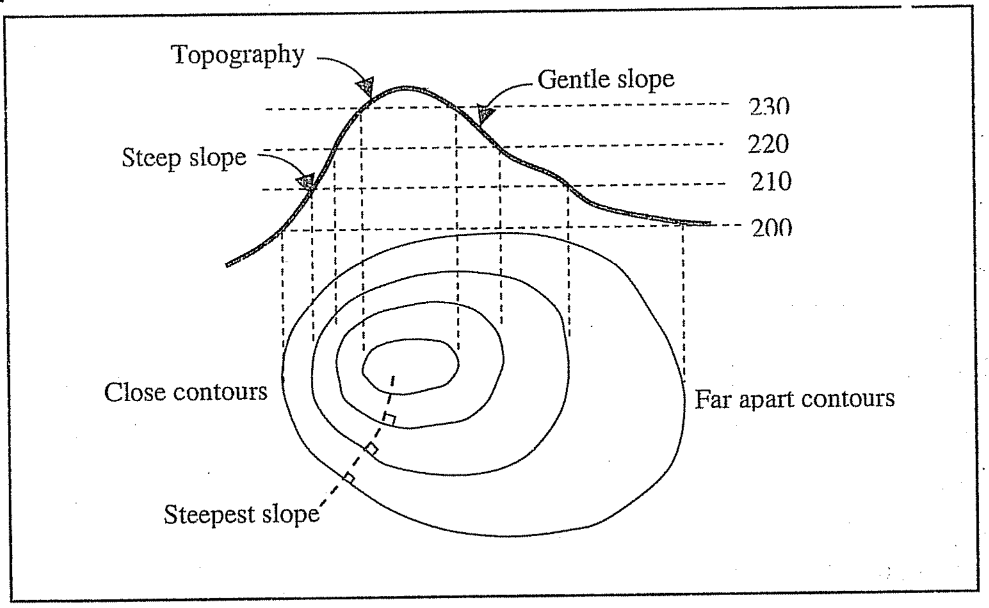


FIGURE 4.20: Relationship between contour spacing and topography slope.

3) Contours of different values do not cross each other nor do they merge except in rare situations where there is a cave or a vertically standing surface such as a wall. Figure 4.21 shows these two exceptions.

4) Contours that portray summits (such as a hill) or depressions (such as a bottom of a lake) are closed lines.

5) A single contour cannot split into two contours of the same value as itself, and it must make a closed circuit although not within the area covered by the contour plan.

6) Irregular contours represent a rough and uneven terrain.

Figure 4.22 shows examples of contours for different types of relief.

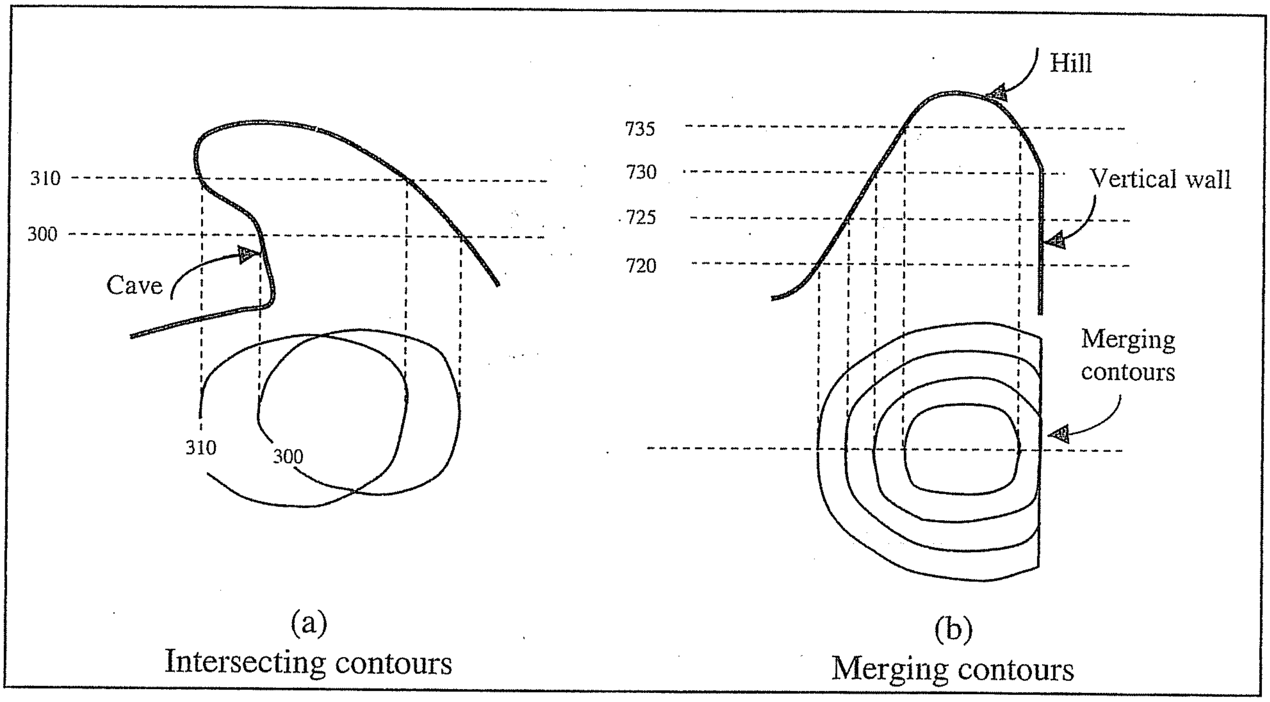


FIGURE 4.21: Examples of intersecting and merging contours.

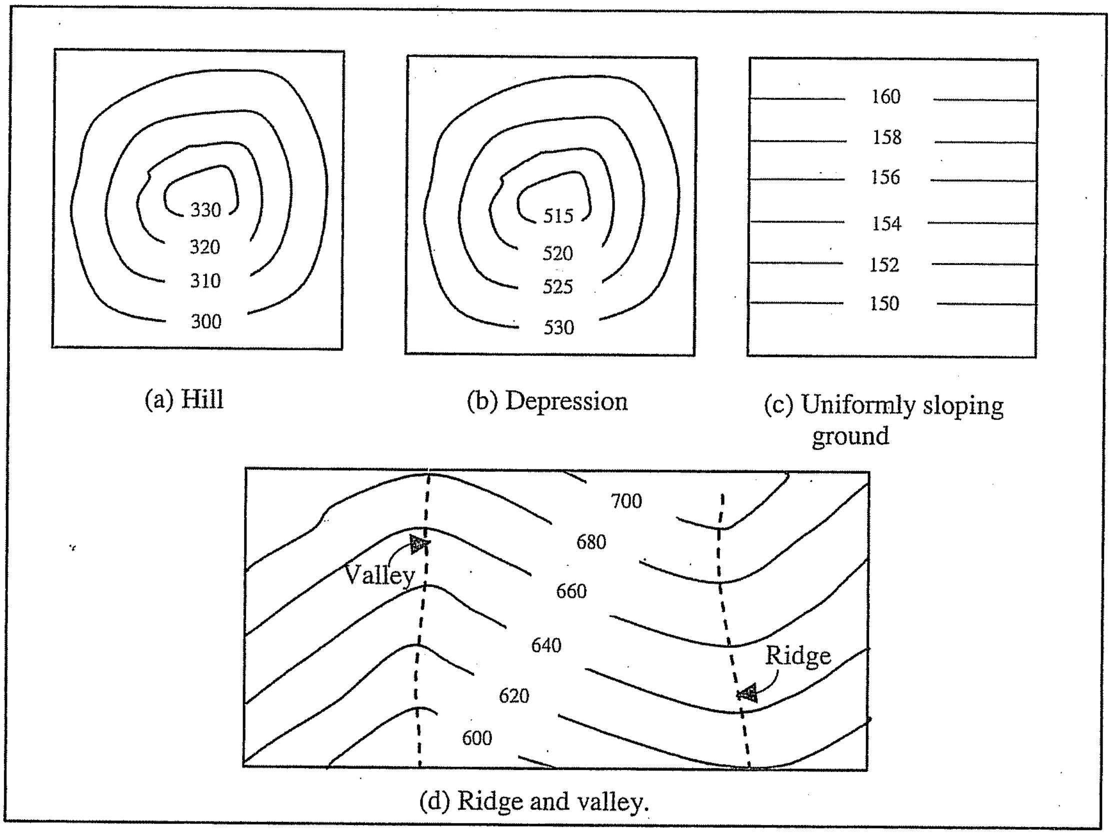


FIGURE 4.22: Examples of contours for different types of terrain.

Factors affecting the choice of contour intervals:

Several factors affect the choice of a contour interval. These include:

1) The scale of the contour plan. As the scale of the contour map becomes smaller, the distance between the contours becomes also smaller, and as a result, the map becomes crowded with lines and difficult to read. To prevent so, the contour interval is made larger.

2) The importance and purpose for which the plan is to be used. When more details are needed in the contour plan for computation of earth volumes and for design purposes, a smaller contour interval should be used. However, if the contour plan aims merely to give an idea about the shape of the relief, a larger contour interval can be used.

3) Accuracy, time and cost of the contour plan. As higher accuracy is needed, a smaller contour interval should be chosen. As a result, the project will be more costly and will require longer time to be accomplished.

4) The topography of the ground. Steep ground requires a larger contour interval to increase the distance between contours, while a small contour interval can be used for flat terrain.

5) The area for which the contour plan is to be made. As the area to be covered becomes larger, a smaller scale is usually chosen to draw the map. As a result, a larger contour interval is used.

Methods of Contouring:

The following are probably the most common methods for contouring:

1) *Griding*. This method is most suitable for flat terrain, especially on comparatively small sites. Rectangles or squares of 5 to 20 m a side are usually set out on the ground in the form of a grid, and levels (staff readings) are taken at the corners (Figure 4.23).

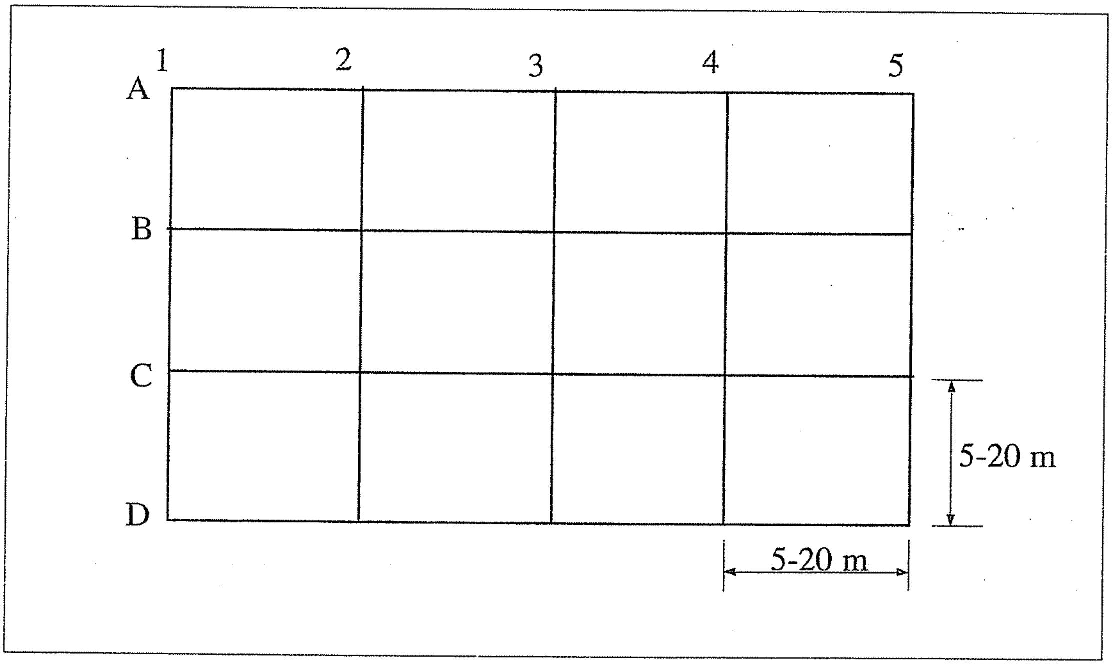


FIGURE 4.23: Griding.

In the booking process, corners are given numbers according to their coordinates in the grid, such as B3 or C5. The reduced levels of these corners are plotted on a plan, which has been girded in the same manner according to some suitable scale. The required contour lines are then plotted by a process called linear interpolation. To illustrate this process, let us assume that the elevations of points D4, D5 and C5 are 13 m, 10 m and 12 m respectively. Let us also assume that the distance between corners D4 and D5 is 12 m and the distance between corners C5 and D5 is 10 m. Now, if the contour line whose elevation is 11 m, is at distance x from corner D5 in the direction of D4, then from Figure 4.24,

$$\frac{x}{12} = \frac{1}{3} \Rightarrow x = \frac{12}{3} = 4 \text{ m}$$

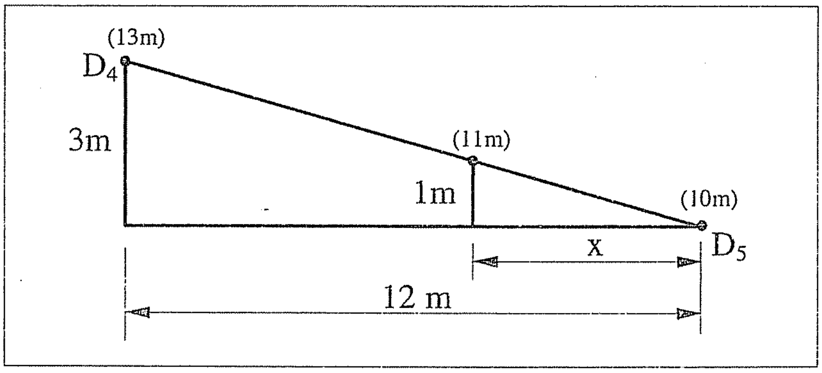


FIGURE 4.24: Linear interpolation.

This means that contour 11 m is 4 m from corner D5 in the direction of D4. By the same way, contour 11 m is found to be midway between corners C5 and D5. Contour 12 m is at 8 m from D5 in the direction of D4. Figure 4.25 shows the 11 m and 12 m contours drawn by a smooth line.

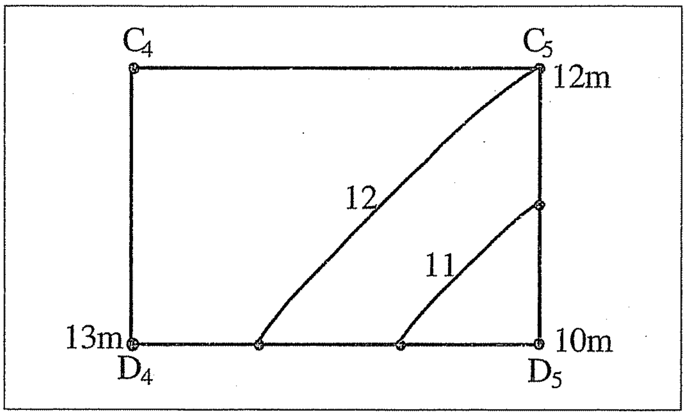


FIGURE 4.25: Plotting contour lines.

Notes:

a) The grid does not need to be made of regular squares or rectangles only. A grid of triangles or any other shapes could also be used. Also, an irregular grid can be used as long as it can be set up on the ground and plotted on a map.

b) The method of linear interpolation explained above is only one of the simplest methods used for contouring. Other several methods are available and can be used. It is beyond the scope of this book to talk about methods of contouring, and the reader can refer to specialized references about the subject.

2) *Radiating Lines.* Rays are set out on the ground from a central point such as the top of a small hill, the directions of these rays being known. Levels are taken along these rays at measured distances from the center (Figure 4.26). Again, linear interpolation is used to give the contour lines.

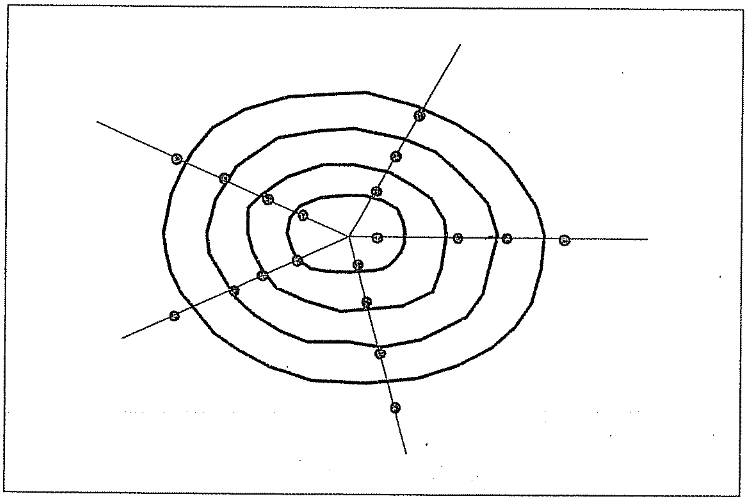


FIGURE 4.26: Contouring by radiating lines.

3) *Cross-Sections Method.* In this method, cross-sections are made on a line or a traverse inside the area for which the contour plan is to be made (Figure 4.27). Levels are then taken at points on the cross-sections where the topography changes. Again contour lines are drawn using the method of linear interpolation as explained earlier.

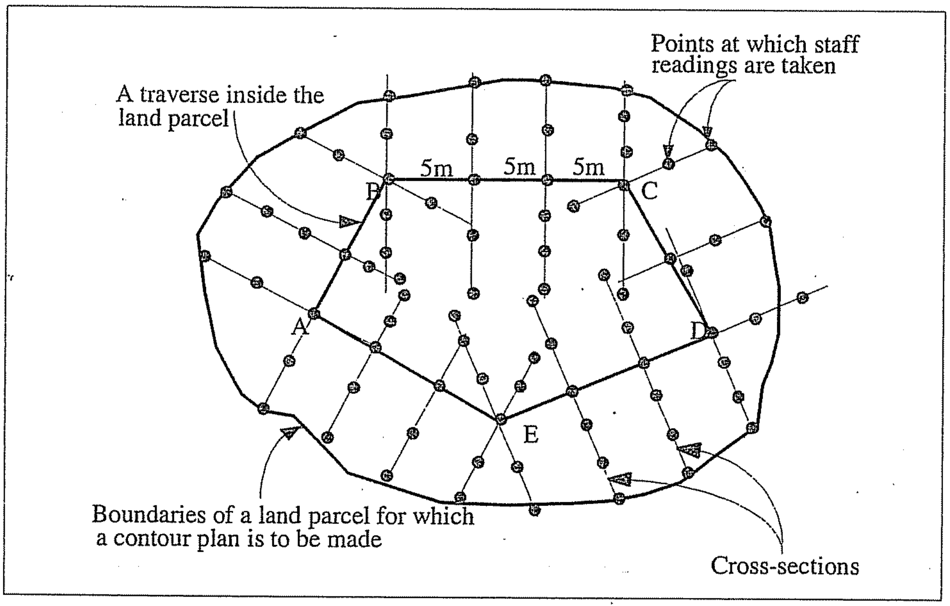


FIGURE 4.27: Contouring by the cross-sections method.

4) *Contouring Using Electronic Total Stations (see Chapter7: Section 7.2.8).* In this method, the coordinates and heights of representative points where topography changes are measured using total stations. These points are then plotted and contour lines are drawn using the method of linear interpolation as explained earlier.

4.14.4 SETTING OUT LEVELS

One of the basic applications of leveling is setting out sight rails, which enable the excavation operators to cut out earth to an even gradient, and enable the pipe-layer to lay the pipes according to this gradient. The following example illustrates the process:
Consider a length of a sewer being laid from manhole A, with an invert level of 30.02 m, to manhole B, 60 m away, and the gradient from A to B being 1 in 100 and falling from A to B (Figure 4.28). Thus, if two rails are fixed over stations A and B about 1 m above ground level, and each a fixed height above the invert level, then an eye sighting from rail A to rail B will be sighting down

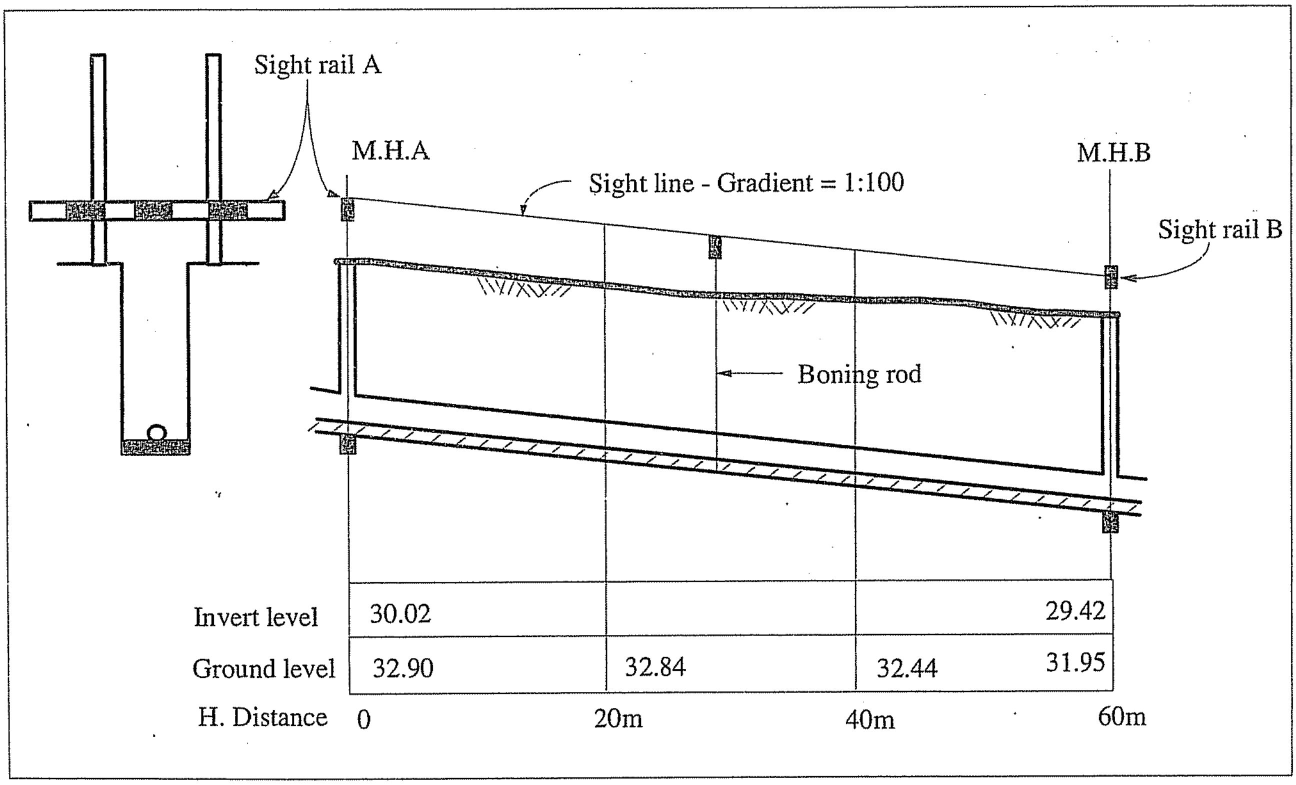


FIGURE 4.28: Setting out levels for laying out a sewage pipe.

a gradient equal to that of the proposed sewer (i.e., 1 in 100 here). In this example, a convenient height above the invert level is taken to be 3.75 m. If while digging, a boning rod (looking like the capital letter T with a sight bar across it) of this length is held vertically somewhere between points A & B so that its sight bar just touches the line of sight between sight rails A and B, it would give at its lower end a point on the sewer invert line.

To fix these sight rails for use with a 3.75 m long boning rod, we drive two posts on either side of the manholes and nail the rails between these at the following levels:

Sight rail A, RL = 30.02 + 3.75 = 33.77m
Distance AB = 60 m
Fall = 60 x 1/100 = 0.60 m
Invert level at B = 30.02 - 0.60 = 29.42 m
⇒ Sight rail height at B, RL = 29.42 + 3.75 = 33.17 m

If a level set up nearby has a height of collimation of 34.85 m, then the staff is moved up and down the posts at MH_A until a reading of 34.85 - 33.77 = 1.08 m is obtained. Pencil marks are made on each post and the black and white sight rail is nailed in position as shown in Figure 4.28. For rail B, the staff reading would be 34.85 - 33.17 = 1.68 m.

PROBLEMS

4.1 The following staff readings were taken during a leveling work: 1.26, 0.82, (1.79), 2.24, (1.25), X, -1.38, (1.42), 0.03, (1.19), 1.35, (2.37). Find the missing reading X, given that the height of the first and last points is 720.00 m AMSL and that the readings in brackets are foresights.

4.2 The staff readings observed with a level were: 2.565, 2.305, 2.115, 3.725, 3.565, 2.570, 1.905, 1.675, 1.565, 1.475, 3.725, 3.250, 3.105 and 1.250 m.

The level was moved after the fourth, ninth and twelfth readings. The first reading was taken on a BM of RL = 100.000 m and a standard deviation of ± 0.01 m. Calculate the difference of levels between the first and last points by both the height of instrument and the rise & fall methods. Check the accuracy of the arithmetic calculations. Also calculate the standard deviation of the RL of the last point given that the standard deviation of a single staff reading = ± 0.005 m.

4.3 A leveling section is run downhill from A to B. The 4m long staff readings taken in order are as follows: 0.53, 1.82, 2.97, 3.75, 1.02, 2.15, 3.36, 3.80, 1.09, 2.70 and 3.94. Given that point B is 500.00 m AMSL, tabulate these readings and calculate the reduced levels (RL) of all the section points and perform the required arithmetic checks.

4.4 A page of a field book is reproduced in the next table with some readings missing. Each station is compared with the one immediately preceding it. Complete the page with all arithmetic checks filling all X marks. (Note: staff readings are in ft).

Point	BS	IS	FS	Rise	Fall	RL	Notes
1	3.65					X	BM
2		X		2.75		X	
3		2.83				X	
4		3.64				X	
5	X		7.42			X	TP
6		12.41			7.32	X	
7		4.32				X	
8		3.00				X	
9		-6.17				X	
10	X		X			108.26	TP
11			X		1.32	X	
Sum	17.66				25.93		

4.5 A level set up in a position 30 m from peg A and 60 m from peg B reads 1.914 m on a staff held at A and 2.237 m on a staff held at B. The bubble has been carefully brought to the center of its run before each reading. It is known that the reduced levels of the tops of the pegs A and B are 87.575 m and 87.280 m respectively. Calculate:

a. The collimation error.
b. The readings that would have been obtained had there been no collimation error.

4.6 The first and last points of a longitudinal section were not given. For this purpose, you started leveling at point A with RL = 625.13 m AMSL and closed at point B with RL = 628.80 m AMSL. The readings are: 2.34, 1.15, 2.86, 0.97, 1.99, 2.68, 3.06, 2.24, 1.16, 1.48, 1.48, 2.90, 2.31, 2.85, 1.62, 0.89, 1.94, 0.92, 2.08, 1.88, 1.12. In each of the fourth and sixth positions of the level, you took two intermediate sights. In the fifth position, you took only one intermediate sight.

a. Tabulate the readings and calculate the RL. How accurate is the work.
b. The longitudinal section starts at the fourth point of the leveling and contains 9 points of equal 30 m distance separations. Draw the

longitudinal section to scales 1:1000 for distances and 1:100 for elevations. Show the road centerline on the same section if the first point has a RL = 627.00 m AMSL and goes upwards at a slope of 1%.

4.7 The following table gives the RL of some grid points. If you know that the grid is comprised of 3m side squares, and that point A1 is located at the top left corner of the grid, and point E1 is at the bottom left corner of the grid, draw the grid at a scale of 1:100, and show all the contour lines at 1 m interval.

Point #	RL (m)	Point #	RL (m)	Point #	RL (m)
A1	19	B5	17	D4	15
A2	20	C1	19	D5	14
A3	20	C2	20	E1	16
A4	19	C3	20	E2	15
A5	17	C4	18	E3	14
B1	20	C5	17	E4	13
B2	23	D1	18	E5	12
B3	23	D2	17		
B4	20	D3	16		

4.8 Proof that the approach discussed in section 4.11 about reciprocal leveling gives an elevation difference, which is free of errors due to the earth's curvature, atmospheric refraction and maladjustment of the level.

4.9 The adjustment of a tilting level was checked by taking the following readings on a vertical staff held in turn at stations X & Y that are 60 m apart:

LEVEL POSITION	STAFF READING at X	STAFF READING at Y
Midway between X and Y	1.514	1.968
90 m from X on XY	2.025	2.439

Comment on these readings. This level was then used without making any adjustments to establish two sight rails at A and B, 90 m apart, for the setting out of a sewer which had a gradient of 1 : 120 falling from A to B. A backsight of 1.06 m was taken on a BM that has a RL of 81.46 m above datum, 60 m away from the level. What staff readings were needed to locate sight rails at A and B, which were 45 m & 75 m respectively away from the level, given that the invert level at A was 78.58 m above datum and that a boning rod of length 3.50 m was to be used?

4.10 In Figure 4.29, a level loop starts and ends at the same BM100 whose elevation is 732.456 m AMSL. Check for closure error and make any required corrections.

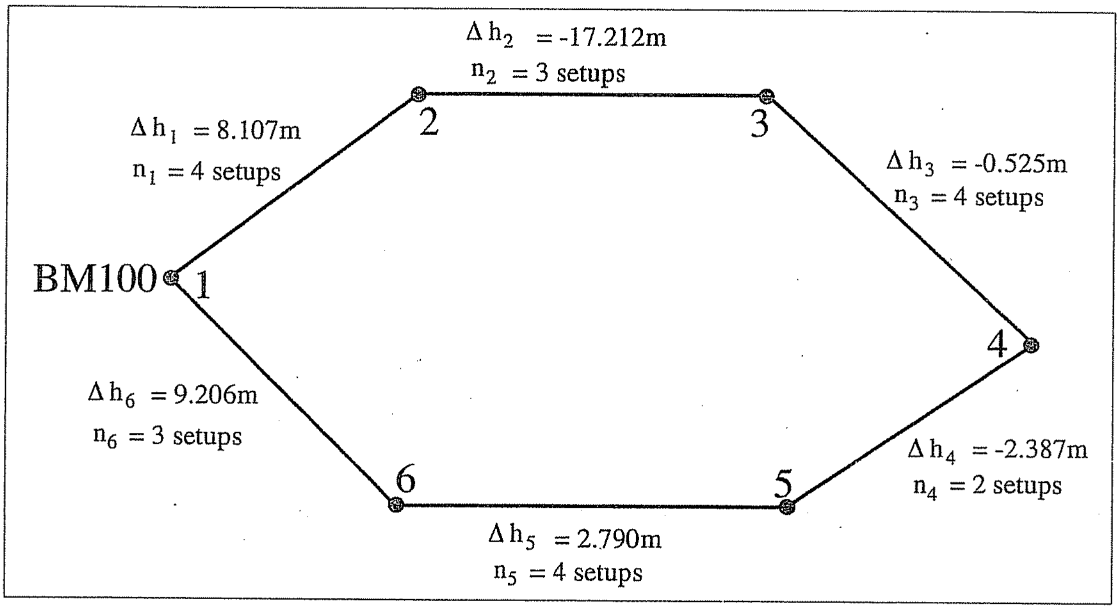


FIGURE 4.29: A level loop.

NAT

5 ANGLES, DIRECTIONS, AND ANGLE MEASURING EQUIPMENT

5.1 INTRODUCTION

Basically, surveying aims to determine the relative location of points on or near the surface of the earth. To locate a point, both distance and angular measurements are usually required. Such angular measurements are either horizontal or vertical, and they are most commonly accomplished with instruments called transits or theodolites.

5.2 HORIZONTAL, VERTICAL AND ZENITH ANGLES

In surveying, angles are measured either in a horizontal plane, yielding horizontal angles, or in a vertical plane, yielding vertical angles. In Figure 5.1, points A, B and C are three points located on the earth's surface. Points A', B' and C' are the projections of points A, B and C onto a horizontal plane. Angles $A'\hat{B}'C'$, $B'\hat{C}'A'$ and $C'\hat{A}'B'$ are the horizontal angles.

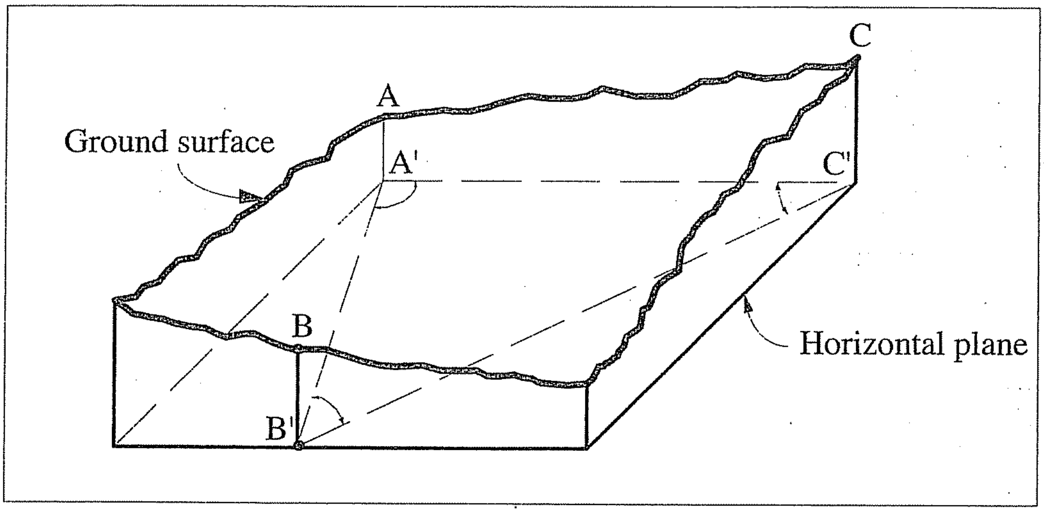


FIGURE 5.1: Horizontal angles.

A vertical angle, on the other hand, is measured in a vertical plane using the horizontal plane as a reference plane. When the point being sighted on is above the horizontal plane, the vertical angle is called an angle of elevation and is considered to be a positive angle. When the point being sighted on is below the horizontal plane, the angle is termed an angle of depression and is considered a negative angle. The value of a vertical angle can range from -90° to +90° (Figure 5.2).

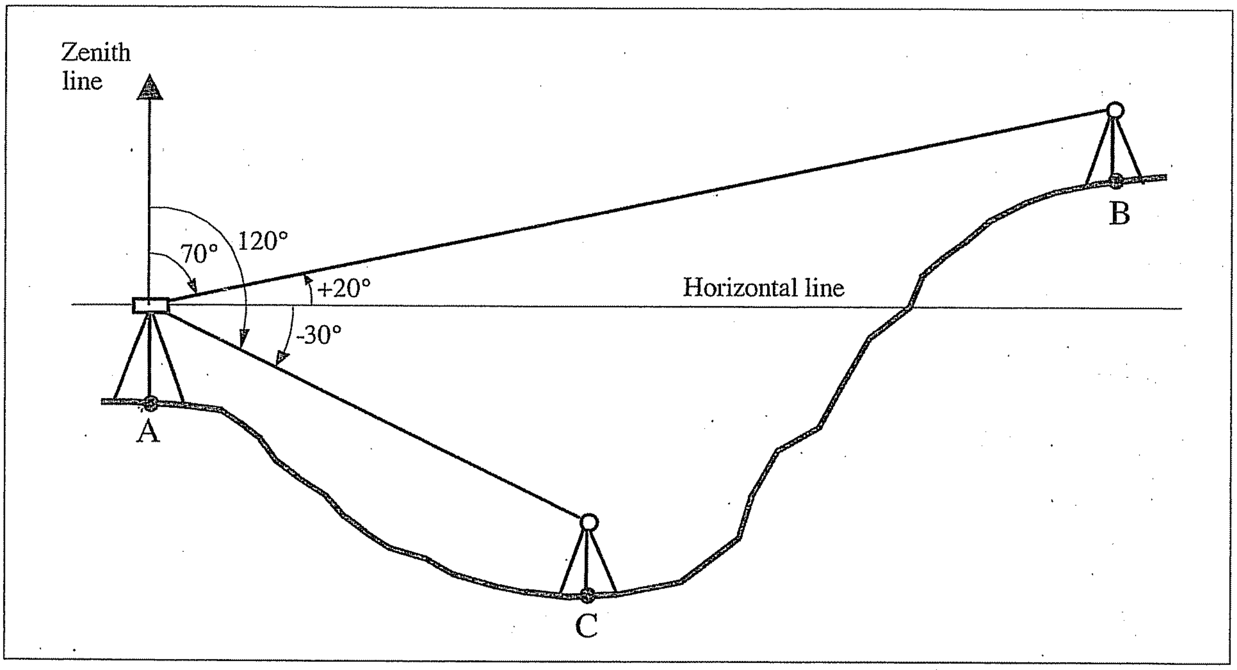


FIGURE 5.2: Vertical and zenith angles.

A zenith angle is also measured in a vertical plane but uses the overhead extension of the plumb line as a reference line (also known as zenith direction). Its value ranges from 0° to 180°. In Figure 5.2, the vertical angles measured at station A to target points B and C are 20° and -30°, respectively. The corresponding zenith angles are 70° and 120°. As can be seen, the summation of the vertical and zenith angles of any line is equal to 90°.

5.3 REFERENCE DIRECTION

It is convenient to choose a reference line to which directions of all the lines of a survey are referenced. Showing this reference direction on the map is also very basic to the map-reader to know how the different elements of the map are oriented in reality. Three different types of direction lines are traditionally used in surveying. These are the true or geographic north, the magnetic north and the assumed north.

5.3.1 TRUE OR GEOGRAPHIC NORTH

The true or geographic north at a point is defined as the direction of the line that connects this point to the North Pole of the earth. This line usually coincides with the great circle (also termed meridian) that passes through the point and the true (geographic) north and south poles of the earth (Figure 5.3). The north direction is usually used in the northern hemisphere, while the south direction is used in the southern hemisphere instead. Sine this direction is steady in a given place and does not change with time, it is usually preferred to be used for referencing maps.

The geographic north at a point can be determined either approximately or precisely through astronomical and global positioning system measurements. It can also be determined using the coordinate geometry principles as will be explained later in chapter 7. However, two approximate methods will be described here.

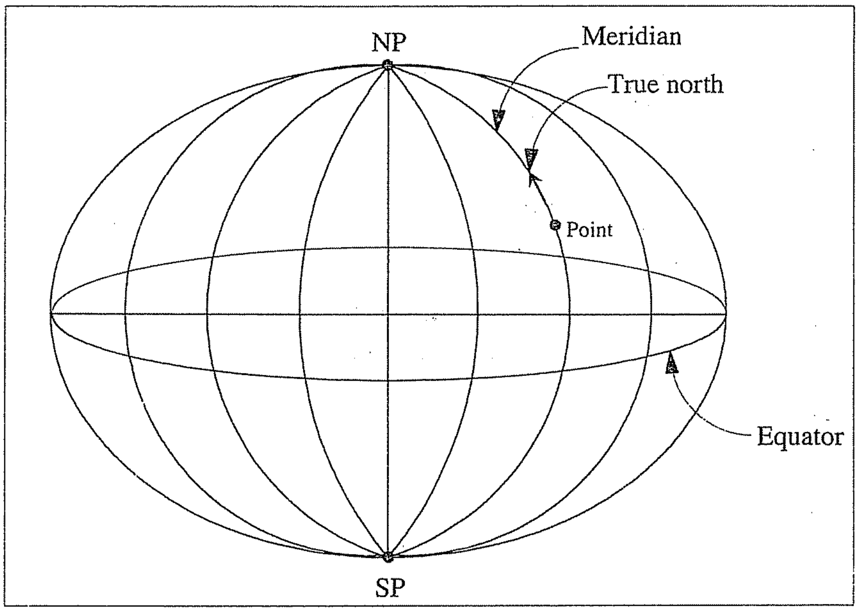


FIGURE 5.3: The true (geographic) north.

a) The watch method:

At noon (around 12:00), the direction of the line joining the observer's eye with the sun points toward the south. At other times, the south direction is determined using a watch with handles (non-digital). To do this, hold the watch in such a way that the hours handle points towards the sun. Bisect the angle between the hour handle and the line joining the center of the watch with the number 12 on the watch (number 1 for summer saving time). This bisecting line will point towards the south, and its opposite should, of course, be towards the north direction (Figure 5.4).

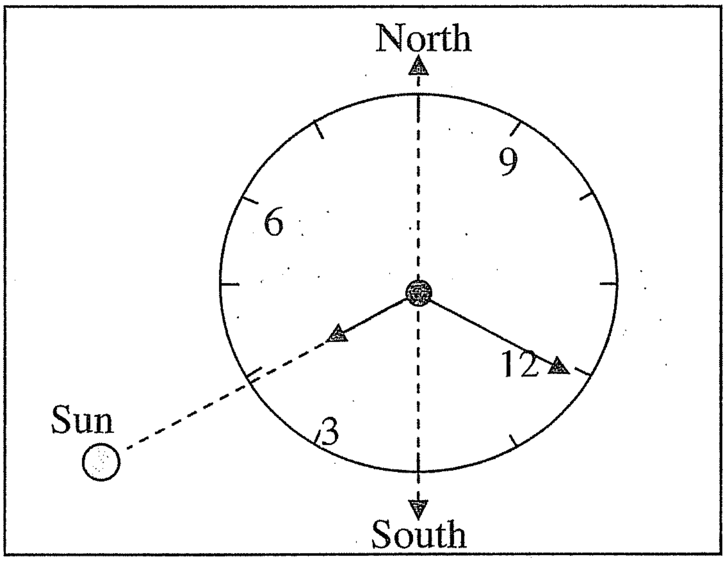


FIGURE 5.4: The watch method for determining the north direction.

b) The shadow method:

In this method, a straight stick 2-3 m long is supported at one of its ends on the ground, and around two-thirds of its length it is supported on an X-like support as shown in figure 5.5. The stick is made to point approximately towards north, and a plumb bob is hung from the free end of the stick to project it down on the ground at point A. Next, and two hours before noon (around 10:00 am), we start watching the shadow of the stick. The end point of the shadow (point B in figure 5.5) is allocated, and a circular arc whose center is at A and radius equal to AB is made using a pin. After this point, the shadow of the stick will start getting shorter and shorter until noon when it will start getting longer again, but towards the other side of the arc. Watch the shadow until it touches the circular arc at point C (should be around 2:00 pm). Bisect line BC at D. The north direction will then coincide with the line AD.

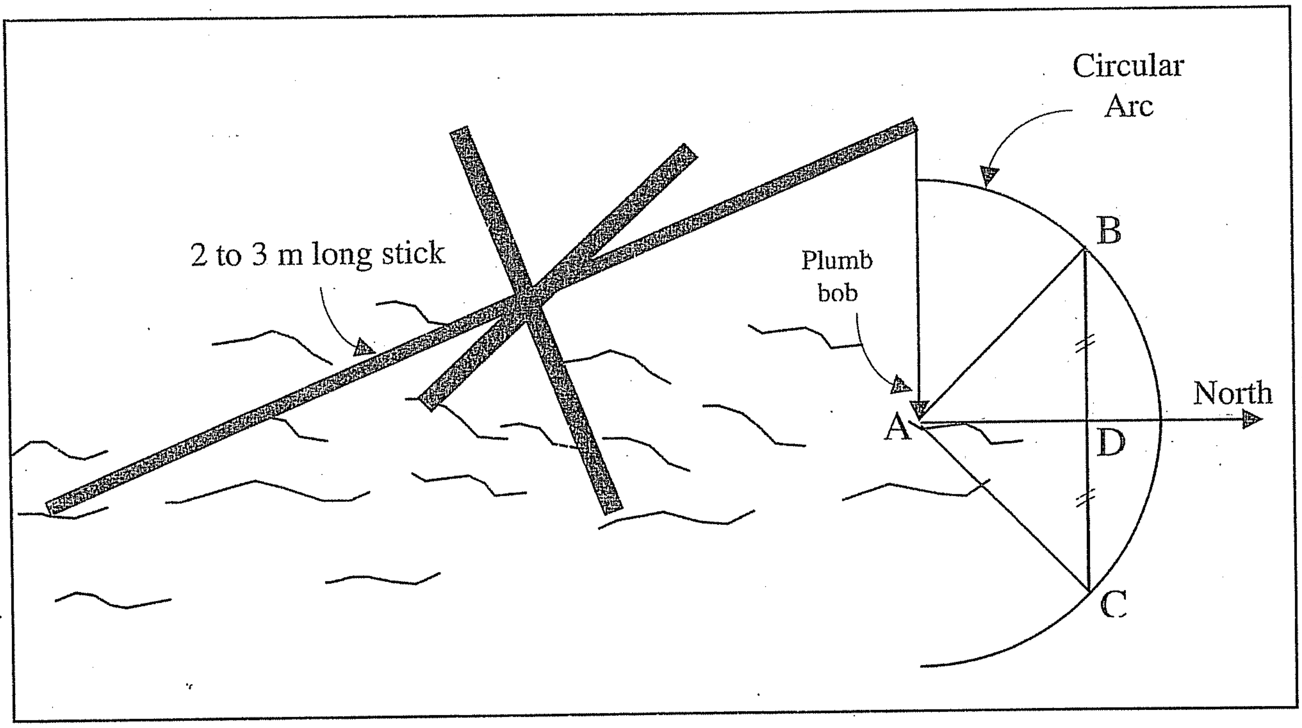


FIGURE 5.5: The shadow method for determining the north direction.

5.3.2 MAGNETIC NORTH

A magnetic needle, when let to come to rest in the earth's magnetic field (away from the effect of other magnetic fields), will point in the direction of the magnetic north pole of the earth. The direction in which the magnetic needle rests is called the magnetic meridian, and this meridian usually does not coincide with the true meridian. They make a small angle with each other called the magnetic declination, which could be to the east or west of the true meridian. Thus a declination of $\gamma°W$ means that the magnetic meridian is $\gamma°$ west of the true meridian (Figure 5.6). As shown in this figure, line AB deflects from the geographic and magnetic norths by the angles α and β respectively.

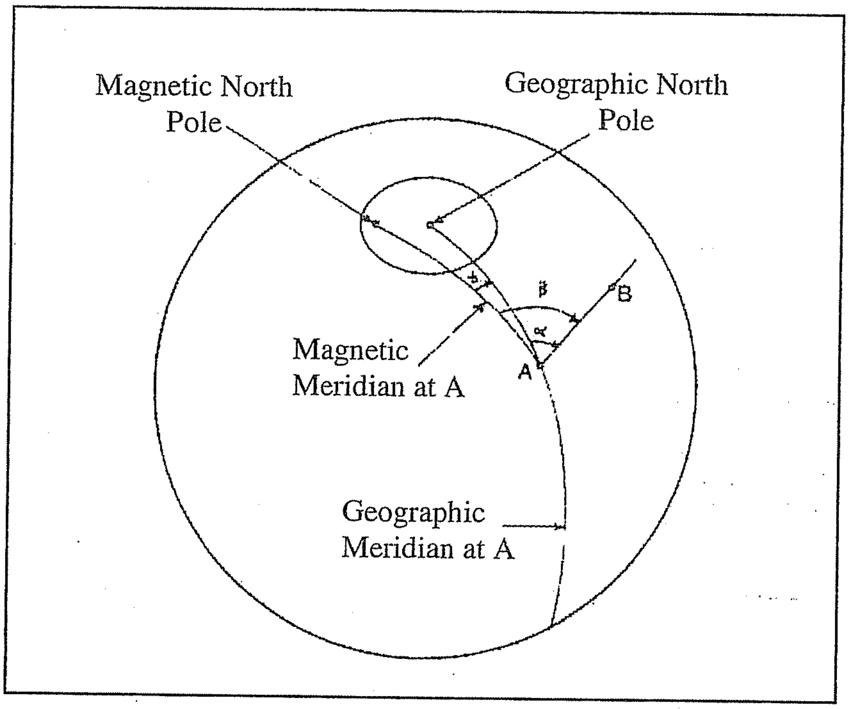
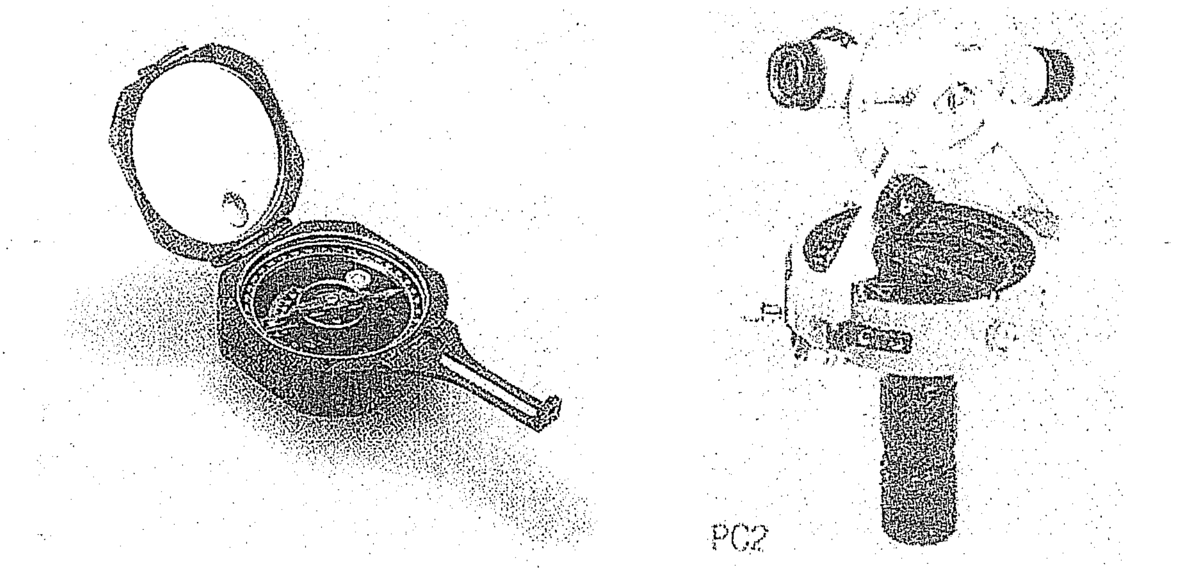


FIGURE 5.6: Relationship between true (geographic) and magnetic norths.

The value of the magnetic declination varies from one location to another, and at a given location, also changes with time. For this reason, it is preferable to reference maps according to the fixed true or geographic north. Usually, maps are prepared which give the value of the magnetic declination at a given location in a given year. If the angle that a line makes with the magnetic north is measured using a magnetic compass (such that shown in Figure 5.7), and if the magnetic declination at this point is known, then the direction of the true north at this location can be found.



(a) Pocket compass (b) Surveyor's compass

FIGURE 5.7: Magnetic compass.

5.3.2 ASSUMED NORTH

If neither the geographic north, nor the magnetic north is known in a certain local area for which the surveyor is making measurements, a line can be arbitrarily chosen and assumed to be in the direction the north. This line is called the assumed north. If later, the angle between the assumed and geographic or magnetic norths is known, then all the obtained survey measurements including any prepared maps can be corrected by simply rotating them to point in the right direction.

5.4 REDUCED BEARING OF A LINE

The reduced bearing of a line is the acute angle that the line makes with the reference meridian (whether being geographic or magnetic). It is expressed as "North (or South) so many degrees to the East (or West)". Thus the true reduced bearing of line OA in Figure 5.8 is N 70° E and for line OC is S 85° 36' 20" W. The true reduced bearing of a line never exceeds 90° and is never referenced to the east or west line.

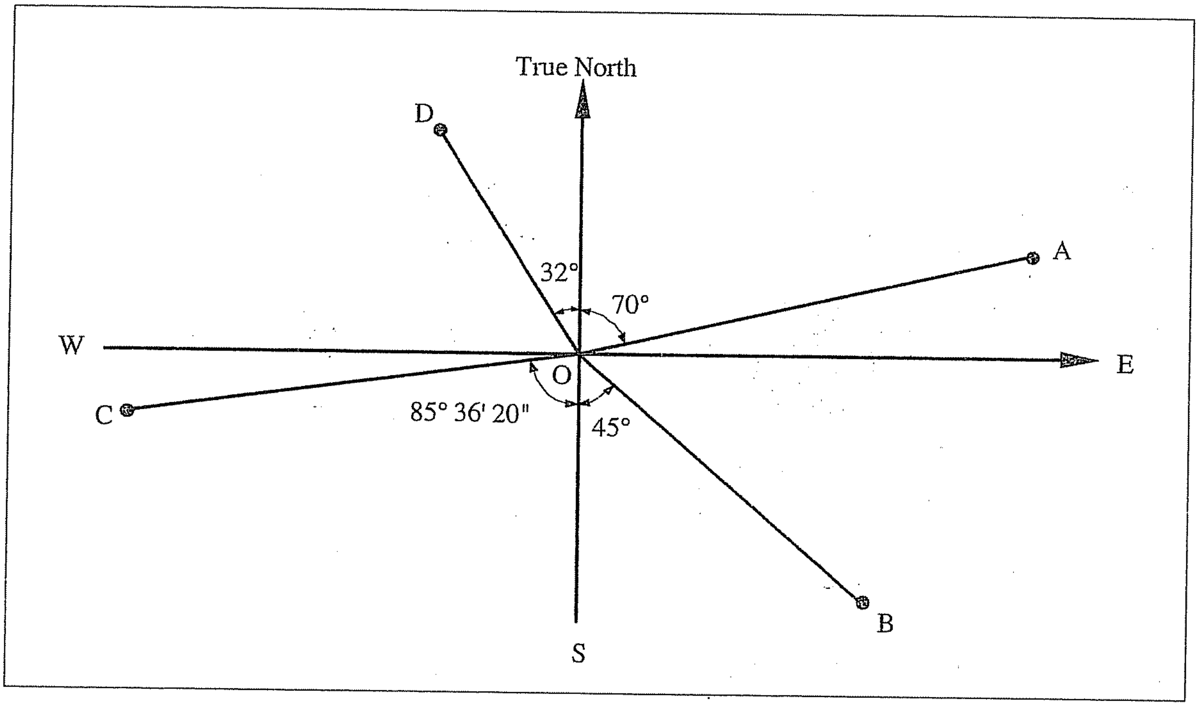


FIGURE 5.8: Reduced bearings.

The magnetic reduced bearing of a line is the acute angle that the line makes with the magnetic meridian. In Figure 5.9, the magnetic reduced bearings of lines OE, OF, OG and OH are: N 40° E, S 63° E, S 81° W and N 47° W, respectively. Since the magnetic meridian is 3° east of the true meridian, the corresponding true reduced bearings for lines OE, OF, OG and OH are: N 43° E, S 60° E, S 84° W and N 44° W, respectively.

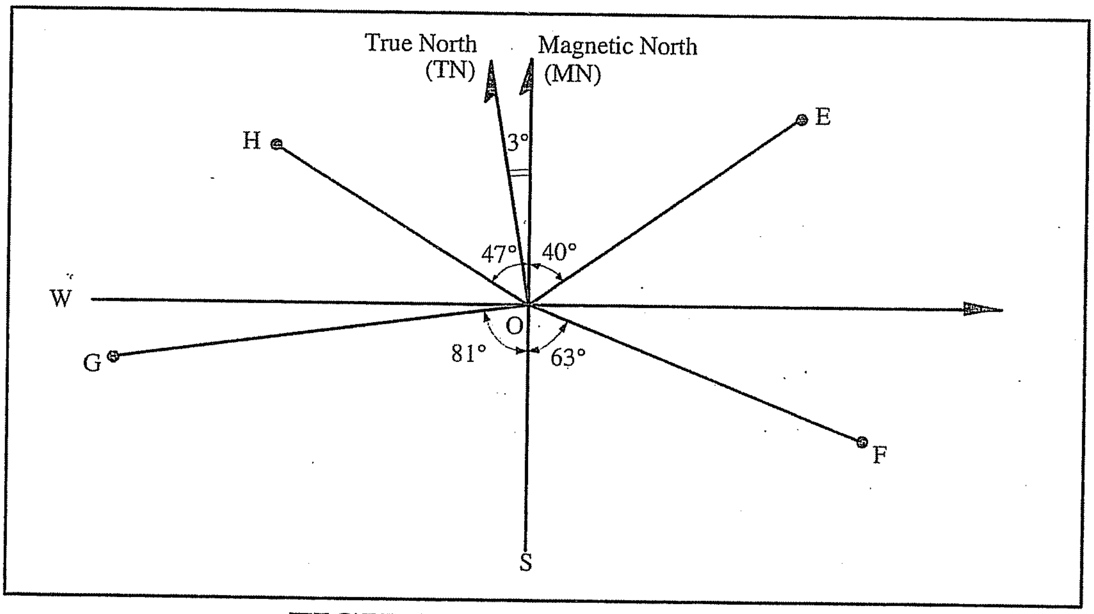


FIGURE 5.9: Magnetic bearings.

5.5 AZIMUTH OR WHOLE CIRCLE BEARING

The direction of a line may also be expressed by its azimuth. The azimuth of a line is the clockwise horizontal angle that the line makes with the north end of the reference meridian. For example, the azimuths of lines OA, OB, OC and OD in Figure 5.10 are: 70°, 135°, 265° 36' 20" and 328° respectively relative to the true north.

It is obvious from the previous values that the magnitude of an azimuth ranges from 0° to 360°. Sometimes azimuths are referred to the south (especially in the southern hemisphere) but all azimuths in a surveying project should refer to the same end of the meridian. In this book, the term azimuth will refer to the north end of the meridian.

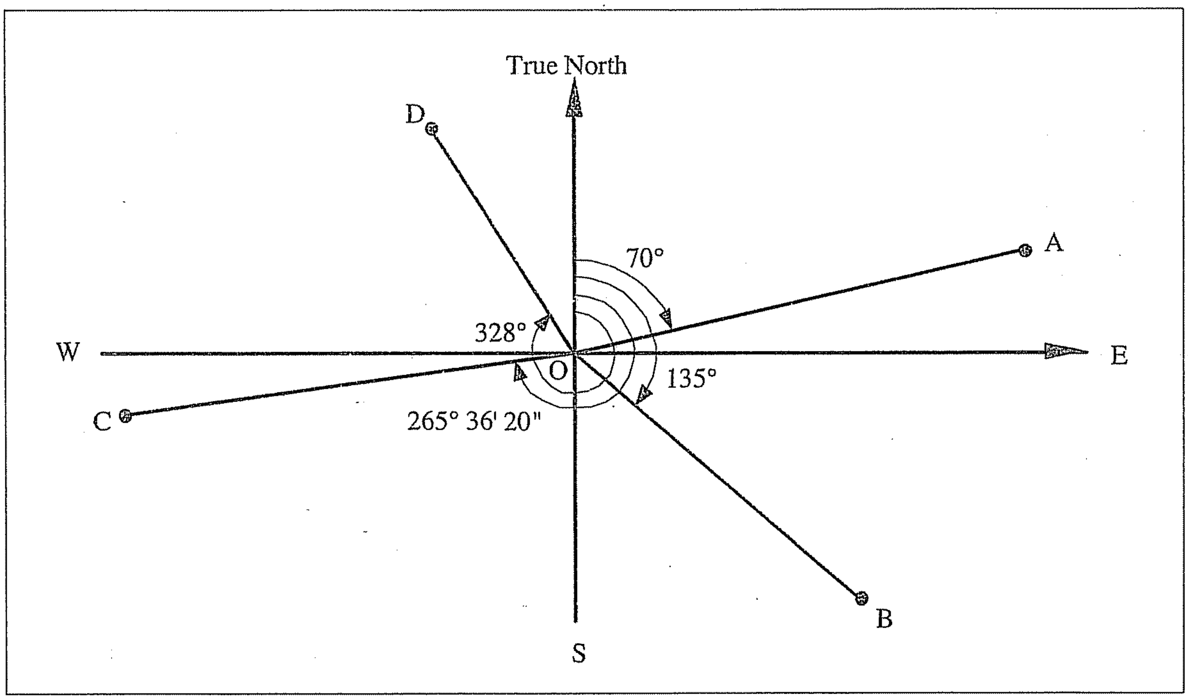


FIGURE 5.10: Azimuth of a line.

5.6 BACK REDUCED BEARING AND BACK AZIMUTH

The back reduced bearing of a line OA is the reduced bearing of the same line going from A to O. In Figure 5.11, the true reduced bearing of line OA going from O to A is N 60° E. The back reduced bearing of the same line

OA (but going from A to O) is S $(60° + \theta)$ W, where θ is an angle due to the convergence of the meridians. The magnitude of θ depends on the east-west distance between the two end points (A and O) and the average latitude along the line. For short lines, θ will be small and can be safely ignored. Similarly, the back azimuth of line OA in Figure 5.11 is $60° + 180° + \theta = 240° + \theta$.

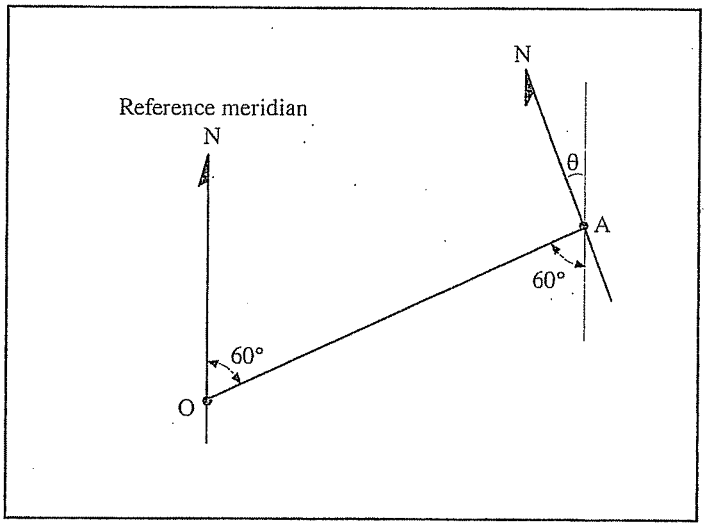


FIGURE 5.11: Back bearing.

5.7 PRINCIPAL ELEMENTS OF AN ANGLE-MEASURING INSTRUMENT

All angle-measuring instruments have the following basic elements:

1. A line of sight,
2. A horizontal axis about which the line of sight revolves,
3. A vertical axis about which the line of sight can be rotated,
4. A graduated horizontal circle for measuring horizontal angles, and
5. A graduated vertical circle for measuring vertical angles.

These five basic elements are illustrated in Figure 5.12. When the instrument is in perfect adjustment and ready for use, the following conditions exist:

1. The line of sight is perpendicular to the horizontal axis.
2. The horizontal axis is perpendicular to the vertical axis.
3. The horizontal axis is perpendicular to the vertical circle.
4. The vertical axis is perpendicular to the horizontal circle.

Figure 5.13 shows an example of a scale-reading (manual) theodolite, while Figure 5.14 shows an example of a digital theodolite.

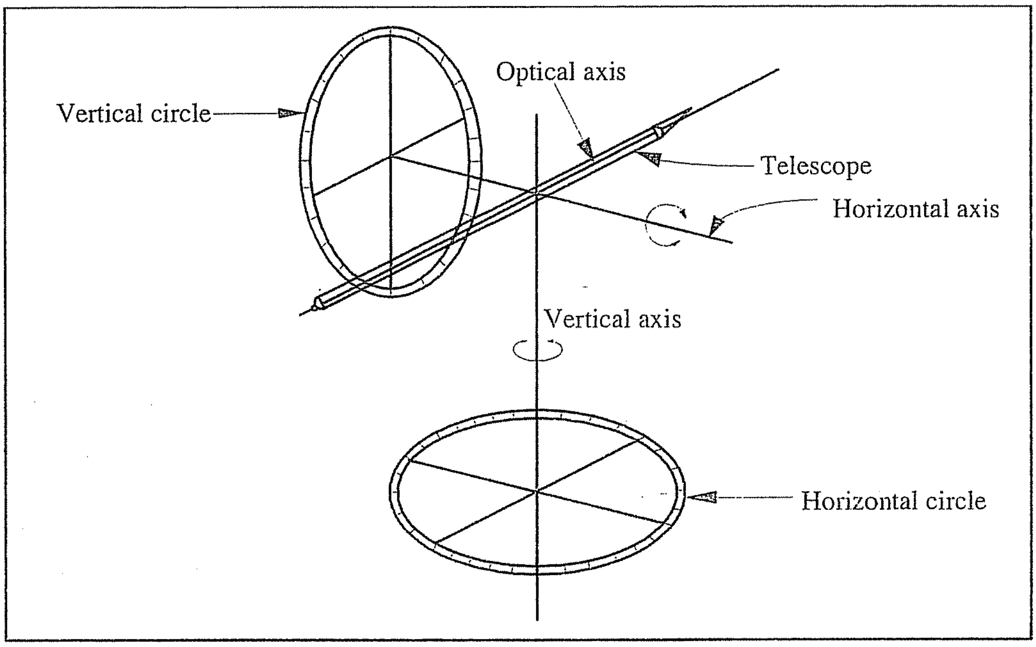


FIGURE 5.12: Principal elements of an angle measuring instrument.

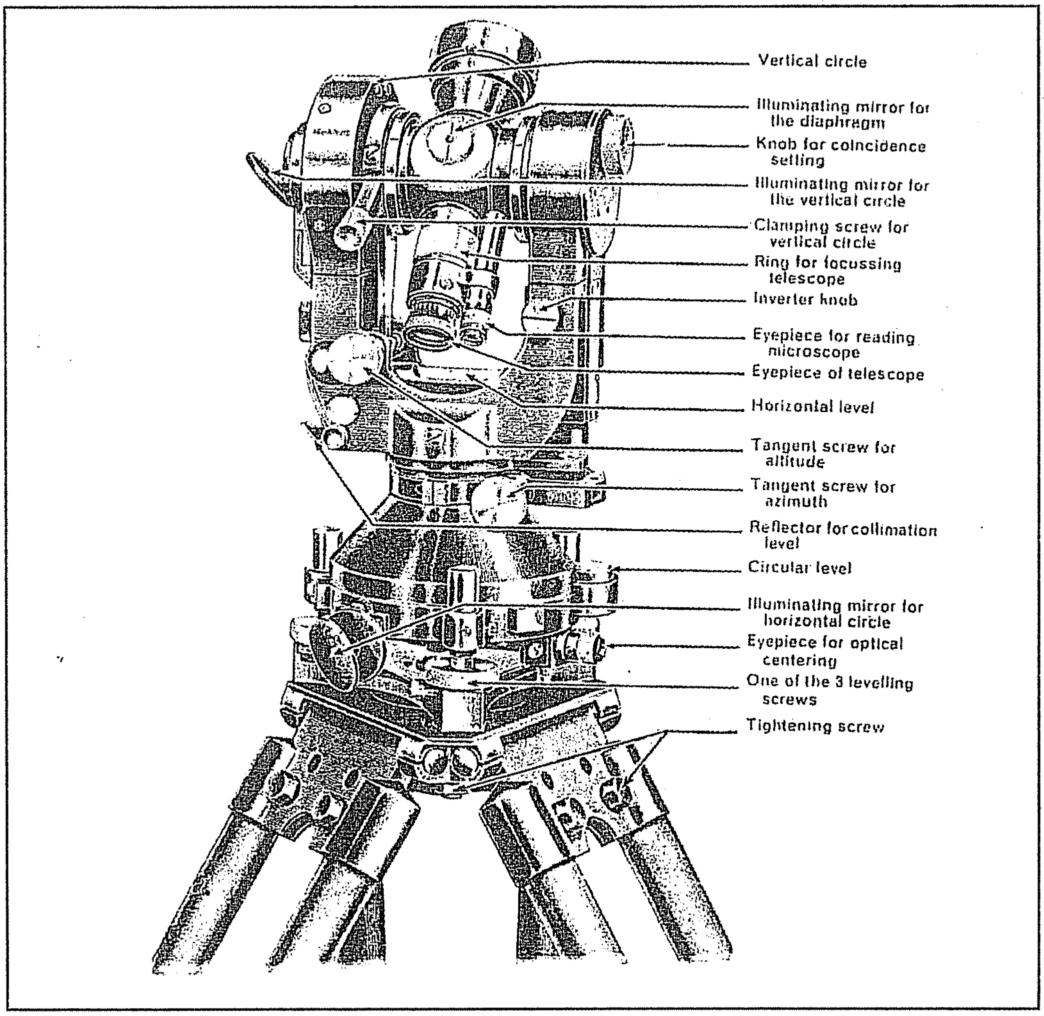


FIGURE 5.13: An example of a scale-reading (manual) Wild-T2 theodolite.

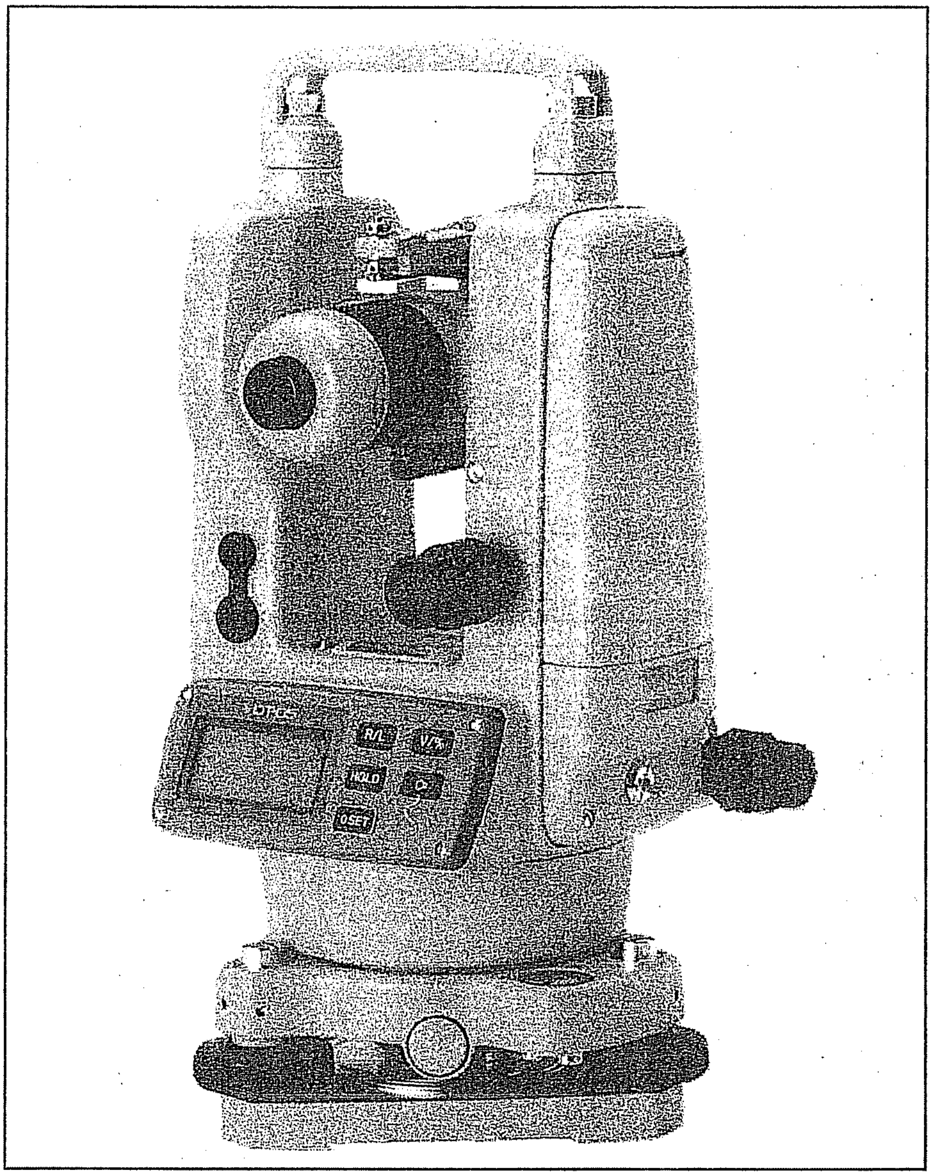


FIGURE 5.14: An example of a digital theodolite.

5.8 SETTING UP A THEODOLITE

A theodolite is usually equipped with an optical plummet for centering over a survey point, a bull's-eye bubble level for preliminary leveling, and one plate bubble for precise leveling of the instrument. The following procedure is used to set up a theodolite equipped with these features and an extension leg tripod:

1. Set the extension legs of the tripod to the proper length for the instrument operator.

2. Attach a plumb bob to the tripod head. Although an optical plummet is available with the instrument, it is usually more convenient to perform the preliminary centering over the survey point using a plumb bob.

3. Keeping the tripod head level, set the tripod approximately over the survey point. Step on the spur of each tripod leg to drive it firmly into the ground.

4. Center the plumb bob over the survey point by adjusting the length of the tripod legs. If the tripod head looks excessively off level, lift the tripod from the ground and repeat steps 3 and 4.

5. Mount the theodolite on the tripod, and level it with the bull's-eye bubble (see section 4.6).

6. If steps 3 and 4 have been performed properly, the survey point should be within the view of the optical plummet. The theodolite is then exactly centered over the survey point by loosening the fastening screw on the tripod head and moving the theodolite around slowly. During this operation the theodolite should always be held firmly by its standard with one hand. Once it is centered properly, the fastening screw is tightened firmly.

7. Check to see if the bull's-eye bubble is still centered. If it has moved off center, repeat steps 6 and 7.

8. Rotate the theodolite so that its plat bubble is parallel to a line joining any two of the foot screws, say screws A and B as shown in Figure 5.15a. The level bubble is centered using the same two foot screws.

9. The theodolite is rotated 180° so that the plate bubble assumes the position shown in Figure 5.15b. If the plate level is in perfect adjustment, the bubble should remain centered. If the bubble has

moved off center, it is brought halfway back towards the center using the same two foot screws (A and B).

10 Rotate the theodolite so that the plate bubble is now perpendicular to the line joining the two previous foot screws (A and B) (see Figure 5.15c). Center the bubble with the third foot screw (C) only.

11. Rotate the theodolite 180° so that the bubble assumes the position shown in Figure 5.15d. If the bubble has moved off center, bring it back halfway using the third foot screw (C).

12. Check the optical plummet to make sure that the instrument is still properly centered over the survey point. If not, repeat steps 6 to 12.

13. The theodolite is now ready for making angle measurements.

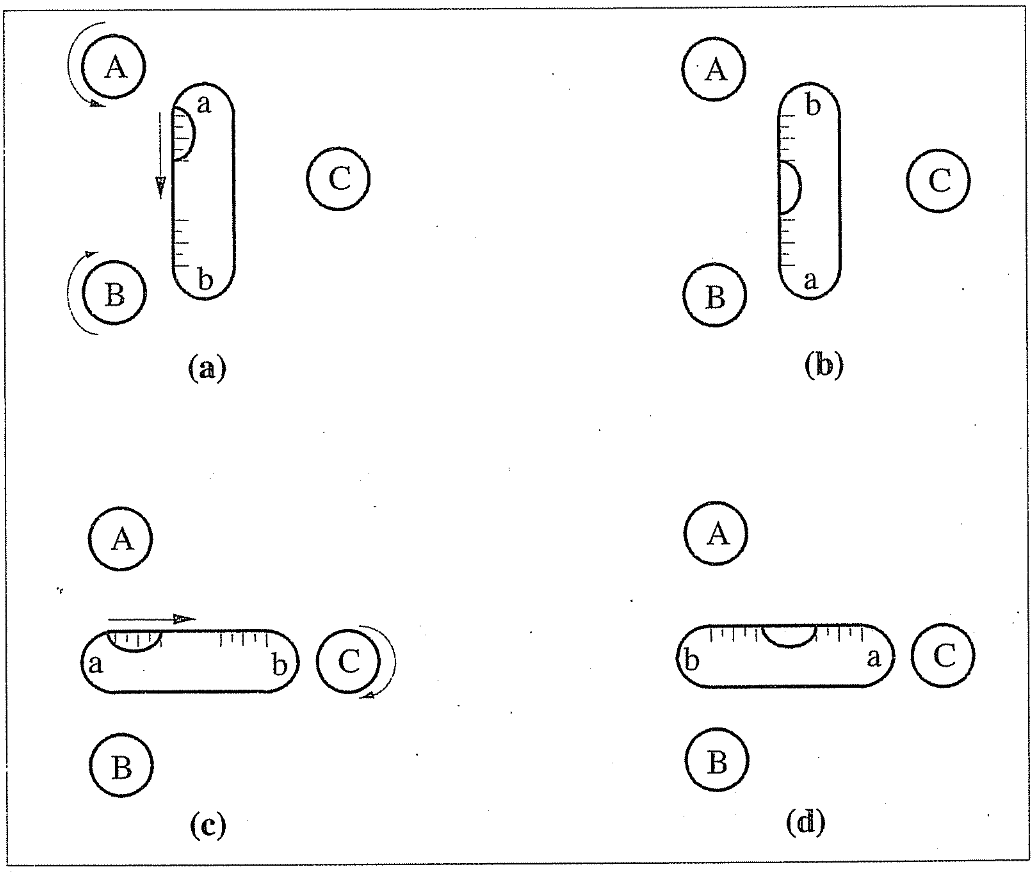


FIGURE 5.15: Leveling a three foot screw instrument.

5.9 MEASUREMENT OF A HORIZONTAL ANGLE

A horizontal angle $A\hat{B}C$ (Figure 5.16) can be simply measured in the field as follows:

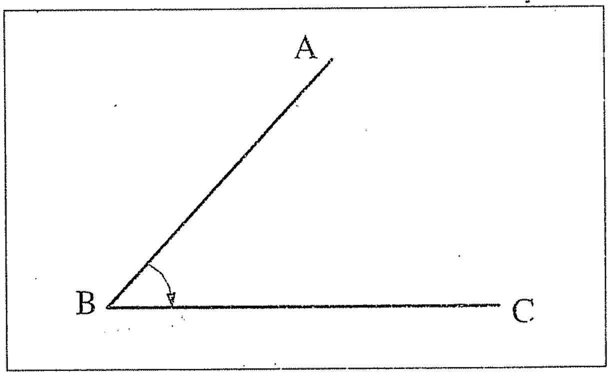


FIGURE 5.16: A horizontal angle.

1) Set up the theodolite over station B according to the steps given in section 5.8.

2) Direct the telescope of the theodolite to sight station A, and set the horizontal circle to read 0° 00' 00", or simply record the initial reading of the horizontal circle (e.g. 50° 20' 15").

3) Direct the telescope in a clockwise direction to sight station C and record the horizontal circle final reading (e.g. 120° 30' 47").

4) The value of the angle will be the difference between the final and the initial readings of the horizontal circle. For the above readings, the angle will be: 120° 30' 47" - 50° 20' 15" = 70° 10' 32".

However, it is possible that small errors will result due to imperfect leveling of the instrument or due to the maladjustment of the instrument when the three principal axes are not mutually perpendicular to each other. These errors can be minimized by using the following procedure for measuring the angle $A\hat{B}C$:

1) Set up the theodolite over station B.

2) With the theodolite in the direct (D) position, backsight to station A and set the horizontal circle to read 0° 00' 00".

3) Rotate the telescope in a clockwise direction and foresight to station C. Read and record the horizontal circle (e.g. 112° 50' 18").

4) Rotate the telescope through 180° in the vertical plane (i.e. about the horizontal axis), and then through 180° in the horizontal plane to sight on station C again. The horizontal circle is read and recorded as reverse (R) reading (e.g. 292° 50' 30"). This reading should differ from the reading obtained from step 3 by 180° plus or minus a few seconds.

5) Rotate the telescope in a counterclockwise direction and backsight to station A. Read and record the horizontal circle reading (e.g., 180° 00' 06").

6) Calculate the angle in both direct (D) and reverse (R) positions as follows:

Direct position: Angle = 112° 50' 18" - 0° 00' 00" = 112° 50' 18"

Reverse position: Angle = 292° 50' 30" - 180° 00' 06" = 112° 50' 24"

7) The value of the angle will be the average of the two values obtained in step 6, i.e.

$$\text{Angle} = \frac{112°\ 50'\ 18'' + 112°\ 50'\ 24''}{2} = 112°\ 50'\ 21''$$

<u>Note 1</u>: In some books, the terms face-right and face-left are used instead of direct and reverse positions

<u>Note 2</u>: When measuring a horizontal angle such as the angle $A\hat{B}C$, it is necessary that the theodolite be stationed exactly over point B. If, however, the theodolite were off station B by a small distance (say positioned at B' in Figure 5.17), the horizontal angle $A\hat{B}'C$ would be mistakably measured instead of angle $A\hat{B}C$.

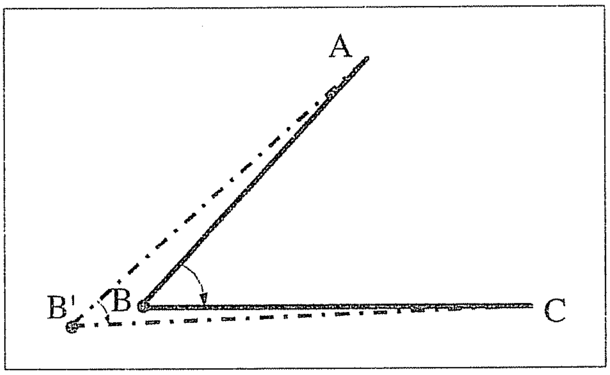


FIGURE 5.17: A wrong setup of the theodolite over station B.

5.10 MAIN APPLICATIONS OF THE THEODOLITE

As explained in the previous sections, the main use of the theodolite is to measure horizontal and vertical angles. These angles, in turn, are used for the calculation of object heights, distances, as well as, point coordinates. The following sections will deal with the main applications of the theodolite. Other applications will be dealt with in the next two chapters.

5.10.1 MEASUREMENT OF OBJECT HEIGHTS

The vertical or zenith angles measured with the theodolite can be used in conjunction with a distance measuring equipment for the measurement of object heights. Two cases can be distinguished here:

1) The determination of the elevation of a point whose horizontal distance from the theodolite can be directly measured.

2) The *determination* of the elevation of a point whose horizontal distance from the theodolite is difficult to be directly measured.

CASE (1): Points whose horizontal distance from the theodolite is directly measured

Assume that in Figure 5.18, the elevation of point C is to be determined by measuring the vertical angle α and the horizontal distance D. Given that the elevation of point A is known (H_A), and the height of the theodolite above A is (i), then the elevation of point C (H_C) is:

$$H_C = H_A + i + D \cdot \tan\alpha \qquad \text{.........................(5.1)}$$

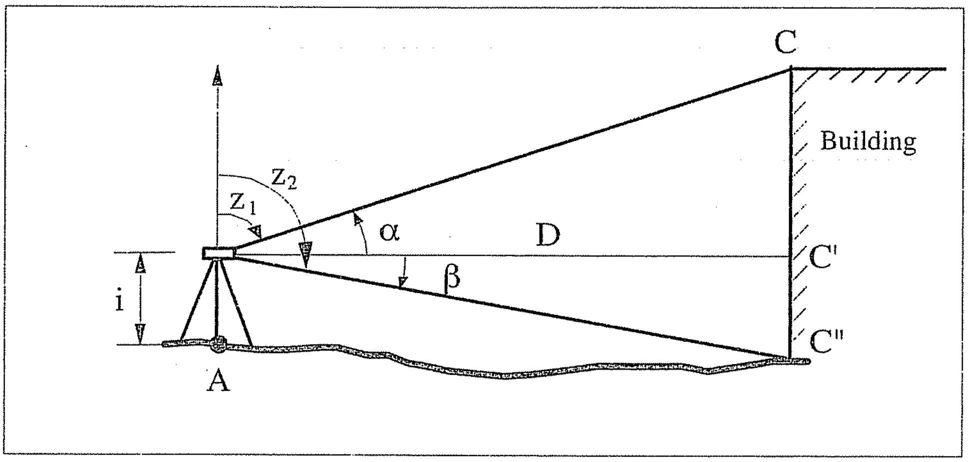


FIGURE 5.18: An object whose horizontal distance (D) from the theodolite can be measured directly.

If point C and C" lie on a vertical line, such as being a vertical edge of a building, then the height difference (ΔH) between these two points can be calculated as follows:

$$\Delta H = D \cdot \tan\alpha - D \cdot \tan\beta = D(\tan\alpha - \tan\beta)$$

$$= D\left(\frac{1}{\tan z_1} - \frac{1}{\tan z_2}\right) \qquad \text{.............(5.2)}$$

Where α is the vertical angle to point C

β is the vertical angle to point C". (*Notice that β is negative in Figure 5.18*),

z_1 and z_2 are the zenith angles to points C and C" respectively.

CASE (2): Points whose horizontal distance from the theodolite is difficult to measure

Assume that the object whose elevation is to be determined is the top of a minaret (C). In this case, it is difficult to measure the horizontal distance D between the theodolite and the top of the minaret. To overcome this problem, the following procedure is used (Figure 5.19):

1) Choose two nearby points A and B such that the horizontal triangle A'B'C' is approximately equilateral.

2) Using a distance measuring equipment such as a tape, measure the horizontal distance AB.

3) Using the theodolite, measure the two horizontal angles a and b. The angle c is then calculated as follows: c = 180° - a - b

4) From the sine law, calculate the horizontal distances A'C' and B'C':

$$\frac{A'C'}{\sin b} = \frac{B'C'}{\sin a} = \frac{A'B'}{\sin c}$$

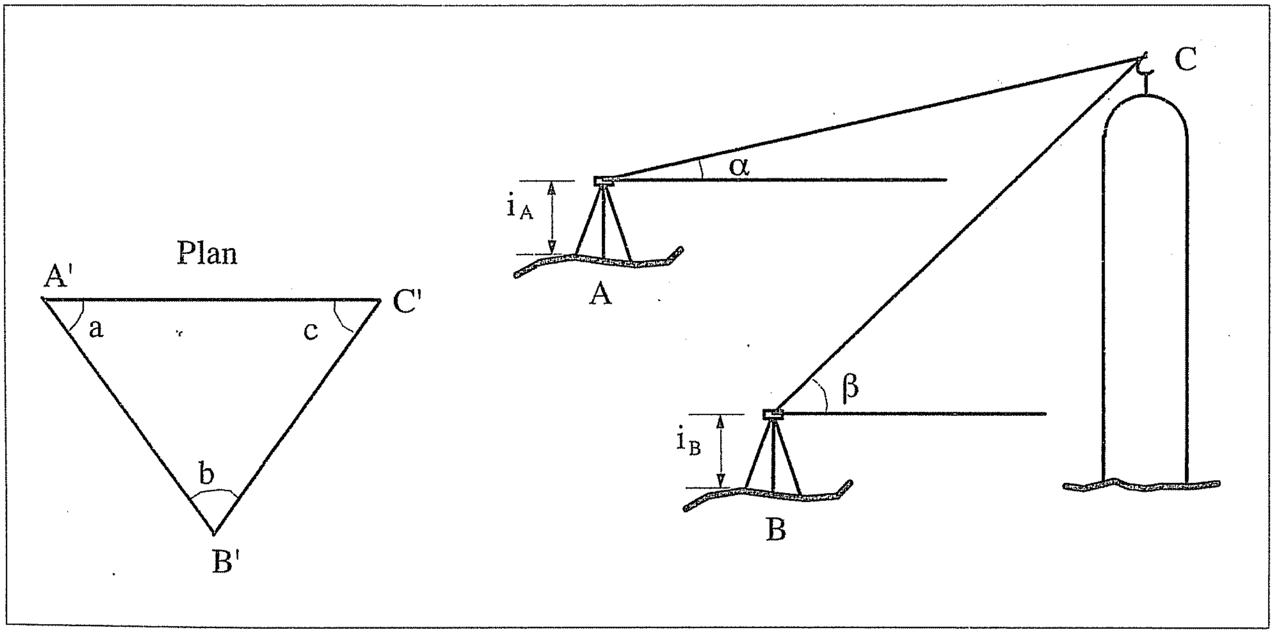


FIGURE 5.19: An object whose horizontal distance from the theodolite is difficult to be measured directly.

$\Rightarrow$

$$A'C' = \frac{A'B'}{\sin c} \cdot \sin b$$
$$B'C' = \frac{A'B'}{\sin c} \cdot \sin a \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.3)$$

5) Measure the two vertical angles α and β as shown in Figure 5.19 (z_A and z_B if the theodolite measures zenith angles instead of vertical angles).

6) The elevation of point C (H_C) can be calculated as follows:

$$H_C = H_A + i_A + A'C' \cdot \tan\alpha = H_A + i_A + \frac{A'C'}{\tan z_A} \qquad \ldots\ldots (5.4)$$

Or

$$H_C = H_B + i_B + B'C' \cdot \tan\beta = H_B + i_B + \frac{B'C'}{\tan z_B} \qquad \ldots\ldots (5.5)$$

5.10.2 TACHEOMETRY

In this branch of surveying, distances and elevation differences are determined from instrumental readings alone, these usually being taken with a specially adapted theodolite. The chaining operation is eliminated, and tacheometry is therefore very useful in broken terrain, e.g. river valleys, standing crops, etc., where direct linear measurements would be difficult and inaccurate.

There are several methods of tacheometry that include: the tangential method, stadia method, subtense bar method and the optical wedge method. Only the tangential and the stadia methods will be discussed here.

5.10.2.1 TANGENTIAL METHOD

In this method, both horizontal distances, as well as, elevation differences can be measured (Figure 5.20). To do this, the theodolite is set up at point A and two vertical angles θ and ϕ are measured on a staff held vertically at B, at two points such M and N respectively. Now, from Figure 5.20:

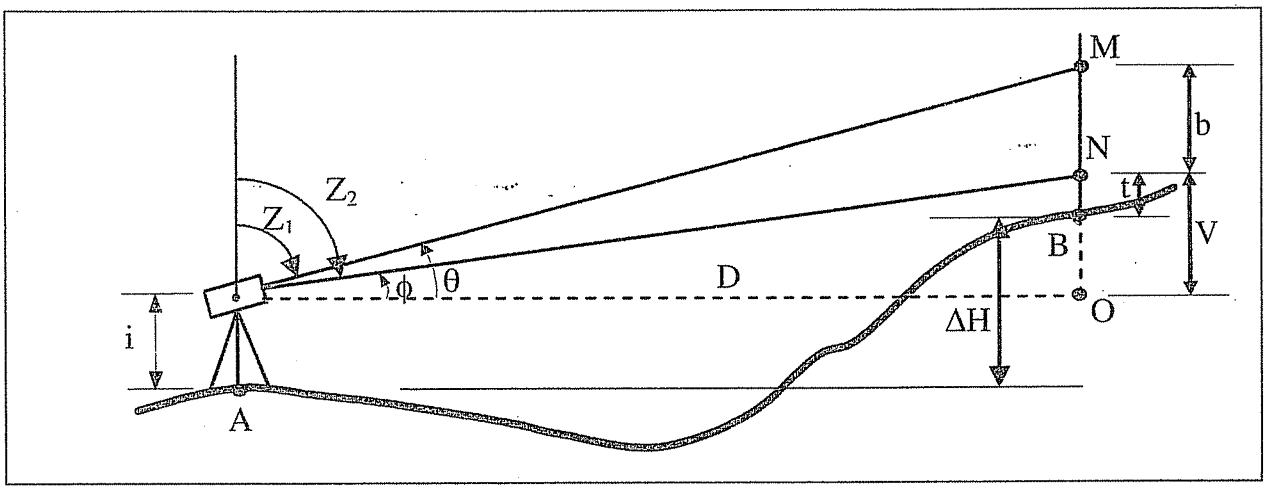


FIGURE 5.20: Tangential method.

$$OM = D \cdot \tan\ \theta$$
$$ON = D \cdot \tan\ \phi$$
$$OM - ON = b = D(\tan \theta\ -\ \tan \phi)$$

$$\Rightarrow \quad D = \frac{b}{(\tan \theta - \tan \phi)} = \frac{b}{\left(\frac{1}{\tan z_1} - \frac{1}{\tan z_2}\right)} \qquad \text{.................... (5.6)}$$

Where D is the horizontal distance between points A and B
b is the difference between the two staff readings

Also from Figure 5.20, the elevation difference (ΔH) between points A and B is calculated as follows:

$$\Delta H = i + V - BN = i + D \cdot \tan \phi - t \qquad \text{.................... (5.7)}$$

Where i is the theodolite height over A
t is the staff reading at N

EXAMPLE 5.1:

The following readings were taken on a staff held vertically at point B.

Vertical Angle	Staff Reading
6° 15' 20" ± 5"	3.50 ± 0.005 m
5° 10' 45" ± 5"	1.00 ± 0.005 m

If you know that the theodolite is 1.65 m above A,

(a) Calculate the horizontal distance and elevation difference between points A and B, as well as, their standard errors.
(b) Do you recommend the tangential method for precise surveying, and why?

SOLUTION:

(a) b $= 3.50 - 1.00 = 2.50$ m

$$D = \frac{b}{(\tan\theta - \tan\phi)} = \frac{2.50}{\tan(6°\ 15'\ 20") - \tan(5°\ 10'\ 45")} = 131.75 \text{ m}$$

$$\Delta H = i + D \cdot \tan\phi - t$$

$$= 1.65 + 131.75 \times \tan(5°\ 10'\ 45") - 1.00 = 12.59 \text{ m}$$

From the Law of propagation of random errors:

$$\sigma_b^2 = (0.005)^2 + (0.005)^2 = 0.00005$$

For small angles θ and ϕ:

$$D = \frac{b}{(\tan\theta - \tan\phi)} \approx \frac{b}{\theta - \phi}$$, where θ and ϕ are in radian,

$$\sigma_D^2 = \left(\frac{\partial D}{\partial b}\right)^2 \cdot \sigma_b^2 + \left(\frac{\partial D}{\partial \theta}\right)^2 \cdot \sigma_\theta^2 + \left(\frac{\partial D}{\partial \phi}\right)^2 \cdot \sigma_\phi^2$$

$$= \left(\frac{1}{\theta - \phi}\right)^2 \cdot \sigma_b^2 + \left(\frac{-b}{(\theta - \phi)^2}\right)^2 \cdot \sigma_\theta^2 + \left(\frac{b}{(\theta - \phi)^2}\right)^2 \cdot \sigma_\phi^2$$

Substitute $\theta = 0.10918$ radian, $\phi = 0.09039$ radian, $b = 2.50$ m,
$\sigma_b^2 = 0.00005$ and $\sigma_\theta = \sigma_\phi = 2.424 \times 10^{-5}$ radian

$$\Rightarrow \sigma_D^2 = 0.2005 \text{ m}^2$$

$$\Rightarrow \sigma_D = \pm\sqrt{0.2005} = \pm 0.45 \text{ m}$$

$$\sigma_{\Delta h}^2 = \sigma_i^2 + \left(\tan\phi\right)^2 \sigma_D^2 + \left(D \cdot \sec^2\phi\right)^2 \sigma_\phi^2 + \sigma_t^2$$

Consider $\sigma_i = 0.0$,

$\Rightarrow$

$$\sigma_{\Delta h}^2 = \left(\tan\left(5° \ 10' \ 45''\right)\right)^2 \cdot \left(0.2005\right) + \left(131.75 \cdot \sec^2\left(5° \ 10' \ 45''\right)\right)^2 \cdot \left(2.424 \times 10^{-5}\right)^2 + \left(0.005\right)^2$$

$$\Rightarrow \sigma_{\Delta h} = \pm 0.04 \text{ m}$$

<u>Final results</u>:

Horizontal distance = D = 131.75 ± 0.45 m

Elevation difference = ΔH = 12.59 ± 0.04 m

(b) From part (a), we notice the high values of σ_D and $\sigma_{\Delta h}$ which makes the tangential method not suitable for precise surveying. In general, tacheometry gives rapid results and is easy to do, but does not give highly accurate results. It is generally used for topographic mapping.

5.10.2.2 STADIA METHOD

For this method, a theodolite equipped with a rectile that has one vertical and three horizontal cross-wires (Figure 5.21) should be available. The upper and lower horizontal cross-wires are called stadia wires.

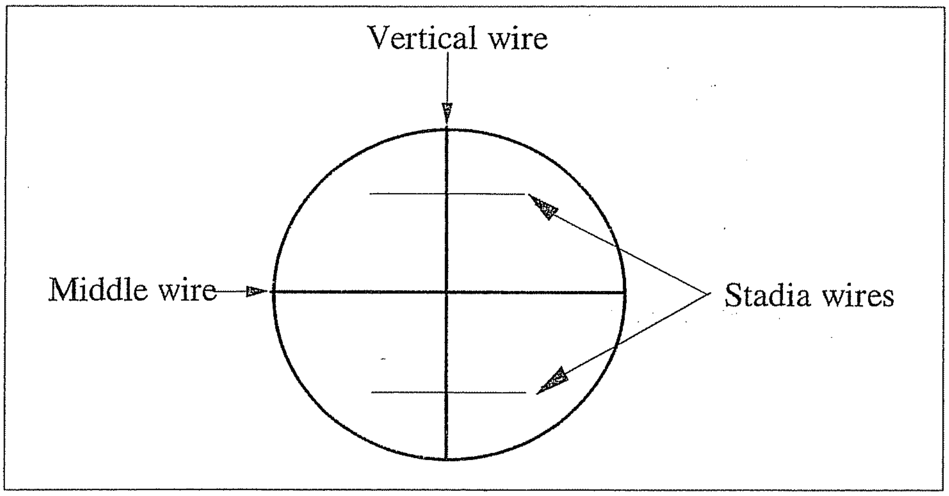


FIGURE 5.21: Stadia wires.

Figure 5.22 illustrates the method. The theodolite is positioned at A and sighted on the rod held at B. The vertical angle θ to the middle hair is recorded. The graduated rod (staff) is also read at all three horizontal cross hairs. The difference between the upper and lower cross-wire readings is called the interval or intercept (r). Its magnitude is a function of the distance between the rod and the theodolite. Thus, the horizontal distance (D) can be computed from this interval. In addition, from the middle cross-wire reading, the difference in elevation between points A and B can be computed by the equations developed for trigonometric leveling.

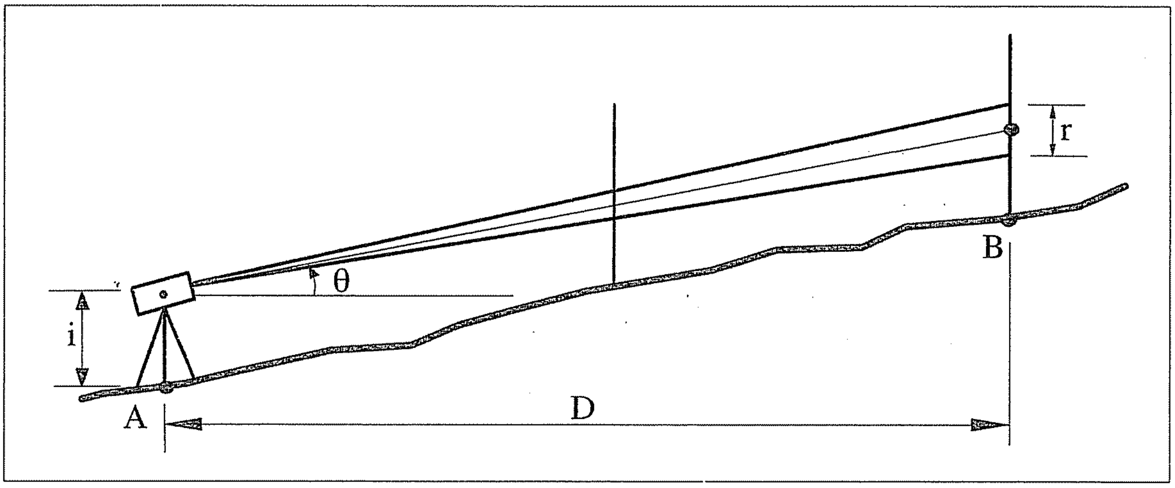


FIGURE 5.22: Stadia method.

Stadia Geometry for Horizontal Sight:

The theory of the stadia method will be first considered for the case of a horizontal sight with an external-focusing telescope. Figure 5.23 illustrates a telescope, the plumb line, and a rod intercept (r). Two rays of vision are shown, namely those emanating from the objective lens and pass through a focal point at a distance F in front of the lens, and intersect the rod as shown.

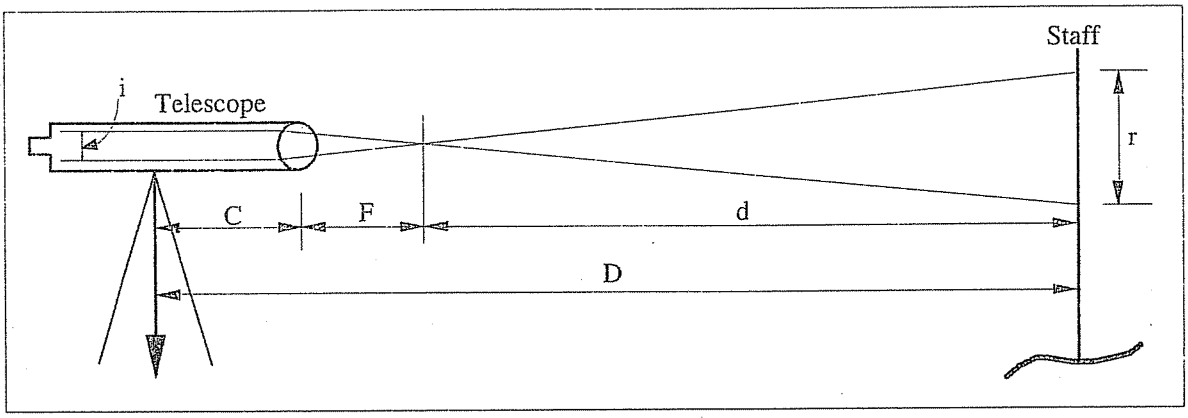


FIGURE 5.23: Stadia geometry for horizontal sight.

If the distance between the stadia cross-wires is represented by (i), then from the similar triangles:

$$\frac{d}{r} = \frac{F}{i} \quad \Rightarrow \quad d = \frac{F}{i} \cdot r$$

From Figure 5.19, the total distance from the staff to the plumb bob is:

$$D = \left(\frac{F}{i}\right) r + (F + C) \quad \text{.............................} \quad (5.8)$$

The distance F is the focal length of the lens and is a constant. Also the quantity (F + C) is practically a constant, since the distance C varies only by a negligible amount when the telescope is focused on different objects. For most instruments the value of (F + C) is about 0.25 - 0.3 m and it can be safely neglected.

Theodolites of recent design have internal focusing telescopes where (F+C) is a constant which can be disregarded under all conditions.

Since F and i are constants for any telescope, (F/i) is also a constant, say k. Then neglecting (F+C), Equation (5.8) may be simplified to:

$$D = k\,r \qquad \dots\dots (5.9)$$

The constant k is called the stadia coefficient and is equal to 100 for most instruments.

Stadia Geometry for Inclined Sight:

Refer to Figure 5.24 and suppose that the theodolite is in position at station A and the staff is held vertically at station B with an intercept = mn = r. The inclination of the line of sight is θ from the horizontal, and m'n' = r' is the intercept that would be read on the staff if it were held perpendicular to the line of sight.

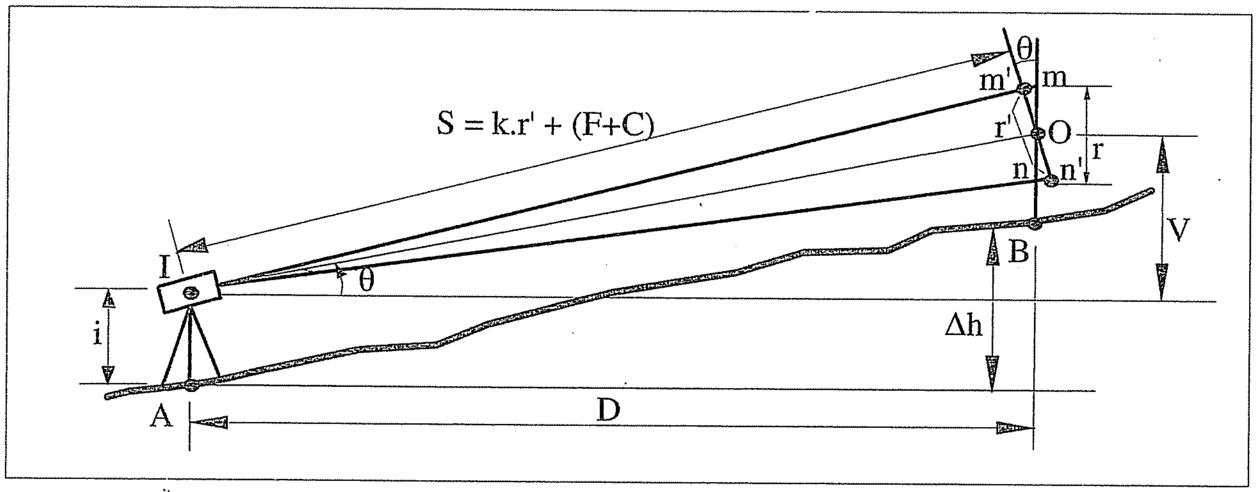


FIGURE 5.24: Stadia geometry for inclined sight.

$$\text{Angle } Om'm = \text{Angle } On'n \approx 90°$$

$$m'n' = mn \cos\theta \text{ , or } r' = r\cos\theta \qquad \dots\dots (5.10)$$

$$S = kr' + (F + C) \qquad \dots\dots (5.11)$$

Substituting Equation (5.10) into (5.11)

$$\Rightarrow \quad S = kr\cos\theta + (F + C) \qquad \text{(5.12)}$$

The difference in elevation between I and O is:

$$V = S\sin\theta = kr\sin\theta\cos\theta + (F + C)\sin\theta$$

From which:

$$V = \frac{1}{2}kr\sin 2\theta + (F + C)\sin\theta \qquad \text{(5.13)}$$

The horizontal distance D between A and B is:

$$D = S\cos\theta = kr\cos^2\theta + (F + C)\cos\theta \qquad \text{(5.14)}$$

From Figure 5.24, it is evident that the difference in elevation (Δh) between A and B is:

$$\Delta h = V + IA - OB$$

If distance OB is taken equal to IA, then, $\Delta h = V$. This is done by setting the middle cross-hair on the rod equal to the height of the instrument (HI).

For reasons stated in the previous section, (F + C) can be disregarded and equations (5.13) and (5.14) become:

$$V = \frac{1}{2}kr\sin 2\theta \qquad \text{(5.15)}$$

$$D = kr\cos^2\theta \qquad \text{(5.16)}$$

Since the zenith angle (z) = 90° - θ, Equations (5.15) and (5.16) may be written as:

$$V = \frac{1}{2}kr\sin 2z \qquad \text{(5.17)}$$

$$D = kr\sin^2 z \qquad \text{(5.18)}$$

When z = 90° (line of sight is horizontal)

$$\Rightarrow \quad V = \frac{1}{2}kr\sin(2 \times 90^\circ) = 0$$

$$D = kr\sin^2(90^\circ) = kr$$

which are the equations derived for the case of horizontal sight.

EXAMPLE 5.2:

The following readings were taken on a vertical staff with a theodolite having a constant k = 100 and F + C = 0.

Staff Station	Azimuth	Stadia Readings	Vertical Angle
A	27° 30'	1.000 1.515 2.025	+ 8° 00'
B	207° 30'	1.000 2.055 3.110	- 5° 00'

Calculate the mean slope between A and B.

SOLUTION:

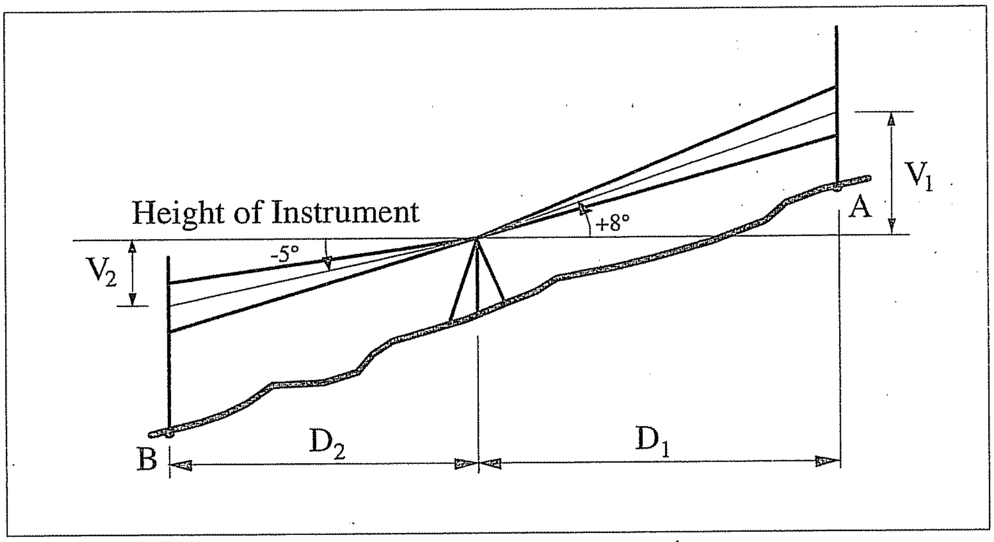


FIGURE 5.25

From Figure 5.25:

(1) Staff at Station A:
Staff intercept r = 2.025 - 1.000 = 1.025 m
Mid-reading = 1.515 m

$$D = kr\cos^2\theta$$

$$V = \frac{1}{2}kr\sin 2\theta$$

$$\Rightarrow D_1 = 100 \text{ x } 1.025 \text{ x } \cos^2(8°) = 100.515 \text{ m}$$

$$V_1 = \frac{1}{2} \times 100 \times 1.025 \times \sin(16°) = 14.126 \text{ m}$$

(2) Staff at Station B:

Staff intercept $r = 3.110 - 1.000 = 2.110$ m

Mid-reading = 2.055 m

$$\Rightarrow \; D_2 = 100 \times 2.110 \times \cos^2(-5°) = 209.397 \text{ m}$$

$$V_2 = \frac{1}{2} \times 100 \times 2.110 \times \sin(-10°) = -18.320 \text{ m}$$

Let h = height of instrument above datum, then

Elevation of point A = h + 14.126 - 1.515 = h + 12.611

Elevation of point B = h - 18.320 - 2.055 = h - 20.375

Elevation difference between B and A (ΔH_{BA}):

$(\Delta H_{BA}) = (h + 12.611) - (h - 20.375) = 32.986$ m

From a consideration of azimuths, it will be seen that A, B and the instrument lie on a straight line (207° 30' - 27° 30' = 180°), so that the mean slope =

$$\frac{\text{Elevation difference}}{D_1 + D_2} = \frac{32.986}{100.515 + 209.397}$$

$$= \frac{1}{9.4} = 1 \text{ in } 9.4$$

$$= 0.1064 = 10.64\%$$

PROBLEMS

5.1 Given that the azimuth of line AB with respect to the geographic north is 153°, and that the magnetic declination at this location is 2°E. Calculate:

- The true back azimuth of line AB.
- The true reduced bearing of line AB.
- The true back reduced bearing of line AB.
- The magnetic azimuth of line AB.
- The magnetic reduced bearing of line AB.

5.2 A horizontal angle was measured using a theodolite in both face right and face left positions. The readings were:

Face right: 0° 00′ 00″ , 45° 19′ 55″
Face left: 180° 00′ 02″ , 225° 20′ 03″

Calculate the accepted value of the angle.

5.3 You are standing on one side of a river (Figure 5.26) with a theodolite, tape and ranging rods. Using this equipment, show how to measure the distance between two points A and B located on the other side of the river without actually crossing to this side. Clarify with a sketch and show all the involved mathematics.

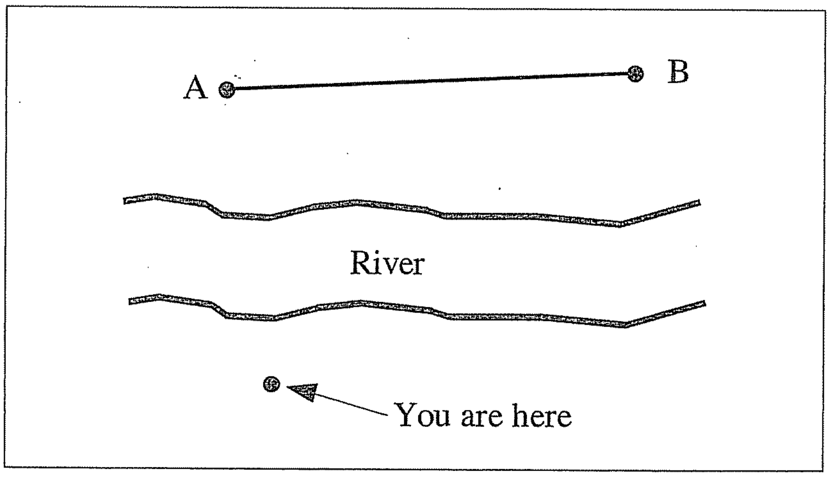


FIGURE 5.26

5.4 Using a theodolite and a tape measure, show how to make a parallel line such as CD to a given line AB and with a separation of 10 m between the two lines.

5.5 Using a theodolite, explain how to allocate on the ground the point of intersection of two intersecting lines AB and CD.

5.6 To measure the elevation of the top of a tower C, two points A and B were chosen near the tower, and the following measurements were made with a theodolite whose stadia coefficient is equal to 100 and (F+C = 0):

- With the theodolite at A, stadia readings at the staff held vertically at B are: 1.00, 1.52, 2.04 m. Vertical angle = +8°
- Height of instrument (i) at A = 1.65 m
- Horizontal angle CAB = 58° 16' 24"
- Horizontal angle ABC = 61° 22' 37"
- Vertical angle measured at A when the theodolite was directed towards C was +30° 18' 00"
- Elevation of point A = 970.34 m AMSL.

Calculate the elevation of the top of the tower C.

5.7 The following data were obtained during a tacheometric survey work using a theodolite whose constants are 100 & 0.

Station	Point	Vertical Circle Reading	Stadia Readings (m)	Instrument Height (m)
A	BM	- 5° 30'	2.15 ,1.95 ,1.75	1.40
A	B	+1° 30'	1.80 ,1.65 ,1.50	
B	C	+12° 00'	2.21 ,2.05 ,1.89	1.30

Knowing that the reduced level of the BM. = 500.00 m, Calculate:
a. The elevations of B & C.
b. Distances AB & BC.

5.8 The following readings were taken on a vertical staff with a theodolite having a constant k=100 and (F+C=0):

Staff Station	Azimuth	Stadia Readings	Zenith Angle
A	30° 20' 40"	1.000 1.515 2.025	82° 00'
B	140° 40' 20"	1.000 2.055 3.110	95° 00'

Calculate the mean slope between points A and B.

6 ELECTRONIC DISTANCE MEASUREMENT (EDM)

6.1 INTRODUCTION [1]

Since the measurement of long distances especially in hilly and mountainous areas with high accuracy (better than 1/20,000) is very difficult with the traditional methods using chain surveying (primarily with the invar tape), the need arose to develop a new system of measurement, which can achieve the required accuracy. Thus, the development of electronic distance measuring (EDM) instruments during the 1960's and 1970's, and later the global positioning systems (GPS) techniques, came to play this role.

This chapter will deal only with the EDM instruments, which can be classified into two types:

a) Electro-optical instruments which use the light and infrared waves as a measurement medium.

b) Microwave instruments, which use the radio waves as the medium.

[1] Most of the material in this chapter has been taken from reference 16 with some modifications.

6.2 ELECTRO-OPTICAL INSTRUMENTS

These instruments use:

a) Visible light for which the human eye is sensitive with wavelength of 0.4 to 0.7 µm.

b) Infrared light which is not visible with wavelength of 0.7 to 1.2 µm. This is used as the carrier wave by most of the newest short-distance EDM instruments.

The basic measurement principle for this instrument, which was first called GEODIMETER (from GEOdetic DIstance METER), is illustrated in Figure (6.1). Two pieces of equipment are placed at the ends of the line to be measured: the transmitter and the reflector. The transmitter generates a frequency modulated light beam and sends it to a reflector mounted on the other end of the line. The reflector, then, sends the light beam back to the transmitter, which in turn measures the travel time (t) of the beam both ways. The slope distance S is determined from the relation:

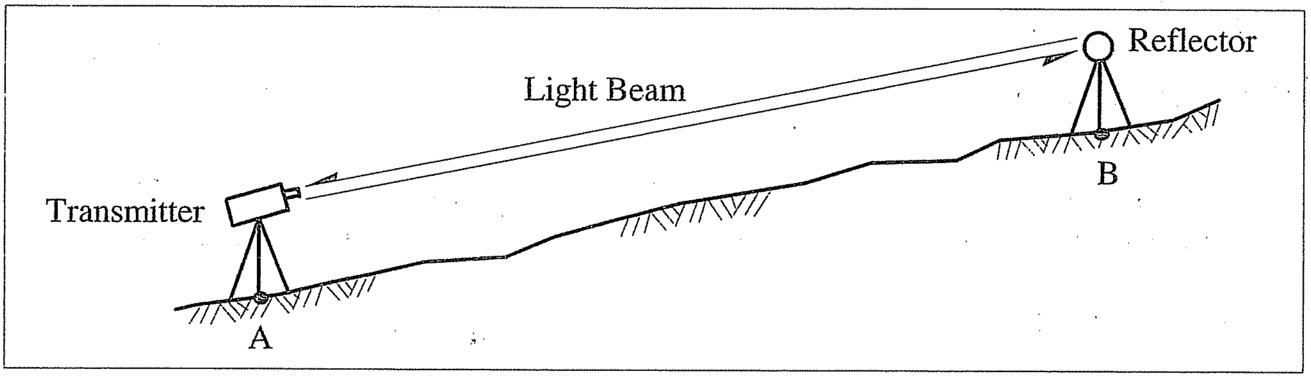


FIGURE 6.1: Measurement principle of an electro-optical EDM.

$$S = \frac{1}{2}Vt \qquad (6.1)$$

Where V is the velocity of light in the atmosphere.

The early models of the GEODIMETER used tungsten or mercury lamps as a light source. They were bulky, expensive, of low accuracy and measurements often had to be performed at night because of the weak light signal. However, developments in electronic engineering, mainly in the 1960's, made low cost electro-optical distance - measuring instruments commonly available.

A wide variety of electro-optical distance - measuring instruments is available in the market today. They differ in the range of distances they measure, accuracy, size, weight, operating temperature and cost. In the late 1970's, advances in electronics led to the manufacture of an instrument called ***Total Station***. This instrument can be used to measure the vertical and horizontal angles as well as distances from a single set up and positioning of the instrument.

6.3 MICROWAVE INSTRUMENTS

These instruments use radio waves, which have length between 10 μm and 100 μm. The basic measurement principle for these devices is as follows: two identical electronic units are mounted one at each end of the line to be measured. One unit serves as a master unit and the other as a remote unit (Figure 6.2). The master unit transmits a frequency - modulated radio wave to the remote unit, which transmits it back to the master unit that measures the double transit time (t). Using Equation (6.1), the distance can be calculated.

When compared to the electro-optical instruments, microwave instruments have several disadvantages that make them of very limited use:

1) Two operators are required to measure a line, one at each end.
2) In some applications, it is either impossible or inconvenient to set up a measuring unit at both ends of the line, especially if the end point of the line is a pole or an edge of a building.
3) The measurement accuracy is severely affected by changes in atmospheric conditions.
4) The accuracy can be degraded by multi-path reflections resulting in an error called ground swing.
5) In general, the accuracy of microwave instruments is lower than that of electro-optical instruments of similar cost.

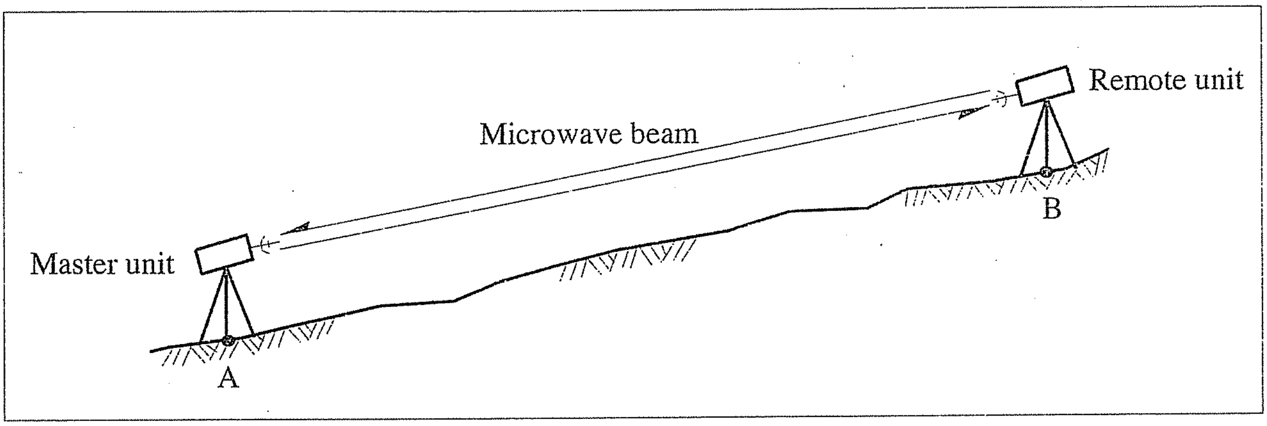


FIGURE 6.2: Measurement principle of a microwave EDM.

6.4 BASIC PRINCIPLE OF ELECTROMAGNETIC MEASUREMENTS

As was explained earlier, the signal from the transmitter to the reflector and back is transmitted in the form of waves. An important relationship in wave motion is:

$$\lambda = \frac{V}{f} \qquad \text{(6.2)}$$

Where λ = wavelength (distance traveled during the period of one cycle)
V = velocity of propagation ($V = 299792.5 \pm 0.4$ km/sec in a vacuum)
f = frequency in Hz (cycle/sec)

Since, in reality, it is difficult to measure the transit time of the light and microwave beams to a high degree of accuracy as was explained earlier, all of the EDM instruments measure time indirectly by measuring the phase difference between transmitted and returning waves. The term ***phase*** refers to a portion of a complete cycle of a wave (Figure 6.3). One complete cycle is represented by 360°, and one-quarter of a cycle would be 90°. ***Phase difference*** is the time in electrical degrees by which one wave leads or lags another.

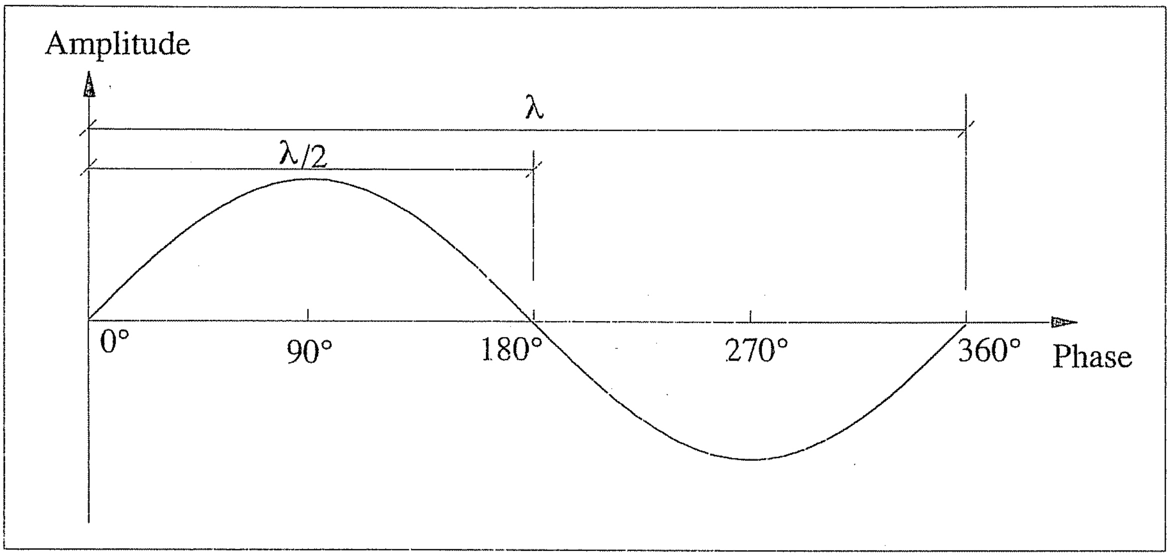


FIGURE 6.3: Phase angle of a sinusoidal wave.

<u>**For example**</u>, suppose that an electromagnetic wave (called the carrier wave) is caused to pulsate at a frequency of f MHz. Suppose also that at a given moment, the wave leaving the EDM instrument is measured to have a phase angle of 50° while the returning wave is measured to have a phase angle of 300 °, as shown in Figure 6.4. The phase difference (Δ) is then equal to (300° - 50°) = 250°. Figure 6.4 shows that the distance (S) between the transmitter and the reflector can be computed as follows:

$$S = \frac{1}{2}\left(n\lambda + \frac{\Delta}{360^\circ}\lambda\right) \qquad \text{...............................(6.3)}$$

Where: λ = wavelength

n = total number of full wavelengths

Δ = phase difference

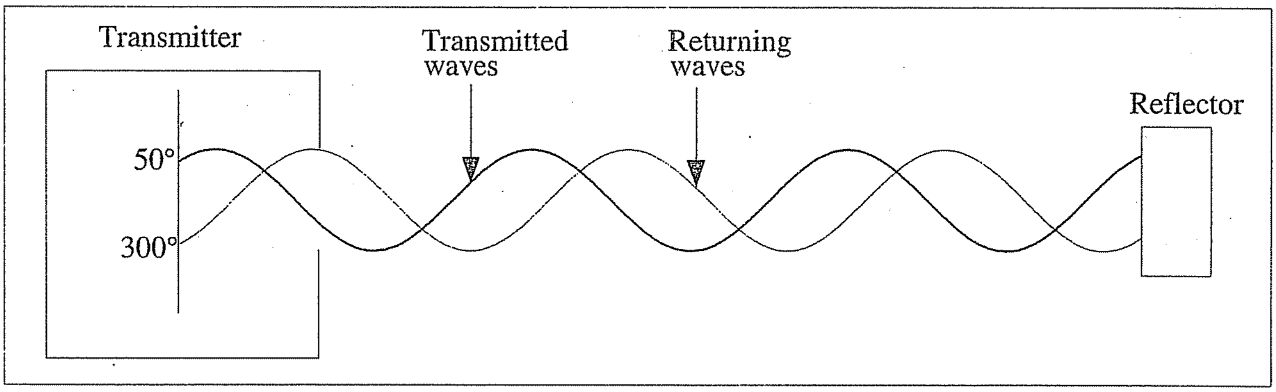


FIGURE 6.4: Measurement of phase difference.

The term $(\Delta/360°)\lambda$ represents the fractional wavelength. For the preceding example, with f = 14.989625 MHz, and taking the speed of light to be 299,792.5 km/sec:

$$\lambda = \frac{V}{f} = \frac{299{,}792{,}500 \text{ m/sec}}{14{,}989{,}625 \text{ cycles/sec}} = 20 \text{ m/cycle}$$

Using Equation (6.3) with a phase difference $(\Delta) = 250°$:

$$S = \frac{1}{2}\left[20n + \frac{250°}{360°} \cdot 20\right] \text{meters} = [10n + 6.944] \text{ meters}$$

Since n cannot be determined using a single pattern frequency, an EDM instrument measures a distance by using several pattern frequencies, which are multiples of 10 of each other. For example, assume that an EDM instrument measures the phase difference along a line using the four frequencies: F_1, F_2, F_3 and F_4 (Table 6.1). Suppose also that the measured phase differences are Δ_1, Δ_2, Δ_3 and Δ_4. The distance S can then be computed as follows:

TABLE 6.1: Calculation of a distance from several pattern frequencies.

Frequency	Measured Phase Difference	λ (m)	$\frac{1}{2}\frac{\Delta}{360°}\lambda$
F_1 = 14.989625 MHz	$\Delta_1 = 250°$	20	6.944 m
F_2 = 1.4989625 MHz	$\Delta_2 = 98°$	200	27 m
F_3 = 149.89625 KHz	$\Delta_3 = 190°$	2000	527 m
F_4 = 14.989625 KHz	$\Delta_4 = 91°$	20000	2527 m
S = 6.944 + 20 + 500 + 2000 = 2526.944 m			

Most EDM instruments available in the market nowadays perform the phase measurements automatically, switching from one frequency to another without operator interference, and display the measured distance directly in feet or meters.

6.5 INDEX OF REFRACTION

The ratio between the velocity of propagation of electromagnetic wave in a vacuum (V_o) and the velocity in the atmosphere (V) is called the index of refraction (N_a); that is:

$$N_a = \frac{V_o}{V} \quad \ldots\ldots (6.4)$$

N_a depends on the wavelength, atmospheric pressure, temperature and relative humidity. For dry atmosphere at 0°C and at sea level (pressure = 760 mm Hg), the refractive index for light waves is given by:

$$N_g = 1 + \left(287.604 + 4.886\,\lambda^{-2} + 0.068\,\lambda^{-4}\right)10^{-6} \quad \ldots\ldots (6.5)$$

Where: λ = wavelength of light in μm
= 0.9 - 0.93 μm for near infrared light from Gallium Arsenide diode
= 0.6328 μm for light generated by helium-neon laser

At other atmospheric conditions:

$$N_a = 1 + \frac{0.359474\,(N_g - 1)P}{273.2 + t} \quad \ldots\ldots (6.6)$$

Where: P = atmospheric pressure in mm Hg
t = air temperature in °C

Thus for λ = 0.9 μm, N_a = 1.000294 at sea level (760 mm Hg) and air temperature of 20°C. In EDM applications, N_a is often expressed as follows:

$$N_a = 1 + 10^{-6} N \quad \ldots\ldots (6.7)$$

Example: If N_a = 1.000294, then N = 294 ppm (part per million).

EDM instruments are designed to display the measured distance for a specific design value of N. The operator is usually given the provision to input manually any correction that is needed for the specific value of N. This correction is called atmospheric correction. For instance, if an EDM instrument uses a design value of N = 294 ppm while the actual N during the measurement is 291 ppm, then the atmospheric correction to be entered into the EDM before the measurement is -3 ppm.

6.6 TYPES OF MOUNTS

An EDM instrument can be mounted either on a tripod so that it may be operated as a unit standing alone, or on top of a theodolite.

The most commonly used types of mounts are:

1) As a stand-alone (separate) unit. Most long-range EDM (20 to 60 km) instruments are used in this mode (Figure 6.5a).
2) On top of the standards of a theodolite (Figure 6.5b)
3) On top of the telescope of a theodolite (Figure 6.5c)
4) Designed as an integral part of an instrument called total station (Figure 6.6).

6.7 RETRO-REFLECTORS

Reflectors used with electro-optical instruments are usually called retro-reflectors, because they are designed to reflect the light ray back along the same direction from which it came. Precisely ground trihedral prisms are used for this purpose. Reflectors can be single or of multi-prism assembly mounted on fixed or tilted housing (Figure 6.7).

The maximum distance that an electro-optical instrument can measure depends on:

1) The design of the instrument
2) The quality and number of prisms used
3) The atmospheric conditions.

In general, the longer the distance to be measured, the larger is the number of required prisms.

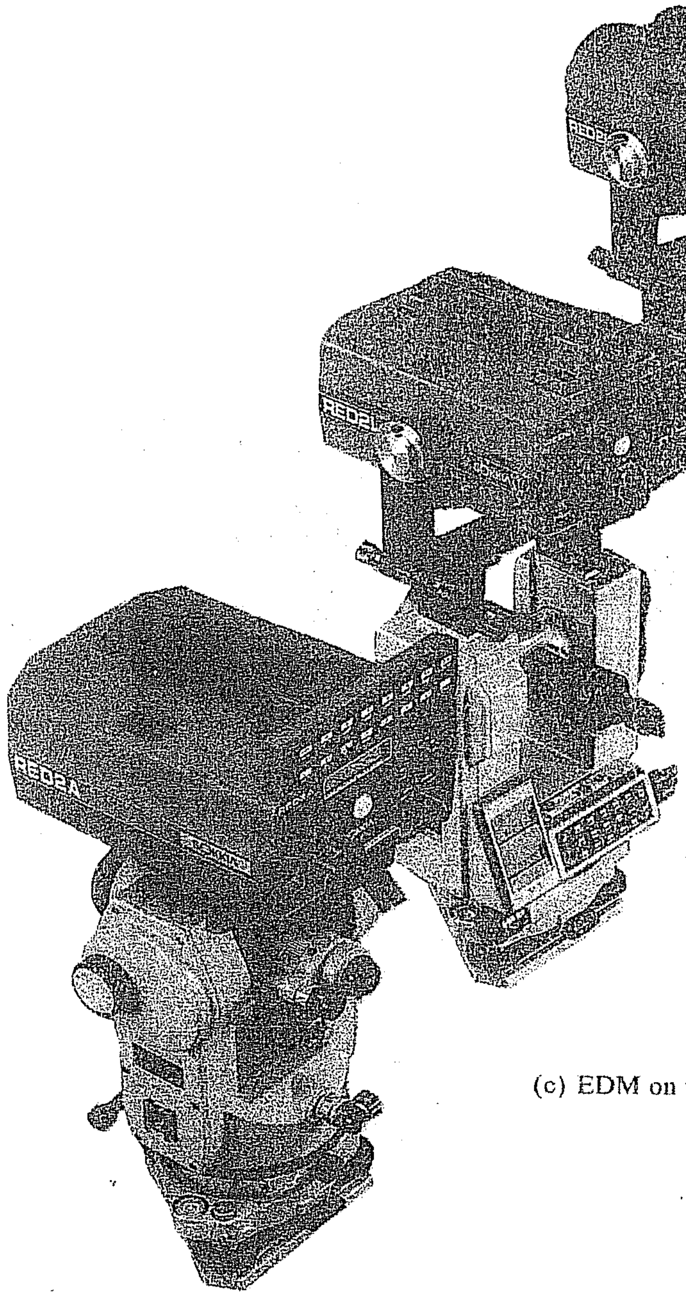


FIGURE 6.5: Examples of three different types of EDM mounts.

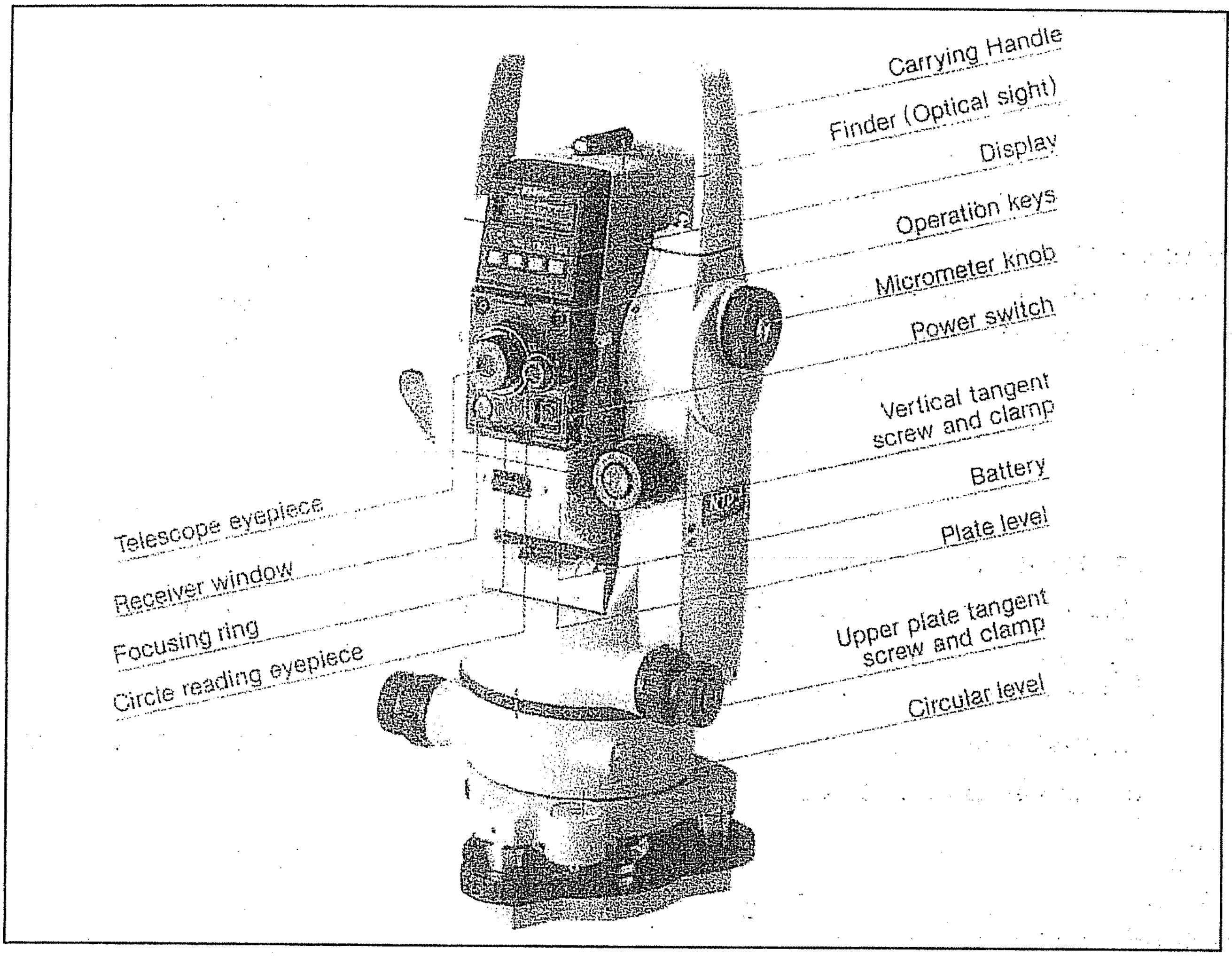


FIGURE 6.6: An example of an electronic total station.

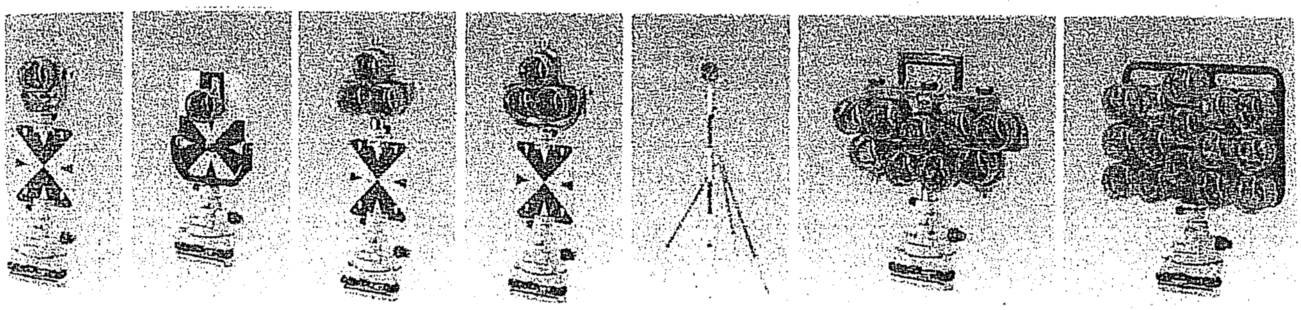


FIGURE 6.7: Examples of retro-reflectors.

6.8 OPERATING PROCEDURE

Usually, the measurement of a distance using an electro-optical instrument requires the following steps:

1) Set up the instrument over the survey station, and record its height (HI) above this station.
2) Set up the reflector at the other end of the line and record its height above the survey station.
3) Point the instrument towards the reflector by means of an alignment telescope.
4) Turn the power on.
5) Adjust the pointing using tangent screws to maximize the strength of the returning signal.
6) Read and record the field temperature and atmospheric pressure.
7) Dial in the atmospheric correction.
8) Press the measure button.
9) Record the distance displayed in numerical digits, either in feet or meters at the option of the operator.

6.9 SOURCES OF MEASUREMENT ERRORS

When using electro-optical instruments for distance measurement, the following sources of errors may occur:

(1) Eccentric error due to inexact centering of the instrument and reflector over the survey stations.
(2) Inexactness of the instrument in performing phase measurements.
(3) The zero point of the light ray used in phase measurement does not coincide exactly with the theoretical center of the instrument.
(4) The actual center of the reflector does not coincide with the theoretical center.
(5) The actual modulating frequencies differ from the theoretical values of these frequencies.
(6) The refraction index (N_a) is not constant through the line to be measured.

Sources (1) and (2) are random while (3) and (4) are constants. The magnitude of errors resulting from these four sources is independent of the length of the line being measured, while errors (5) and (6) are directly proportional to the length of the measured line. As a result of these types of errors, the manufacturers of EDM instruments provide the standard error for a measured distance in two parts:

1) A constant part that is independent of the length of the distance.

2) A variable part given as parts per million (ppm) of the measured distance.

Example: If an EDM has a measurement accuracy of ±(10mm + 4ppm), then a distance of 1843.56m measured with this instrument would have a standard deviation equal to $\pm(0.01 + 4 \times 10^{-6} \times 1843.56) = \pm 0.02$ m.

6.10 CALIBRATION PROCEDURES

Modern theodolites and automatic levels usually remain in excellent calibration for many years under normal use conditions. However, the situation is different with EDM instruments that need to be calibrated from time to time against systematic errors. This is usually done using calibration base lines with accurately known lengths.

Calibration procedures for determining the systematic errors due to zero centering and pattern frequency are performed as follows:.

1) Determination of corrections for zero centering:

The combined error for zero centering at the instrument and reflector is a constant error that appears in all the distances measured by that combination of instrument and reflector. To find the correction, a several hundred meters line AB is used with point C on the line as shown in Figure 6.8.

FIGURE 6.8: Layout for determination of correction for zero centering errors.

- Let the actual calibrated lengths be AB, AC & CB, and the measured lengths by the EDM instrument be $\overline{AB}$, $\overline{AC}$ & $\overline{CB}$.
- Let the unknown correction be (c).

Then: $AB = \overline{AB} + c$

$AC = \overline{AC} + c$

$CB = \overline{CB} + c$

But: $AB = AC + CB \Rightarrow \overline{AB} + c = \overline{AC} + \overline{CB} + 2c$

$\Rightarrow c = \overline{AB} - \overline{AC} - \overline{CB}$ ....(6.8)

EXAMPLE 6.1:

To determine the zero centering correction for an EDM, the following values for AB, AC and CB were measured by the EDM:

$\overline{AB} = 313.647$ m, $\overline{AC} = 112.556$ m, and $\overline{CB} = 201.088$ m.

(a) Find the correction for zero centering (c).

(b) A distance was recorded to be 718.128 m when using the same instrument. Compute the correct distance.

SOLUTION:

(a) $c = \overline{AB} - \overline{AC} - \overline{CB} = 313.647 - 112.556 - 201.088 = +0.003$ m

(b) Corrected distance = 718.128 + 0.003 = 718.131 m

2) Correction for error in modulation frequency:

If the theoretical frequency is f and the measured frequency is f ', then the correct distance should be equal to the measured distance multiplied by the scale factor (f '/f). To do this correction, follow one of two methods:

1 - Measure f ' in the laboratory, or

2 - If this is not possible, measure two known distances such as AB and AC in Figure 6.8. Then:

$$AB = g \cdot \overline{AB} + c \qquad \text{(6.9)}$$

$$AC = g \cdot \overline{AC} + C \qquad \text{(6.10)}$$

Where: AB and AC are the known distances,
$\overline{AB}$ and $\overline{AC}$ are the measured distances,
$g = f\,'/f$ = scale factor,
c = correction for zero centering

EXAMPLE 6.2:

The following data belongs to Figure 6.8:

AB = 1499.8635 m, AC = 149.9921 m

$\overline{AB}$ = 1499.9000 m, $\overline{AC}$ = 149.9935 m.

Determine g and c.

SOLUTION:

$$1499.8635 = 1499.9000\,g + c$$
$$149.9921 = 149.9935\,g + c$$

Solving these two equations for g and c: $\Rightarrow$ $g = 0.999974$
$c = +0.0025$ m

Note: When using (g) later for correcting a measured distance, it should be used as it is without approximating it to 1.

6.11 THE EFFECT OF EARTH CURVATURE ON THE REDUCED HORIZONTAL DISTANCES

For short distances measured with an EDM instrument, the effect of atmosphere and curvature of the earth can be ignored. From Figure 6.9, it is clear that:

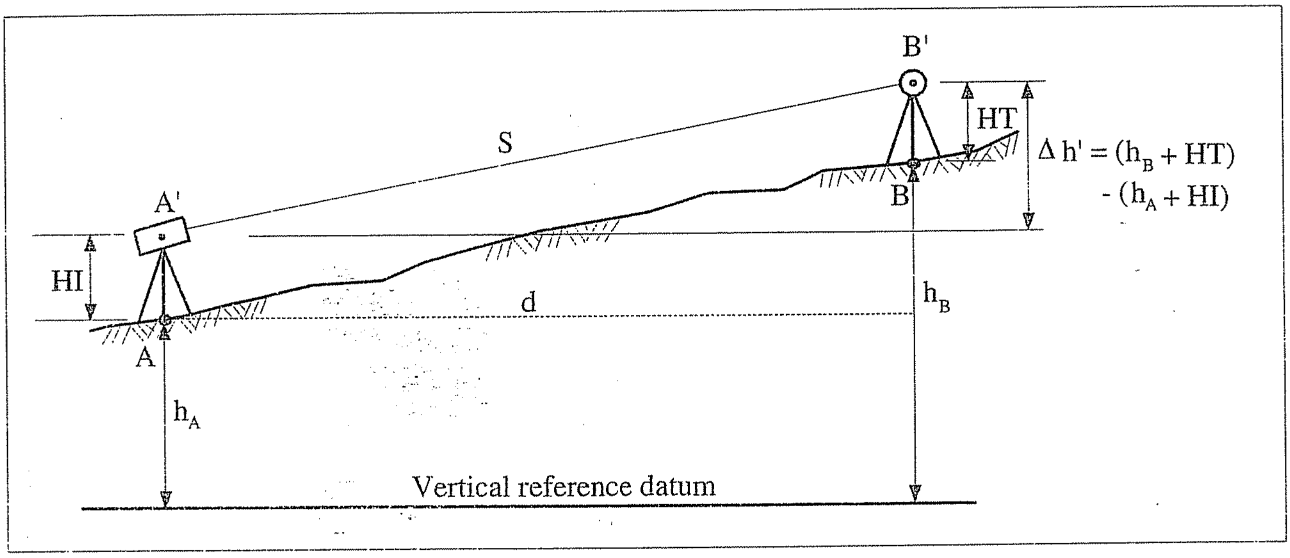


FIGURE 6.9: Reduction to horizontal distance using height difference.

$$\Delta h' = \left(h_B + HT\right) - \left(h_A + HI\right) \quad \ldots\ldots\ldots\ldots (6.11)$$

Where: $\Delta h'$ = height difference between A' and B'
h_A, h_B = heights of points A and B above the chosen datum
HI = height of instrument above A
HT = height of reflector above B

Simply, the horizontal distance d is:

$$d = \sqrt{S^2 - \Delta h'^2} \quad \ldots\ldots\ldots\ldots (6.12)$$

Using the law of propagation of errors:

$$\sigma^2_{\Delta h'} = \sigma^2_{h_B} + \sigma^2_{HT} + \sigma^2_{h_A} + \sigma^2_{HI} \quad \ldots\ldots\ldots\ldots (6.13)$$

And,

$$\sigma^2_d = \left(\frac{S^2}{S^2 - \Delta h'^2}\right)\sigma^2_S + \left(\frac{\Delta h'^2}{S^2 - \Delta h'^2}\right)\sigma^2_{\Delta h'} \quad \ldots\ldots\ldots\ldots (6.14)$$

However, for long distances measured with an EDM instrument, the effect of earth curvature must be considered when computing horizontal distances from measured slope distances. Figure 6.10, though exaggerated in scale, illustrates the effect of earth curvature. The chord distance (d_1) can be computed from the following expression:

$$d_1 = \sqrt{\frac{S^2 - \left(h_B + HT - h_A - HI\right)^2}{\left(1 + \dfrac{HI}{R + h_A}\right)\left(1 + \dfrac{h_B - h_A - HT}{R + h_A}\right)}} \quad \ldots\ldots\ldots\ldots (6.15)$$

Where R is the mean radius of the earth and can be approximated to be 6,372,200 m. Since HI and HT are usually much smaller than $(R + h_A)$, Equation (6.15) can be simplified to the following form without any degradation in accuracy:

$$d_1 = \sqrt{\frac{S^2 - \left(h_B + HT - h_A - HI\right)^2}{\left(1 + \dfrac{h_B - h_A}{R + h_A}\right)}} \quad \ldots\ldots\ldots\ldots (6.16)$$

The actual horizontal distance (L_1) along the curved surface of the earth is given by:

$$L_1 = d_1 + \frac{d_1^3}{24\left(R + h_A\right)^2} \quad \ldots\ldots\ldots\ldots (6.17)$$

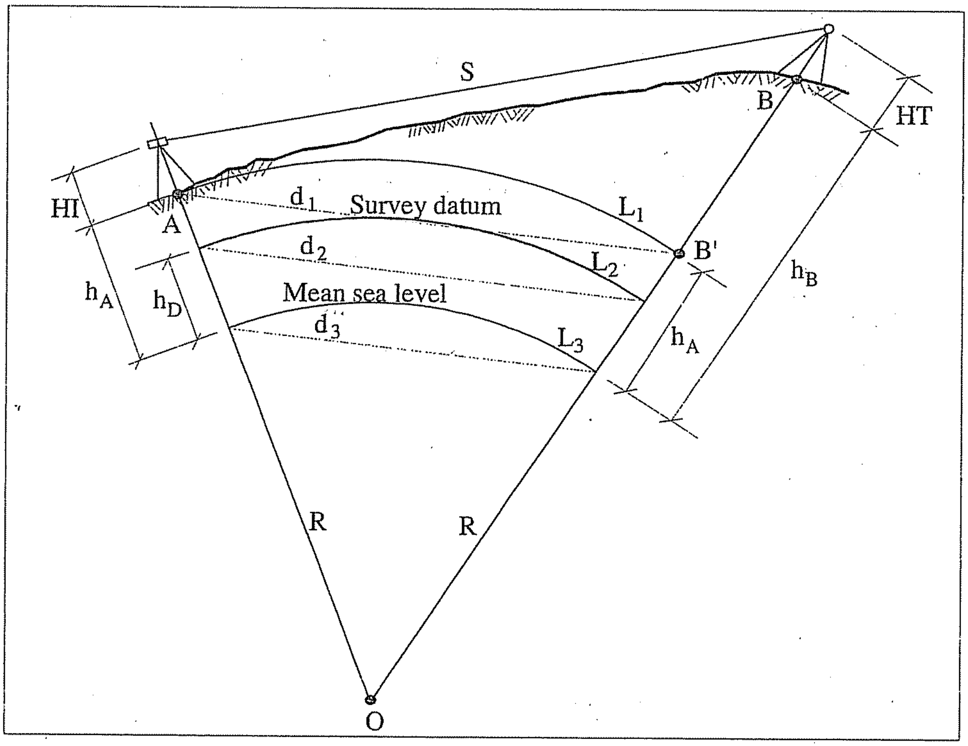


FIGURE 6.10: Effect of earth curvature on slope reduction.

EXAMPLE 6.3:

In Figure 6.9, the following measurements were taken:

$S = 3781.298$ m, with RMS $= \pm$ (5 mm + 5 ppm)

$h_A = 432.564 \pm 0.100$ m, HI $= 1.624 \pm 0.005$ m

$h_B = 487.273 \pm 0.050$ m, HT $= 1.431 \pm 0.005$ m

Compute:

a) The horizontal distance (d) and its standard error if earth curvature effect is to be ignored.

b) The horizontal distance (d_1) if earth curvature effect is to be considered.

SOLUTION:

a) $$\Delta h' = (h_B + HT) - (h_A + HI)$$

$$= (487.273 + 1.431) - (432.564 + 1.624) = 54.516 \text{ m}$$

$$d = \sqrt{S^2 - \Delta h'^2} = \sqrt{(3781.298)^2 - (54.516)^2} = 3780.905 \text{ m}$$

$$\sigma^2_{\Delta h'} = \sigma^2_{h_B} + \sigma^2_{HT} + \sigma^2_{h_A} + \sigma^2_{HI}$$

$$= (0.050)^2 + (0.005)^2 + (0.100)^2 + (0.005)^2 = 0.01255 \text{ m}^2$$

$$\sigma_S = \pm\left(0.005 \text{ m} + \frac{5}{10^6} \times 3781.298\right) = \pm 0.024 \text{ m}$$

$$\sigma^2_d = \frac{S^2}{(S^2 - \Delta h'^2)} \cdot \sigma^2_S + \frac{\Delta h'^2}{(S^2 - \Delta h'^2)} \cdot \sigma^2_{\Delta h'} = 0.00057873$$

$$\Rightarrow \sigma_d = \pm\sqrt{0.00057873} = \pm 0.024 \text{ m}$$

$$\Rightarrow d = 3780.905 \pm 0.024 \text{ m}$$

b) From Equation (6.16)

$$d_1 = \sqrt{\frac{(3781.298)^2 - (487.273 + 1.431 - 432.564 - 1.624)^2}{\left(1 + \frac{487.273 - 432.564}{6372200 + 432.564}\right)}}$$

$= 3780.889$ m.

And from Equation (6.17),

$$L_1 = 3780.889 + \frac{(3780.889)^3}{24(6372200 + 432.564)^2}$$

$$= 3780.889 + 0.00006 = 3780.889 \text{ m}$$

6.12 REDUCTION TO DATUM

Referring to Figure 6.10, assume that the survey datum has been chosen to be at an elevation h_D above the mean sea level (MSL). The chord distance d_2 at the datum surface can be computed from the chord distance d_1 at an elevation h_A by proportion:

$$d_2 = \frac{R + h_D}{R + h_A} \cdot d_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.18)$$

Where $R \cong 6{,}372{,}200$ m. Similarly, the chord distance d_3 at sea level can be computed from chord distance d_1 by the following expression:

$$d_3 = \frac{R}{R + h_A} \cdot d_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.19)$$

Equations (6.18) and (6.19) can be used in a similar manner to calculate the actual distances L_2 and L_3 once L_1 is known. The distance L_1, in this case, replaces d_1 in these equations.

EXAMPLE 6.4:

A 7-km distance in Nablus (500 m AMSL) is to be drawn on a map that uses MSL as a datum. If you know that the radius of the earth is 6372.2 km, how long the distance to be drawn is going to be?

SOLUTION:

$$d = \frac{R}{R + h_A} \cdot d_1 = \frac{6372200}{6372200 + 500} \cdot 7000.00 = 6999.45\ \text{m}$$

EXAMPLE 6.5:

In the previous example, if the distance is to be drawn on a map that uses Jerico as a reference datum (300 m BMSL), how long should be the distance to be drawn?

SOLUTION:

$$d = \frac{R + h_D}{R + h_A} \cdot d_1 = \frac{6372200 - 300}{6372200 + 500} \cdot 7000.00 = 6999.12\ \text{m}$$

6.13 TRIGONOMETRIC LEVELING - SHORT LINE

Trigonometric leveling is the process of determining height differences by the measurement of distances and vertical or zenith angles. Figure 6.11 illustrates the geometry of trigonometric leveling when the earth's curvature and atmospheric refraction are ignored.

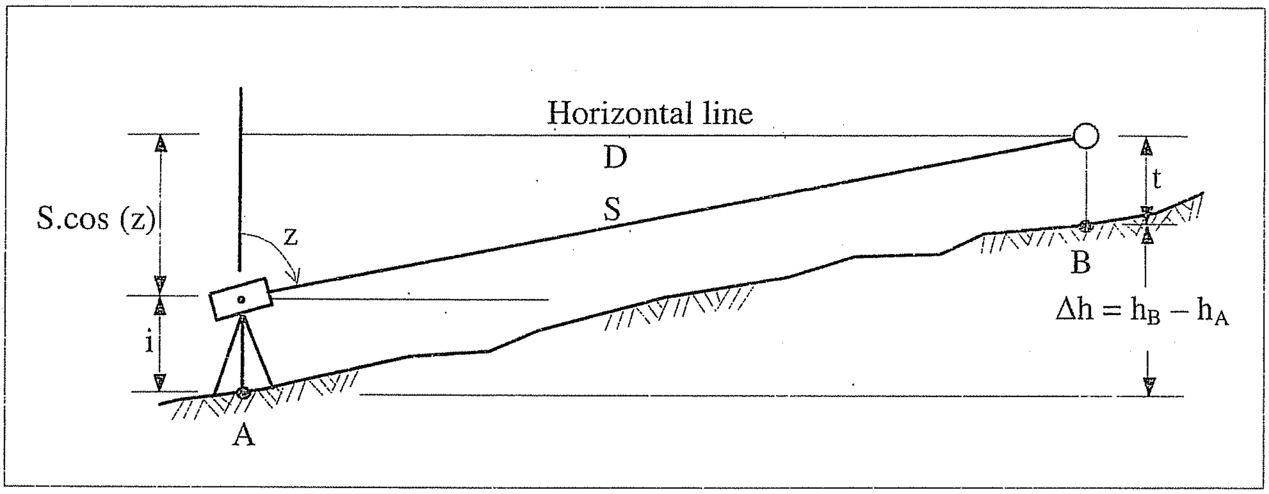


FIGURE 6.11: Trigonometric leveling – short line.

The following relation exists:

$\Delta h = h_B - h_A = s \cos z + i - t$ ....(6.20)

Also, the horizontal distance (D):

$D = s \sin z$ ....(6.21)

Where:

h_A and h_B are the elevations of points A and B

S = the slope distance along the line of sight

z = zenith angle

i = the height of the instrument (total station)

t = the height of the sight target (reflector)

EXAMPLE 6.6:

In Figure 6.11, the following values were recorded:

$h_A = 369.27 \pm 0.00$ m

$z = 86° 20' \pm 1'$

$i = 1.330 \pm 0.005$ m

$t = 1.436 \pm 0.005$ m

$S = 478.256 \pm 0.011$ m

Calculate h_B , σ_{h_B}

SOLUTION:

$$h_B - h_A = S \cdot \cos z + i - t$$

$$\Rightarrow h_B = 369.27 + 478.256 \cos(86° \ 20') + 1.330 - 1.436 = 399.75 \text{ m}$$

From the law of propagation of random errors:

$$\sigma_{\Delta h}^2 = \cos^2 z \cdot \sigma_s^2 + (-\sin z \cdot S)^2 \sigma_z^2 + \sigma_i^2 + \sigma_t^2$$

Where σ_z is in radian

$$\Rightarrow \sigma_z = \frac{1}{60} \text{ x } \frac{\pi}{180} = \frac{\pi}{10800}$$

$$\Rightarrow \sigma_{\Delta h}^2 =$$

$$\cos^2(86° \ 20')(0.011)^2 + (-478.256 \cdot \sin(86° \ 20'))^2 \left(\frac{\pi}{10800}\right)^2 + 2(0.005)^2$$

$$= 0.01933$$

$$\Rightarrow \sigma_{\Delta h} = \pm\sqrt{0.01933} = \pm \ 0.139 \text{ m}$$

$$h_B = h_A + \Delta h \qquad \Rightarrow \qquad \sigma_{h_B}^2 = \sigma_{h_A}^2 + \sigma_{\Delta h}^2$$

$$\Rightarrow \sigma_{h_B}^2 = (0.00)^2 + (0.139)^2 \qquad \Rightarrow \qquad \sigma_{h_B} = \pm \ 0.139 \text{ m}$$

$$\Rightarrow h_B = 399.75 \pm 0.14 \text{ m}$$

EXAMPLE 6.7:

For the data in Example 6.6, if z was found to be 117° 36' 15" instead (Figure 6.12), find h_B and the horizontal distance (D) between points A and B.

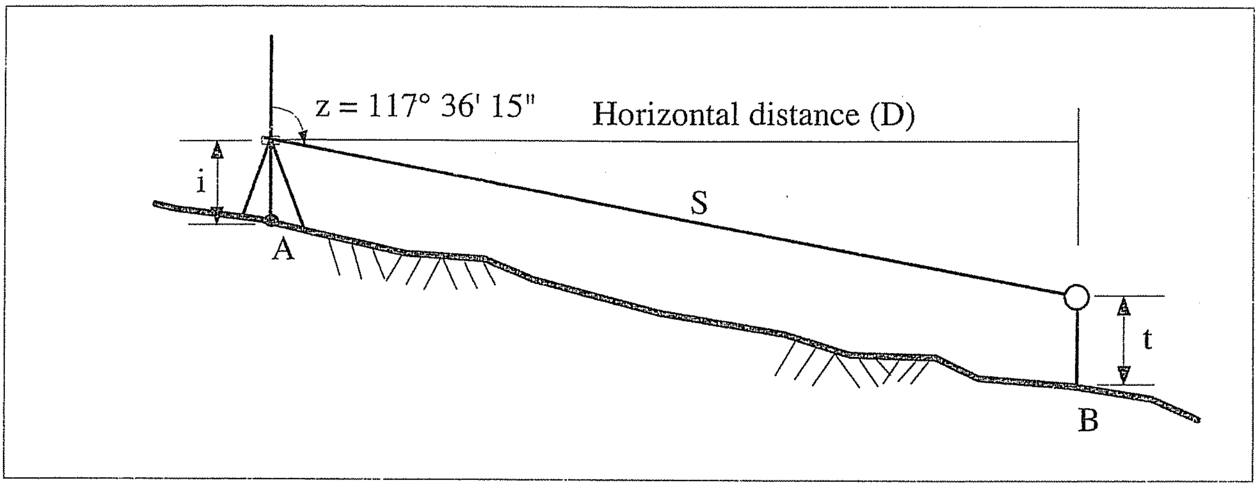


FIGURE 6.12

SOLUTION:

$$h_B - h_A = \Delta h = S \cdot \cos z + i - t$$
$$= 478.256 \cos(117° \ 36' \ 15") + 1.330 - 1.436$$
$$= -221.71 \text{ m (B is lower than A)}$$

$$h_B = h_A + \Delta h = 369.27 + (-221.71) = 147.56 \text{ m}$$

$$\text{Horizontal distance} = D = S \cdot \sin z$$
$$= 478.256 \sin(117° \ 36' \ 15")$$
$$= 423.816 \text{ m}$$

Usually, when the EDM is mounted on top of the standards of the theodolite, the measured slope distance may not coincide with the line of sight in the angular measurement. The geometry of this general case is illustrated in Figure 6.13.

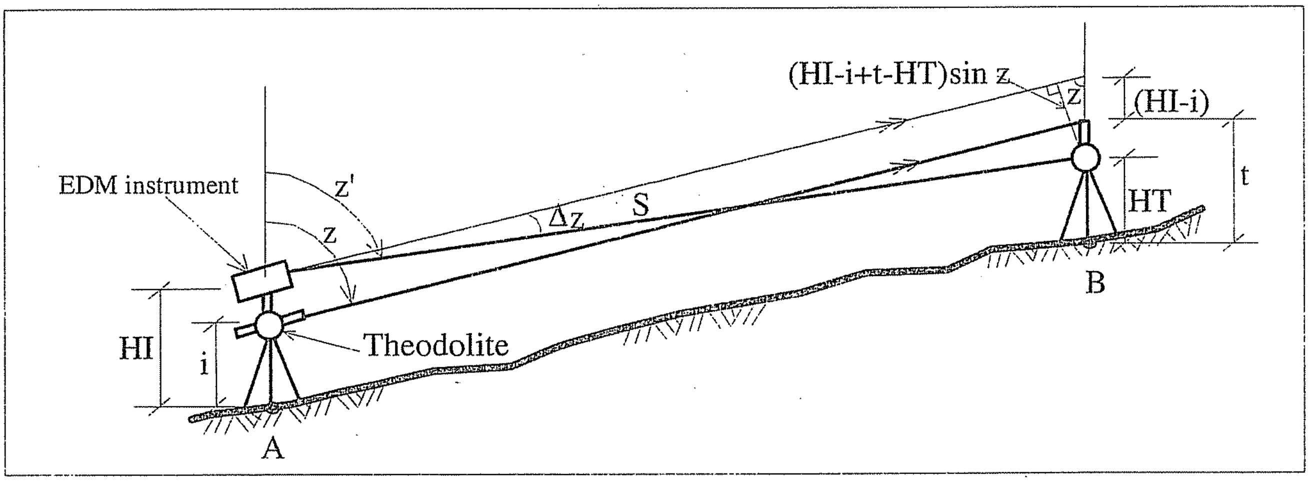


FIGURE 6.13: Trigonometric leveling with the EDM mounted on top of the standards of the theodolite.

The following relation can be written:

$$S \cdot \Delta z \cdot \frac{\pi}{180} = \left(HI - i + t - HT\right) \sin z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(6.22)}$$

Where:

Δz	=	angular difference in degrees between the measured slope distance and the line of sight of the angular measurement. Δz can be either positive or negative.
HI & HT	=	respective heights of instrument (EDM) and reflector used in the distance measurement.
i & t	=	respective heights of instrument (theodolite) and sight target used in angular measurement
z	=	measured zenith angle

Rearranging the terms in Equation (6.22) yields:

$$\Delta z = \frac{180\left(HI - i + t - HT\right) \sin z}{\pi \cdot S} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(6.23)}$$

The corrected zenith angle (z') is then computed as follows:

$$z' = z + \Delta z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(6.24)}$$

The height difference between the end stations A and B can then be computed as follows:

$$\Delta h = h_B - h_A = S \cos z' + HI - HT \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.25)$$

By applying the law of propagation of random errors on Equation (6.20), the following expression can be derived for the estimated standard error ($\sigma_{\Delta h}$) of the height difference:

$$\sigma_{\Delta h}^2 = \cos^2 z \cdot \sigma_S^2 + \left(23.503 \times 10^{-12}\right)\left(S \sin z\right)^2 \sigma_z^2 + \sigma_{HI}^2 + \sigma_{HT}^2 \quad \ldots\ldots (6.26)$$

Where σ_z is the estimated standard error of the measured zenith angle in *seconds*. The measured angle z is used in Equation (6.26) instead of the adjusted angle z' without introducing any appreciable error in the calculations.

EXAMPLE 6.8

Referring to Figure 6.13, the slope distance S between points A and B was measured using an EDM that has a measurement accuracy (RMS) of ± (10 mm + 2 ppm). The measured slope distance, after correction for atmospheric refraction, was 478.256 m. The other relevant data are given below:

HI	= 1.365	± 0.005 m
HT	= 1.482	± 0.005 m
h_A	= 369.27	± 0.00 m (assumed to be error free)
z	= 86° 20' 00" ± 1'	
i	= 1.330	± 0.005 m
t	= 1.436	± 0.005 m

Compute the height difference between points A & B and its estimated standard error. Also calculate h_B and σ_{h_B}.

SOLUTION:

$$\Delta z = \frac{180\left(HI - i + t - HT\right)\sin z}{\pi \cdot S}$$

$$= \frac{180\left(1.365 - 1.330 + 1.436 - 1.482\right)\cdot \sin\left(86°\ 20'\ 00''\right)}{\pi \times 478.256} = -0°\ 00'\ 05''$$

$$z' = z + \Delta z = 86°\ 20'\ 00'' - 0°\ 00'\ 05'' = 86°\ 19'\ 55''$$

From Equation (6.25):

$$\Delta h = h_B - h_A = S \cos z' + HI - HT$$

$$= 478.256 \cos(86°\ 19'\ 55'') + 1.365 - 1.482 = 30.480 \text{ m}$$

$$\sigma_S = \pm\left(10 \text{ mm} + \frac{2}{1{,}000{,}000} \cdot S\right)$$

$$= \pm\left(0.010 + \frac{2}{1{,}000{,}000} \times 478.256\right) \text{m} = \pm 0.011 \text{ m}$$

From Equation (6.26):

$$\sigma^2_{\Delta h} = \cos^2\left(86°20'\right)\left(0.011\right)^2 + \left(23.503 \times 10^{-12}\right)\left(478.256 \sin\left(86°20'\right)\right)^2 (60)^2$$

$$+\left(0.005\right)^2 + \left(0.005\right)^2$$

$$\therefore \quad \sigma_{\Delta h} = \pm 0.139 \text{ m}$$

Therefore, $\Delta h = 30.480 \pm 0.139$ m

$h_B = h_A + \Delta h = 369.27 + 30.480 = 399.75$ m

$$\sigma_{h_B} = \pm\sqrt{\sigma^2_{h_A} + \sigma^2_{\Delta h}} = \sqrt{\left(0.00\right)^2 + \left(0.139\right)^2} = \pm 0.139 \text{ m}$$

$$h_B = 399.75 \pm 0.14 \text{ m}$$

6.14 TRIGONOMETRIC LEVELING - LONG LINE [2]

Atmospheric refraction is the primary factor that affects the accuracy of trigonometric leveling for long lines. Its effect on the zenith angle measurement can be minimized by making simultaneous reciprocal zenith angle measurements from both ends of the line. Special sighting targets can be mounted on the standards of the two theodolites so that the heights of the targets coincide with the horizontal axes of the theodolites. In this arrangement, the theodolite at one end of the line can sight on the theodolite located on the other end while zenith angle measurements are being made from both ends.

Figure 6.14 illustrates the geometry involved. An EDM instrument is set up over point A, with the height of the instrument being represented by HI. Point C is the center of the EDM instrument over point A. Point O is the center of the earth, and line CO represents the vertical line (or direction of gravity) at point C. Line CF is the horizontal line at C, and is perpendicular to CO. The zenith angle z_A is measured at a height HI above point A. If the actual height of the theodolite is not equal to HI, then the measured zenith angle must be corrected according to Equations (6.23) and (6.24).

Angle $E\hat{C}D$, is the small angular displacement caused by atmospheric refraction and is equal to $m\gamma$: where m is the coefficient of refraction at point A, and γ is the angle subtended at the center of the earth by the line of sight.

A second theodolite is assumed to be set up over point B, with the height of the theodolite being equal to HT. If the actual height differs from HT, then the measured zenith angle must also be corrected according to Equations (6.23) and (6.24). Point D is at a height of HT above point B, and the zenith angle at point D is represented by z_B.

By assuming that the coefficient of refraction at B is the same as at A, angle $E\hat{D}C = m\gamma$. Since OC = OB', the triangle OCB' is an isosceles triangle and line OF is perpendicular to CB' at point G. The radius R should be the radius of the earth at the average latitude along the line joining stations A and

[2] This section is almost entirely copied from reference 16.

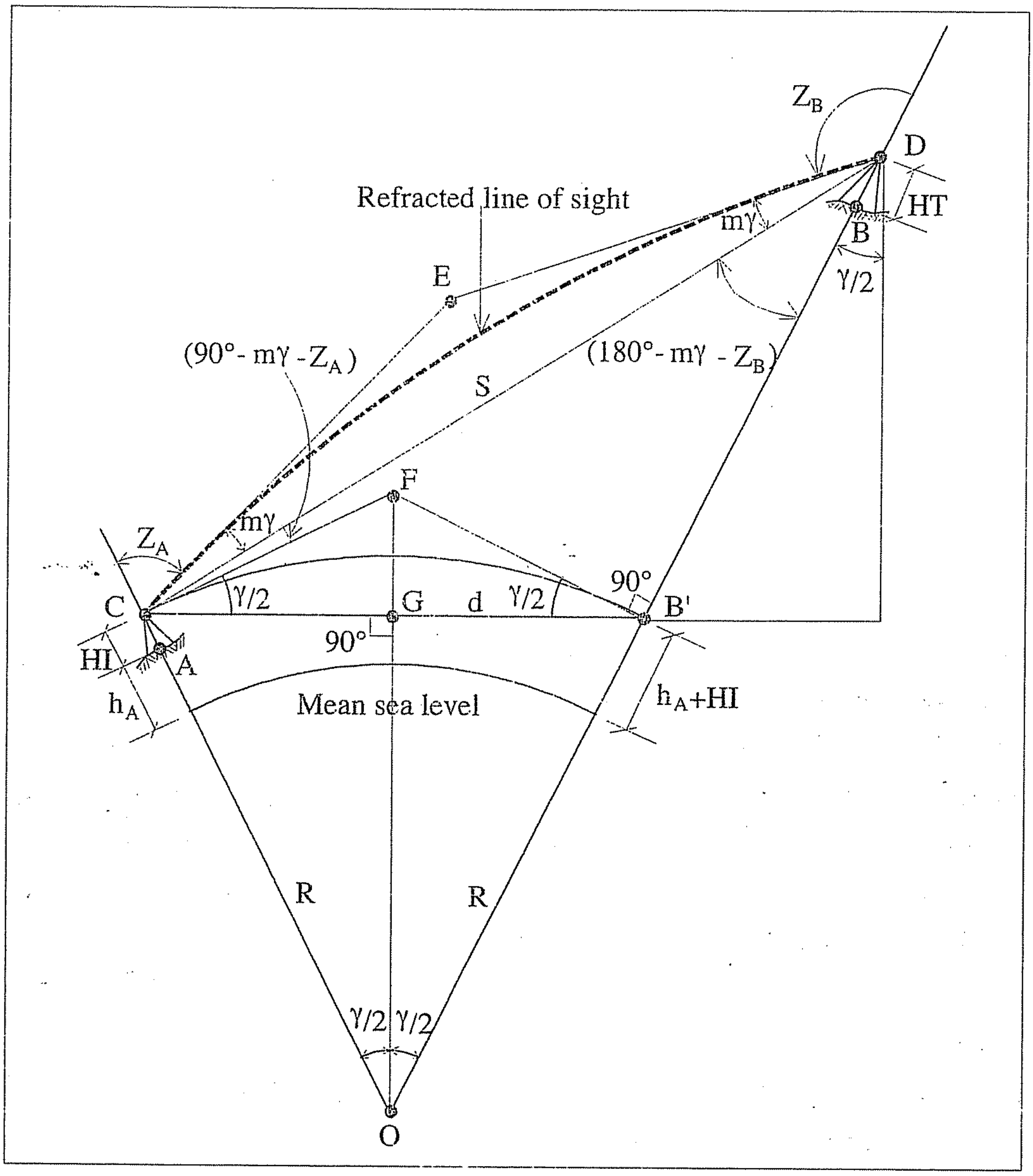


FIGURE 6.14: Trigonometric leveling – long line.

B. In most cases it is sufficiently accurate to use the mean radius of the earth: that is R = 20,906,000 ft or 6,372,200 m.

In triangle CDB', the sum of the three interior angles must add up to 180°; that is,

$$\left(90 - z_A - m\gamma + \frac{\gamma}{2}\right) + \left(180 - m\gamma - z_B\right) + \left(90 + \frac{\gamma}{2}\right) = 180° \quad \text{.........................} (6.27)$$

By manipulating the terms in Equation (6.27), the following expression can be derived:

$$m = \frac{1}{2}\left[1 - \frac{z_A + z_B - 180^\circ}{\gamma}\right] \qquad \text{.....................(6.28)}$$

Furthermore,

$$D\hat{C}B' = 180^\circ - C\hat{D}B' - D\hat{B}'C$$

Therefore, from Figure 6.14,

$$D\hat{C}B' = 180^\circ - (180^\circ - m\gamma - z_B) - \left(90^\circ + \frac{\gamma}{2}\right) \qquad \text{................(6.29)}$$

By substituting Equation (6.28) into Equation (6.29), the following expression can be derived:

$$D\hat{C}B' = \frac{1}{2}(z_B - z_A) \qquad \text{............................(6.30)}$$

Then, by applying the sine law to triangle CDB', the following expression is derived:

$$DB' = \frac{S \cdot \sin\frac{1}{2}(z_B - z_A)}{\sin\left(90^\circ + \frac{\gamma}{2}\right)} \qquad \text{............................(6.31)}$$

Since DB' = $(h_B + HT) - (h_A + HI)$,

$$(h_B + HT) - (h_A + HI) = \frac{S \cdot \sin\frac{1}{2}(z_B - z_A)}{\sin\left(90^\circ + \frac{\gamma}{2}\right)} \qquad \text{...........(6.32)}$$

An expression can be derived for angle γ by using triangle CGO:

$$\sin\frac{\gamma}{2} = \frac{d}{2(R + h_A + HI)} \qquad \text{.........................(6.33)}$$

The term HI is much smaller than the mean radius of the earth (R) and can be omitted. The distance d can be approximated by the following expression without introducing appreciable error in γ:

$$d \approx S \cdot \sin z_A \qquad \text{..................................(6.34)}$$

Then, from Equations (6.33) and (6.34),

$$\gamma = 2\sin^{-1}\left[\frac{S \sin z_A}{2(R + h_A)}\right] \qquad \text{(6.35)}$$

Equations (6.32) and (6.35) can be used to calculate the height difference (h_B - h_A) from the measured slope distance S and the zenith angles z_A and z_B. The coefficient of refraction (m) is not involved in any of these two equations. Thus, by making reciprocal zenith angle measurements from both ends of the line, the need for knowing the value of m is eliminated.

If the zenith angle is measured only from one end of the line, say from point C in Figure 6.14, the following expression can be derived by applying the sine law to triangle CDB':

$$(h_B + HT) - (h_A + HI) = \frac{S \sin\left[90° - z_A + (0.5 - m)\gamma\right]}{\sin(90° + 0.5\gamma)} \qquad \text{(6.36)}$$

Then Equations (6.35) and (6.36) are used to compute the elevation difference (h_B - h_A). However, in this case a value must be assumed for the coefficient of refraction (m).

EXAMPLE 6.9

The elevation difference between points A and B is to be determined by simultaneous reciprocal zenith angle measurements. The following field data were obtained:

z_A = 88° 01' 33" , i_A = 1.63 m

z_B = 92° 07' 11" , i_B = 1.60 m

The slope distance between points A and B was also measured with the EDM instrument located at A, and the target at B. The following field data were recorded:

S=19289.40 m, HI at A = 1.56 m, HT at B = 1.46 m

The elevation (h_A) of point A is known to be 433.87 m. Determine the elevation of point B. Use R = 6372200 m.

SOLUTION:

Step 1: Compute the correction for z_A using Equation (6.23)

$$\Delta z_A = \frac{180(1.56-1.63+1.60-1.46)\sin(88°\,01'\,33")}{\pi \text{ x } 19289.40} = +00°\ 00'\,0.8"$$

$$z'_A = 88°\ 01'\ 33" + 00°\ 00'\ 0.8" = 88°\ 01'\ 33.8"$$

Step 2: Compute the correction for z_B using Equation (6.23)

$$\Delta z_B = \frac{180(1.46-1.60+1.63-1.56)\sin(92°\,07'\,11")}{\pi \text{ x } 19289.40} = -00°\ 00'\,0.8"$$

$$z'_B = 92°\ 07'\ 11" - 00°\ 00'\ 0.8" = 92°\ 07'\ 10.2"$$

Step 3: Compute angle γ using Equation (6.35)

$$\gamma = 2\sin^{-1}\left[\frac{19289.40\sin(88°\,01'\,33.8")}{2(6372200+433.87)}\right] = 0°\ 10'\ 24"$$

Step 4: Compute the elevation difference using Equation (6.32)

$$(h_B + HT) - (h_A + HI) = \frac{19289.40\sin\frac{1}{2}(92°\,07'\,10.2" - 88°\,01'\,33.8")}{\sin\left[90° + \frac{1}{2}(0°\,10'\,24")\right]}$$

$$= 688.91 \text{ m}$$

Step 5: Compute the elevation of station B

H_B = 688.91 + h_A + HI – HT = 688.91 + 433.87 + 1.56 – 1.46 = 1122.88 m

PROBLEMS

6.1 Recent developments in optical and electromagnetic distance measuring equipment have produced new possibilities in surveying techniques. Discuss the impact that these developments have made, referring to the major changes that have taken place in surveying methodology.

6.2 An EDM instrument uses the following modulating frequencies:

F_1 = 14.989625 MHz, F_2 = 1.4989625 MHz,
F_3 = 149.89625 KHz, F_4 = 14.989625 KHz.

Knowing that the speed of light (V) = 299,792,500 m/sec, compute the distances in meters for the following sets of phase differences:

a. $\Delta_1 = 285.95°$ $\Delta_2 = 244.1°$ $\Delta_3 = 168.1°$ $\Delta_4 = 0°$
b. $\Delta_1 = 151.56°$ $\Delta_2 = 233.9°$ $\Delta_3 = 0°$ $\Delta_4 = 0°$

6.3 Determine the correction for zero centering (c) and its standard error (σ_c) of a certain electro-optical instrument from the following length measurements:

Line	Length (m)	Line	Length (m)
AB	191.733	BG	313.466
AC	222.247	CG	282.953
AD	252.740	DG	252.461
AE	283.262	EG	221.940
AF	313.673	FG	191.526

ABCDEFG are collinear, and AG was measured as 505.091 m.

6.4 The stations A, B, and C are located along a straight line. The calibrated lengths of lines AB and AC were AB = 1,480.306 m and AC = 150.046 m. These two known distances were measured using an EDM instrument and a triple prism reflector and were found to be as follows: AB = 1,480.253 m and AC = 150.038m.

a. Determine the scale factor (g) and correction constant (c) for zero centering for the EDM instrument-reflector combination.

b. The same EDM instrument-reflector combination was later used to measure the horizontal distance and elevation difference between two points D & E. The following set of data was obtained:

z_D = 85° 44' 35" , i = 1.76 m , t = 1.43 m

Slope distance (S) measured with EDM at station D = 241.66 m

Calculate the correct horizontal distance and elevation difference between points D & E. Ignore earth curvature and refraction.

6.5 In the previous question, if the elevation of point D (h_D) was 750.00 m AMSL, what will be the horizontal distance between D & E if the earth curvature is to be considered (use R=6372.2 km)? If this distance were to be drawn at a datum that coincides with MSL, what will be the length to be drawn?

6.6 For each of the following sets of data from trigonometric leveling, calculate the difference in elevation between the end stations and its estimated standard error. The instrument used is an EDM mounted on the standards of a theodolite. Ignore earth curvature and refraction.

a. From station A to station B:
Zenith angle measured at station A
z_A = 91° 23' 44" ± 10"
i = 5.44' ± 0.01' *HI* at station A = 5.71' ± 0.01'
t = 4.61' ± 0.01' *HT* at station B = 4.89' ± 0.01'
Slope distance (s) measured with EDM at station A = 461.24' ± 0.02'

b. From station C to station D:
Zenith angle measured at station C
z_C = 85° 44' 35" ± 20"
i = 1.76 m *HI* at station C = 1.885 m ± 0.005 m
t = 1.43 m *HT* at station D = 1.431 m ± 0.005 m
Slope distance (s) measured with EDM at station C = 241.66 m ± 0.01 m

6.7 The elevation differences must be determined with an estimated standard error of ±0.05 ft or better in problem 6.6a and ±0.02 m or better in problem 6.6b. Determine the maximum permissible standard error for the measured zenith angle in each case.

6.8 Compute the elevation of station B as well as distance AB for the following set of data from trigonometric leveling by simultaneous reciprocal zenith angle measurements:

z_A = 88° 57' 25" i_A = 1.55m
z_B = 91° 02' 51" i_B = 1.63m
s = 35,711.26 m
HI at station A = 1.47 m
HT at station B = 1.32 m
Elevation of station A = 863.21 m above mean sea level
Use R = 6,372,200 m

7 COORDINATE GEOMETRY, AND TRAVERSE SURVEYING

This soft-copy scanned by Eng. Mohammed F. Al-Hallaq & Eng. Samer T. Saqallah

7.1 INTRODUCTION

The rapid advances in surveying technology and the increasing use of computers in all aspects of engineering planning and design made the use of coordinates to define geographic positions of survey points a necessity, rather than just being convenient. There are now computer programs available for performing many of the basic surveying calculations by the use of coordinates. Examples of these program packages are the PC SURVEY for Microsoft Windows written by Soft-Art, Inc., Softdesk 8 and the COCOLA package written by the author.

The fundamental principles of coordinate geometry and traverse surveying will be discussed in this chapter, and emphasis will be given to horizontal coordinates only. It should be noted here that this book uses the coordinate system utilized by the Palestinian Survey Department where the x-axis is taken to coincide with the north direction, while the y-axis coincides with the east direction (see Figure 7.1).

7.2 COORDINATE GEOMETRY

Several methods are used to locate and calculate the coordinates of a point with respect to a known line (a known line is one for which the coordinates of the end points are known or the coordinates of the beginning point and the azimuth of the line are known). These methods include: location by angle and distance, distance and offset, intersection by angles, intersection by distances and resection.

7.2.1 THE INVERSE PROBLEM

If the y and x coordinates of two points are known, the horizontal distance and azimuth of the line joining them can be computed. Figure 7.1 illustrates the problem. The horizontal distance d_{ij}:

$$d_{ij} = \sqrt{(y_j - y_i)^2 + (x_j - x_i)^2} \quad \dots\dots\dots (7.1)$$

The azimuth α_{ij} of the line going from *i* to *j* is:

$$\alpha_{ij} = \tan^{-1}\frac{(y_j - y_i)}{(x_j - x_i)} + c \quad \dots\dots\dots (7.2)$$

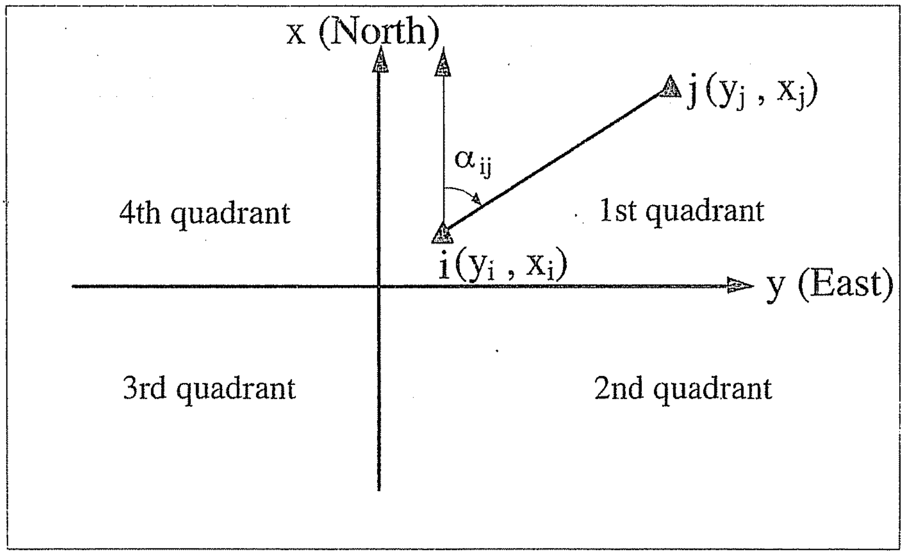


FIGURE 7.1: The inverse problem.

Where

$c = 0°$ if Δy is positive and Δx is positive (1st quadrant)
$c = 180°$ if Δy is positive and Δx is negative (2nd quadrant)
$c = 180°$ if Δy is negative and Δx is negative (3rd quadrant)
$c = 360°$ if Δy is negative and Δx is positive (4th quadrant)

Traditionally, $(\Delta y = y_j - y_i)$ is called the departure of line *ij*, and $(\Delta x = x_j - x_i)$ is called the latitude of line *ij*.

EXAMPLE 7.1:

Given the following horizontal coordinates for two points *i* and *j* in Nablus area:

$y_i = 174410.56$ m $\quad x_i = 181680.76$ m
$y_j = 174205.31$ m $\quad x_j = 181810.22$ m

Compute the horizontal distance (d_{ij}) and azimuth (α_{ij}) of line *ij*.

SOLUTION:

$$\Delta y = y_j - y_i = 174205.31 - 174410.56 = -205.25 \text{ m}$$

$$\Delta x = x_j - x_i = 181810.22 - 181680.76 = 129.46 \text{ m}$$

$$\Rightarrow d_{ij} = \sqrt{(-205.25)^2 + (129.46)^2} = 242.67 \text{ m}$$

Since Δy is negative and Δx is positive $\Rightarrow$ the line is in the 4th quadrant (c=360°),

$$\Rightarrow \alpha_{ij} = \tan^{-1}\frac{\Delta y}{\Delta x} + c = \tan^{-1}\frac{-205.25}{129.46} + 360 = 302°\ 14'\ 29''$$

7.2.2 LOCATION BY ANGLE AND DISTANCE

Referring to Figure 7.2, let *i* and *j* be two points of known coordinates. The horizontal coordinates of a new point, such as *k*, can be determined by measuring the horizontal angle β and the distance d_{ik}. The azimuth α_{ij} is calculated from Equation (7.2).

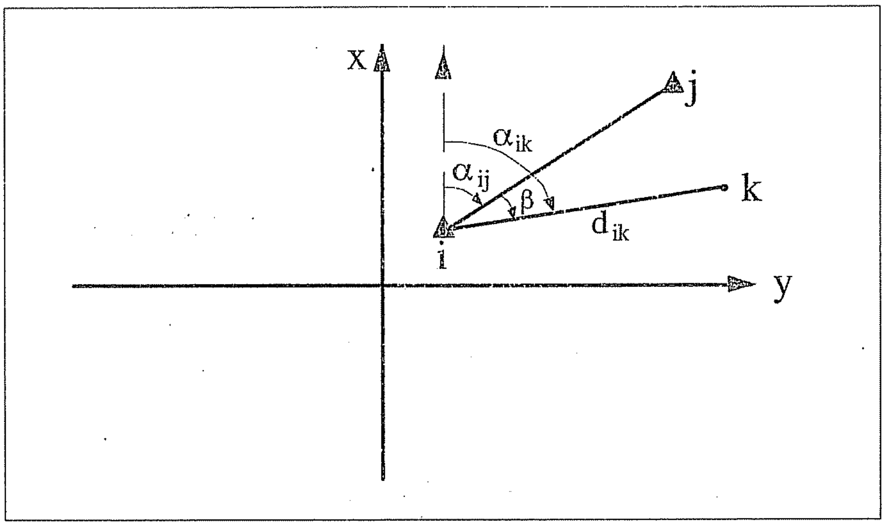


FIGURE 7.2: Location by angle and distance.

Then,

$$\alpha_{ik} = \alpha_{ij} + \beta \qquad (7.3)$$

Note: If α_{ik} is found to be more than 360°, then subtract 360° from it.

From Figure 7.2, the coordinates (y_k, x_k) of point *k* are computed as follows:

$$y_k = y_i + d_{ik} \sin \alpha_{ik} \qquad (7.4)$$

$$x_k = x_i + d_{ik} \cos \alpha_{ik} \qquad (7.5)$$

EXAMPLE 7.2:

For the two points *i* & *j* in example 7.1, a total station was set up at point *i* and directed towards point *j*. The following horizontal angle and distance were then measured to point *k*:

$\beta = 111° 27' 45''$, $\quad d_{ik} = 318.10$ m

Compute the horizontal coordinates of point *k*.

SOLUTION:

From example 7.1, $\alpha_{ij} = 302° 14' 29''$

$$\alpha_{ik} = \alpha_{ij} + \beta = 302° 14' 29'' + 111° 27' 45'' = 413° 42' 14'' - 360°$$
$$= 53° 42' 14''$$

From equations (7.4) and (7.5):

$$y_k = 174410.56 + 318.10 \sin(53° 42' 14'') = 174666.94 \text{ m}$$
$$x_k = 181680.76 + 318.10 \cos(53° 42' 14'') = 181869.06 \text{ m}$$

7.2.3 LOCATING THE NORTH DIRECTION AT A POINT

Suppose that you are standing with your theodolite or total station at point *i* whose coordinates are known, and facing another point *j* whose coordinates are also known. Now, in order to locate the direction of the north at point *i*, perform the following steps:

1. Calculate the azimuth of line *ij* (α_{ij}).
2. Let the horizontal circle reading of your instrument read the value of α_{ij} while sighting point *j*.
3. Rotate the instrument in a counterclockwise direction. The horizontal circle reading will start decreasing, and when it becomes 0°, the telescope of the instrument will point in the north direction.

Now, let us assume that the coordinates of the point at which the north direction is to be located are not known. For this case, measurements are needed to calculate the coordinates of this point using the procedure of location by angle and distance in the previous section, or any of the procedures that will be explained in the following sections. Now that the coordinates of this point are known, set up the instrument over the point and direct it towards another point of known coordinates, and then repeat the previous three steps.

7.2.4 LOCATION BY DISTANCE AND OFFSET

Referring to Figure 7.3, let *i* and *j* be two points of known coordinates. The horizontal coordinates of a new point, such as *p*, can be determined by measuring the horizontal distance *im* along the line *ij* and the offset o_1 to point *p*. The azimuth α_{ij} is calculated from Equation (7.2).

The coordinates of point *p*, which lies to the left of line *ij* are calculated from the following equations:

$$y_p = y_i + d_{im}.\sin\alpha_{ij} + o_1.\sin(\alpha_{ij} - 90°)$$

$$x_p = x_i + d_{im}.\cos\alpha_{ij} + o_1.\cos(\alpha_{ij} - 90°)$$

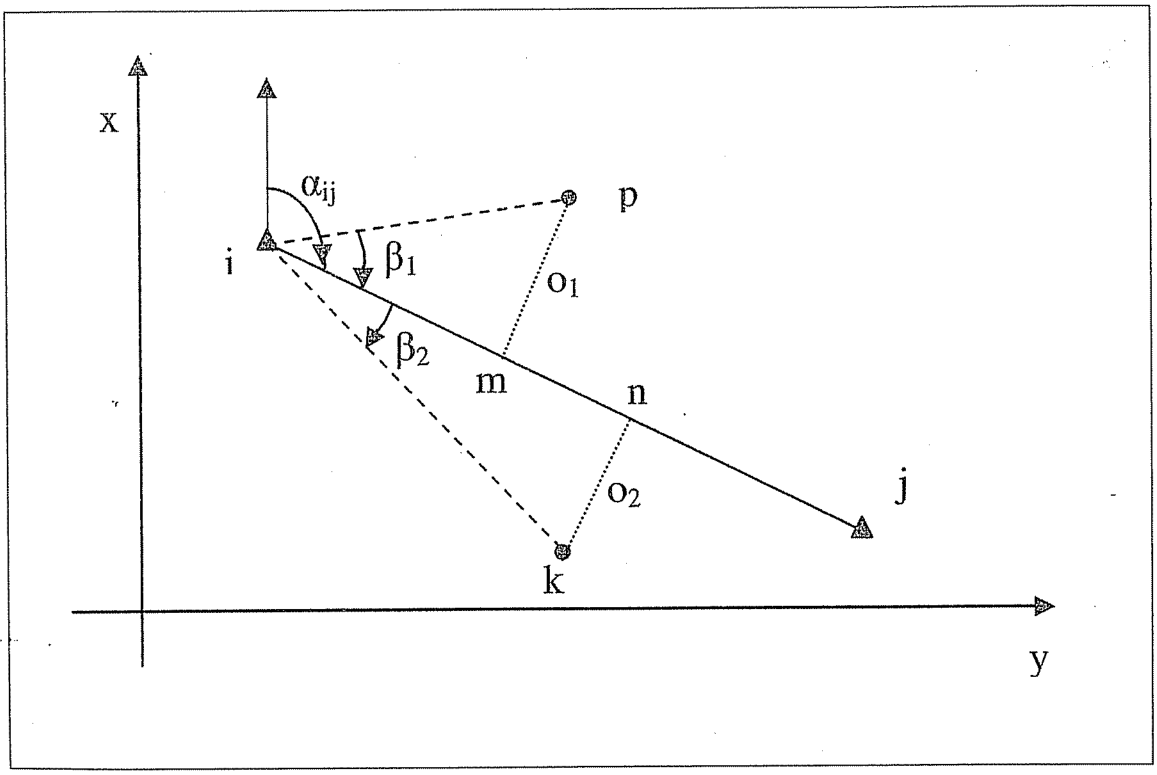


FIGURE 7.3: Location by distance and offset.

But, $\sin(\alpha_{ij} - 90°) = -\cos\alpha_{ij}$ and $\cos(\alpha_{ij} - 90°) = \sin\alpha_{ij}$, therefore:

$$\begin{aligned} y_p &= y_i + d_{im}.\sin\alpha_{ij} - o_1.\cos\alpha_{ij} \\ x_p &= x_i + d_{im}.\cos\alpha_{ij} + o_1.\sin\alpha_{ij} \end{aligned} \qquad \ldots\ldots\ldots\ldots (7.6)$$

However, if the point lies to the right of line *ij*, such as point *k* (Figure 7.3), the coordinates are calculated from the following equations:

$$\begin{aligned} y_k &= y_i + d_{in}.\sin\alpha_{ij} + o_2.\sin(\alpha_{ij} + 90°) \\ x_k &= x_i + d_{in}.\cos\alpha_{ij} + o_2.\cos(\alpha_{ij} + 90°) \end{aligned}$$

But, $\sin(\alpha_{ij} + 90°) = \cos\alpha_{ij}$ and $\cos(\alpha_{ij} + 90°) = -\sin\alpha_{ij}$, therefore:

$$\begin{aligned} y_k &= y_i + d_{in}.\sin\alpha_{ij} + o_2.\cos\alpha_{ij} \\ x_k &= x_i + d_{in}.\cos\alpha_{ij} - o_2.\sin\alpha_{ij} \end{aligned} \qquad \ldots\ldots\ldots\ldots (7.7)$$

Alternatively, the coordinates of points *p* and *k* can be calculated as follows:

A) For points which lie on the left side of the line ij (such as point *p* in Figure 7.3):

1. Calculate the horizontal distance from *i* to *p* ($d_{ip} = \sqrt{d_{im}^2 + o_1^2}$)
2. Calculate angle $\beta_1 = \tan^{-1}\dfrac{o_1}{d_{im}}$
3. Calculate the azimuth of line ip ($\alpha_{ip} = \alpha_{ij} - \beta_1$)
4. Calculate the coordinates of point p as follows:

$$y_p = y_i + d_{ip}.\sin\alpha_{ip}$$
$$x_p = x_i + d_{ip}.\cos\alpha_{ip}$$

B) For points which lie on the right side of the line ij (such as point *k* in Figure 7.3):

1. Calculate the horizontal distance from i to k ($d_{ik} = \sqrt{d_{in}^2 + o_2^2}$)
2. Calculate angle $\beta_2 = \tan^{-1}\dfrac{o_2}{d_{in}}$
3. Calculate the azimuth of line ik ($\alpha_{ik} = \alpha_{ij} + \beta_2$)
4. Calculate the coordinates of point k as follows:

$$y_k = y_i + d_{ik}.\sin\alpha_{ik}$$
$$x_k = x_i + d_{ik}.\cos\alpha_{ik}$$

Now, let us assume that you are given the coordinates of the end points of a line such as *i* and *j* as well as the coordinates of a third point *p*, and you want to know if point *p* is located to the left or right of line *ij* and at what distance and offset. The following procedure is followed (refer to Figure 7.3):

1. Calculate both azimuths of lines *ip* and *ij* (i.e. α_{ip} & α_{ij}).
2. Calculate the angle β between the lines *ip* and *ij* ($\beta = \alpha_{ip} - \alpha_{ij}$). If angle β is negative, this means that point *p* is to the left of the line *ij*, else it is to the right.
3. Calculate the distance between points *i* and *p* ($d_{ip} = \sqrt{(y_p - y_i)^2 + (x_p - x_i)^2}$).
4. The distance *im* and offset o (Figure 7.3) are calculated as follows:
   $$d_{im} = d_{ip}.\cos\beta$$
   $$o = d_{ip}.\sin\beta$$

Again, a negative offset means that point *p* lies on the left side of line *ij*, while a positive offset means that point *p* is on the right side of line *ij*.

EXAMPLE 7.3:

The coordinates of the end points of a chain line *ij* are as follows:

y_i = 1000.00 m x_i = 1000.00 m
y_j = 1050.00 m x_j = 975.00 m

An edge of a building *k* is located at a distance of 30.00 m and offset of 10.00 m to the right of line *ij*. Calculate the coordinates of this edge point *k*.

SOLUTION:

$$\alpha_{ij} = \tan^{-1}\frac{1050.00 - 1000.00}{975.00 - 1000.00} + 180^\circ = 116^\circ\ 33'\ 54'' \quad \text{(second quadrant)}$$

Using equations (7.7):

$$y_k = 1000.00 + 30.00 \times \sin(116^\circ\ 33'\ 54'') + 10.00 \cdot \cos(116^\circ\ 33'\ 54'')$$
$$= 1022.36 \text{ m}$$
$$x_k = 1000.00 + 30.00 \times \cos(116^\circ\ 33'\ 54'') - 10.00 \cdot \sin(116^\circ\ 33'\ 54'')$$
$$= 977.64 \text{ m}$$

7.2.5 INTERSECTION BY ANGLES

The horizontal coordinates of a new point can be determined by measuring angles from two points of known coordinates. In Figure 7.4, the coordinates of points *i* and *j* are known and the coordinates of point *k* are to be calculated. This procedure is mostly used when the surveyor has a theodolite and does have access to an EDM, or when point *k* is difficult to be reached such as the top of a mosque minaret or a church.

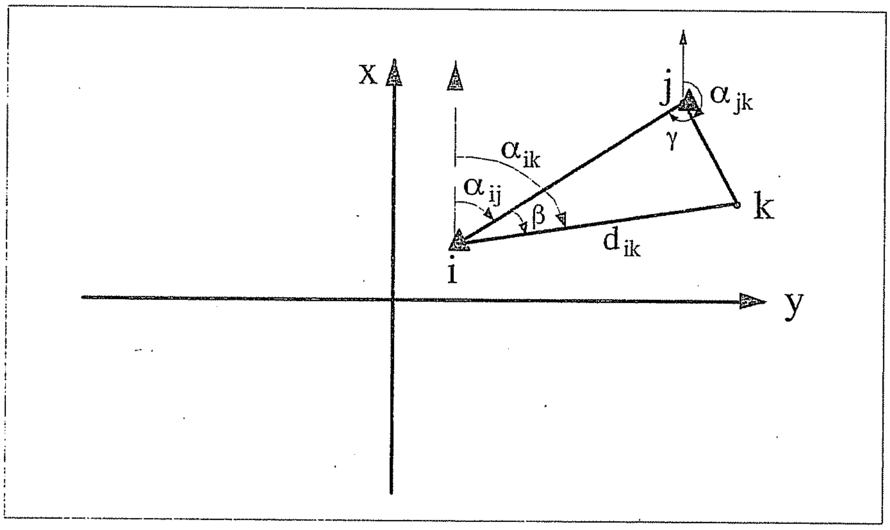


FIGURE 7.4: Intersection by angles.

The azimuth α_{ij} and distance d_{ij} can be first be computed by equations (7.1) and (7.2). The azimuths α_{ik} and α_{jk} can then be computed from the azimuth α_{ij} and the measured angles β and γ. Let d_{ik} and d_{jk} represent the lengths of lines *ik* and *jk* respectively. By the Sine law:

$$\frac{d_{ik}}{\sin\gamma} = \frac{d_{ij}}{\sin(180 - \gamma - \beta)}$$

$$\Rightarrow \quad d_{ik} = \frac{d_{ij}\sin\gamma}{\sin(180 - \gamma - \beta)} \quad \ldots\ldots (7.8)$$

Then,

$$y_k = y_i + d_{ik}\sin\alpha_{ik} \quad \ldots\ldots (7.9)$$

$$x_k = x_i + d_{ik}\cos\alpha_{ik} \quad \ldots\ldots (7.10)$$

Similarly,

$$d_{jk} = \frac{d_{ij}\sin\beta}{\sin(180 - \gamma - \beta)} \quad \ldots\ldots (7.11)$$

$$y_k = y_j + d_{jk}\sin\alpha_{jk} \quad \ldots\ldots (7.12)$$

$$x_k = x_j + d_{jk}\cos\alpha_{jk} \quad \ldots\ldots (7.13)$$

EXAMPLE 7.4:

Referring to Figure 7.4, let *i* & *j* be two points of known coordinates, and point *k* be the cross of a church whose coordinates are to be calculated. Given:

y_i = 175329.41 m $\quad$ x_i = 184672.66 m
y_j = 176321.75 m $\quad$ x_j = 185188.24 m
β = 31° 26' 30" $\quad$ γ = 42° 33' 41"
Compute the horizontal coordinates y_k and x_k.

SOLUTION:

$$y_j - y_i = 176321.75 - 175329.41 = 992.34 \text{ m}$$
$$x_j - x_i = 185188.24 - 184672.66 = 515.58 \text{ m}$$

$$\Rightarrow d_{ij} = \sqrt{(992.34)^2 + (515.58)^2} = 1118.29 \text{ m}$$

$$\alpha_{ij} = \tan^{-1}\frac{992.34}{515.58} + 0 = 62° \; 32' \; 44'' \quad (1^{st} \text{ quadrant})$$

$$\alpha_{ik} = \alpha_{ij} + \beta = 62° \; 32' \; 44'' + 31° \; 26' \; 30'' = 93° \; 59' \; 14''$$
$$180° - \beta - \gamma = 180° - 31° \; 26' \; 30'' - 42° \; 33' \; 41'' = 105° \; 59' \; 49''$$

$$d_{ik} = \frac{1118.29 \sin(42° \; 33' \; 41'')}{\sin(105° \; 59' \; 49'')} = 786.86 \text{ m}$$

Then,

$$y_k = 175329.41 + 786.86 \sin(93° \; 59' \; 14'') = 176114.37 \text{ m}$$
$$x_k = 184672.66 + 786.86 \cos(93° \; 59' \; 14'') = 184617.95 \text{ m}$$

7.2.6 INTERSECTION BY DISTANCES

The coordinates of a new point can also be determined by measuring distances from (or to) two points of known coordinates. In Figure 7.4, the coordinates (y_k, x_k) of new point k can be determined by measuring the distances d_{ik} and d_{jk} instead of β and γ. This procedure is preferable when point *k* is accessible; especially when measuring distances is easier and faster. This method is also widely used when calculating the coordinates of unknown boundary points when there is a need to fix boundaries of land parcels on the ground.

The solution here is similar to that used for the method of intersection by angles. The angle β is computed using the Cosine law.

$$d_{jk}^2 = d_{ij}^2 + d_{ik}^2 - 2d_{ij}d_{ik}\cos\beta$$

$$\Rightarrow \quad \beta = \cos^{-1}\left[\frac{d_{ij}^2 + d_{ik}^2 - d_{jk}^2}{2d_{ij}\cdot d_{ik}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.14)$$

The coordinates of point *k* are then calculated by using equations (7.3), (7.4) and (7.5).

EXAMPLE 7.5:

In example 7.4 and referring to Figure 7.4, the following distance measurements were made from points *i* & *j* to point *k:*

$d_{ik} = 888.86$ m , $\quad d_{jk} = 950.55$ m

Compute the horizontal coordinates y_k and x_k.

SOLUTION:

From equation (7.14)

$$\beta = \cos^{-1}\left[\frac{d_{ij}^2 + d_{ik}^2 - d_{jk}^2}{2d_{ij}\cdot d_{ik}}\right]$$

$$= \cos^{-1}\left[\frac{1118.29^2 + 888.86^2 - 950.55^2}{2\text{x}1118.29\text{x}888.86}\right] = 55^\circ\, 06'\, 42''$$

From example 7.4, $\alpha_{ij} = 62^\circ\, 32'\, 44''$

$\alpha_{ik} = \alpha_{ij} + \beta = 62^\circ\, 32'\, 44'' + 55^\circ\, 06'\, 42'' = 117^\circ\, 39'\, 26''$

Then,

$y_k = y_i + d_{ik}\sin\alpha_{ik} = 175329.41 + 888.86\sin(117^\circ\, 39'\, 26'')$
$\quad = 176116.71$ m

$x_k = x_i + d_{ik}\cos\alpha_{ik} = 184672.66 + 888.86\cos(117^\circ\, 39'\, 26'')$
$\quad = 184260.07$ m

7.2.7 RESECTION

The horizontal position of a new point can also be determined by measuring angles from a point to three points of known coordinates. This method is called *resection*. It is mainly used when the surveyor is standing at a point of unknown coordinates and is facing three points of known coordinates that may be far away or inaccessible. In Figure 7.5, A, B and C are points of known coordinates, and hence distances b and c can be calculated. The coordinates of point P are to be found by measuring angles M and N.

Let,

$$J = \beta + \gamma \qquad \text{.............................. (7.15)}$$

Then,

$$J = 360° - (M + N + R) \qquad \text{.............................. (7.16)}$$

By the Sine law,

$$\frac{AP}{\sin\,\beta} = \frac{c}{\sin M} \quad \Rightarrow \quad AP = \frac{c \cdot \sin\beta}{\sin M} \qquad \text{.......................... (7.17)}$$

Similarly,

$$AP = \frac{b \cdot \sin\gamma}{\sin N}$$

Therefore,

$$\frac{c \cdot \sin\beta}{\sin M} = \frac{b \cdot \sin\gamma}{\sin N} \qquad \text{.............................. (7.18)}$$

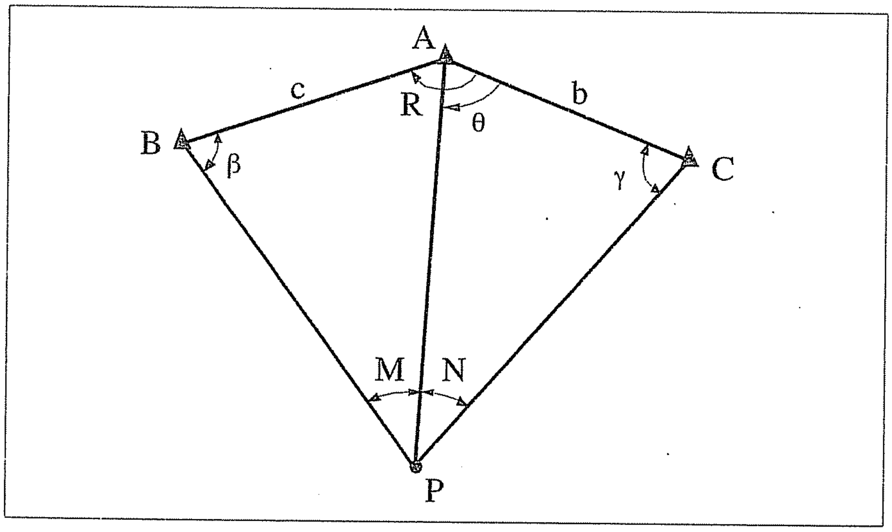


FIGURE 7.5: Resection.

Let,

$$H = \frac{\sin \beta}{\sin \gamma} \qquad \text{(7.19)}$$

Then, from Equation (7.18)

$$\Rightarrow \quad H = \frac{b \sin M}{c \sin N} \qquad \text{(7.20)}$$

Also from Equation (7.19)

$$H \sin \gamma = \sin \beta = \sin(J - \gamma)$$

Or $\quad H \sin \gamma = \sin J \cos \gamma - \cos J \sin \gamma$

Dividing both sides by cos γ and rearranging the terms:

$$\Rightarrow \quad \tan \gamma = \frac{\sin J}{H + \cos J} \qquad \text{(7.21)}$$

Also, $\quad \theta = 180° - N - \gamma \qquad$ (7.22)

The following general procedure is used to compute the coordinates of point P:

1. Compute b, c, azimuths α_{AB} and α_{AC}, and R from the known coordinates of points A, B and C.
2. Compute J using Equation (7.16).
3. Compute H using Equation (7.20)
4. Compute the angle γ using Equation (7.21).
5. Compute the angle θ using Equation (7.22).
6. Compute the azimuth α_{AP} of line AP, $\alpha_{AP} = \alpha_{AC} + \theta$
7. Compute AP, from Equation (7.17).
8. Compute y_P and x_P as follows:

$$y_P = y_A + AP \sin \alpha_{AP}$$
$$x_P = x_A + AP \cos \alpha_{AP}$$

EXAMPLE 7.6:

Given the following coordinates for points A, B and C in Figure 7.5:

Point	y-coordinate	x-coordinate
A	146732.41 m	138111.26 m
B	142139.65 m	136781.33 m
C	149822.47 m	137266.32 m

The measured angles are: $M = 37°\ 21'\ 33''$ $N = 41°\ 03'\ 56''$

Compute the coordinates (y_P, x_P) of point P.

SOLUTION:

$$c = \sqrt{(142139.65 - 146732.41)^2 + (136781.33 - 138111.26)^2}$$
$$= 4781.44 \text{ m}$$

$$b = \sqrt{(149822.47 - 146732.41)^2 + (137266.32 - 138111.26)^2}$$
$$= 3203.50 \text{ m}$$

$$\alpha_{AB} = \tan^{-1}\left(\frac{142139.65 - 146732.41}{136781.33 - 138111.26}\right) + 180° = 253°\,51'\,02''$$

$$\alpha_{AC} = \tan^{-1}\left(\frac{149822.47 - 146732.41}{137266.32 - 138111.26}\right) + 180° = 105°\,17'\,35''$$

$$R = \alpha_{AB} - \alpha_{AC} = 148°\ 33'\ 27''$$

$$J = 360° - (M + N + R) = 133°\ 01'\ 04''$$

$$H = \frac{b \sin M}{c \sin N} = \frac{3203.50 \sin(37°\ 21'\ 33'')}{4781.44 \sin(41°\ 03'\ 56'')} = 0.61887727$$

$$\tan\gamma = \frac{\sin J}{H + \cos J} = \frac{\sin(133°\,01'\,04'')}{0.61887727 + \cos(133°\,01'\,04'')} = -11.5416786$$

$\Rightarrow \gamma = 94°\ 57'\ 07''$ (because the tangent is negative, then γ has to be between $90°$ & $180°$)

$$\theta = 180° - N - \gamma = 43°\ 58'\ 57''$$

$$AP = \frac{b \sin\gamma}{\sin N} = \frac{3203.50 \sin(94°57'07")}{\sin(41°03'56")} = 4858.33 \text{ m}$$

$$\alpha_{AP} = \alpha_{AC} + \theta = 105°\ 17'\ 35" + 43°\ 58'\ 57"$$
$$= 149°\ 16'\ 32"$$

$$y_P = y_A + AP \sin \alpha_{AP}$$
$$= 146732.41 + 4858.33 \sin(149°\ 16'\ 32") \quad = 149214.58 \text{ m}$$

$$x_P = x_A + AP \cos \alpha_{AP}$$
$$= 138111.26 + 4858.33 \cos(149°\ 16'\ 32") \quad = 133934.87 \text{ m}$$

7.2.8 MAPPING DETAILS USING EDM

To illustrate how a theodolite-EDM combination or a total station can be used to measure and map field details, let us refer to Figure 7.6 which shows a sketch for a land parcel which is to be surveyed and mapped. To do so, the instrument is set up at a known station, such as P which can be inside or outside the area to be surveyed, and from which all points can be seen. The telescope is then directed towards another known point so that the azimuth between station P and this known point can be calculated from the known coordinates. The horizontal circle of the theodolite or total station is then set to read zero. If no points of known coordinates were available in the area, or if national grid coordinates for the area were not required, a station P with assumed coordinates and an arbitrary azimuth can be chosen. The resulting coordinates with respect to the assumed datum can be transformed to national grid coordinates using the equations in the next section if the national grid coordinates of two points were known, in addition to their local coordinates.

The instrument is then directed to points 1, 2, 3, etc., and for each point the following data are recorded: point number, slope distance, horizontal circle reading, vertical circle reading, reflector height and any notes about the point such as being an iron angle, pole, etc. Sometimes, the horizontal distance and elevation difference ($\Delta H'$) between the instrument and the reflector are recorded instead of the slope distance and the vertical circle reading. Table 7.1 shows a typical page from a field book used to record the EDM data.

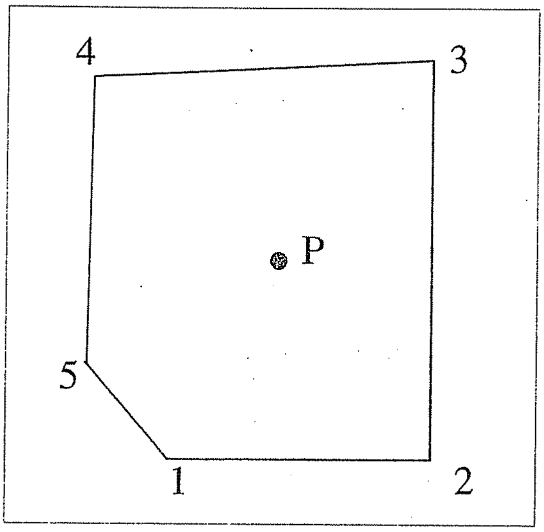


FIGURE 7.6: A sketch for a land parcel.

EXAMPLE 7.7

For the land parcel shown in Figure 7.6, the instrument was stationed at point P and the following measurements were taken:

Station No. (محطة الجهاز) P **H.I.** (ارتفاع الجهاز) 1.50 (m)

2002 / 4 /12 : التاريخ مراد معروف نجار : اسم المساح

رقم النقطة Point #	المسافة المائلة S (m)	الزاوية الأفقية (H.A) °	'	"	الزاوية السمتية (Z.A.) °	'	"	ارتفاع العاكس HT (m)	ملاحظات Notes
1	16.85	0	00	00	83	20	55	1.60	+ (صليب)
2	22.29	294	51	07	88	33	25	1.60	سور
3	32.75	188	05	03	92	11	42	1.60	سور
4	31.22	133	58	47	89	05	11	1.60	سور
5	15.07	48	26	24	82	23	24	1.60	A.I.

Given that the coordinates and elevation of point P are (Y = 100.00, X = 100.00, H = 300.00) and that the azimuth of the line from P to 1 is 195° 00' 00", calculate the elevations and coordinates of all the points.

SOLUTION:

1) Elevations: the elevation (H) of any point is calculated from the following equation:

$$H = H_p + S.\cos z + i - t$$

The calculations are shown in the next table:

Point #	Slope Dist. S	Zenith Angle (z) ° ' "	Horizontal Dist. D = S.sin z	t	S.cos z ($\Delta H'$)	Elev. Diff. $\Delta H = \Delta H' + i - t$	Elevation $H = H_p + \Delta H$
1	16.85	83 20 55	16.74	1.60	1.95	1.85	301.85
2	22.29	88 33 25	22.28	1.60	0.56	0.46	300.46
3	32.75	92 11 42	32.73	1.60	-1.25	-1.35	298.65
4	31.22	89 05 11	31.22	1.60	0.50	0.40	300.40
5	15.07	82 23 24	14.94	1.60	2.00	1.90	301.90

2) Coordinates: the coordinates (y and x) of any point *i* are calculated from the following equations:

$$y_i = y_p + d_{pi} \sin \alpha_{pi} \quad , \text{ and}$$

$$x_i = x_p + d_{pi} \cos \alpha_{pi} \quad ,$$

where α_{pi} is the azimuth of line *pi* and is equal to the azimuth of line *p1* (given as 195° 00' 00") plus the horizontal angle between lines *p1* and *pi*. The calculations are shown in the next table:

Point #	Horiz. Dist. (D)	Horiz. Angle ° ' "	Azimuth (α_{pi}) ° ' "	$\Delta y = d_{pi}.\sin\alpha_{pi}$	$\Delta x = d_{pi}.\cos\alpha_{pi}$	$y = y_p + \Delta y$	$x = x_p + \Delta x$
1	16.74	00 00 00	195 00 00	-4.33	-16.17	95.67	83.83
2	22.28	294 51 07	129 51 07	17.11	-14.28	117.11	85.72
3	32.73	188 05 03	23 05 03	12.83	30.11	112.83	130.11
4	31.22	133 58 47	328 58 47	-16.09	26.75	83.91	126.75
5	14.94	48 26 24	243 26 24	-13.36	-6.68	86.64	93.32

TABLE 7.1: A typical page from an EDM field book.

Station No. ____________ H.I. (ارتفاع الجهاز) ____________ (m)

/ / : التاريخ ______________________________ : اسم المساح

رقم النقطة Point #	المسافة المائلة S.D. (m)	المسافة الأفقية H.D. (m)	ΔH' (m)	(H.A.) الزاوية الأفقية			(Z.A.) الزاوية السمتية			ارتفاع العاكس HT (m)	ملاحظات Notes
				°	'	"	°	'	"		

Now, assume that some points (features) cannot be seen from the selected instrument location (such as P). To illustrate this problem, let us refer again to Figure 7.6 and assume that an obstacle between station P and point 3 (such as a building) prevents seeing the reflector at this point (Figure 7.7). The following procedure solves this problem:

1. Choose another station (such as Q) from which the remaining points (which could not be seen from station P) can be seen.

2. With the instrument still located at P, take measurements to this new station so that its coordinates can be calculated.

3. Move the instrument and set it up at the new station Q (the coordinates of Q are known at this stage).

4. Direct the instrument towards the old station P or any other point with known coordinates and set the horizontal angle to read zero. Now rotate the instrument in a clockwise direction and start taking measurements at the points that have not been observed from the previous station (point 3 in Figure 7.7).

5. Calculate the coordinates of these points in the same manner as explained in example 7.7.

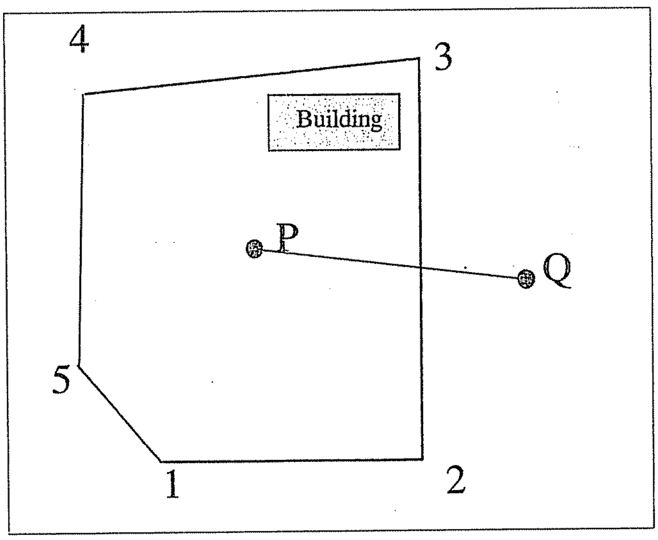


FIGURE 7.7: A land parcel with an obstacle.

6. It is a good practice to repeat the readings at a point that has been read from the previous station. The coordinates computed for this point from both stations should be identical. If they differ by a big amount, this means that an error has occurred when moving from the first station to the second.

7. If there are still some points, which could not be observed from the first and second stations, a third station can be chosen, and the previous process repeated until all points are observed.

7.2.9 TRANSFORMATION OF COORDINATES

When horizontal control points of known coordinates are not available in the area to be surveyed, the surveyor can set up his instrument (total station or theodolite - EDM combination) at any suitable station (national grid coordinates are unknown) and direct it to an assumed azimuth. In this case, the calculated coordinates of all the surveyed points will be referenced to the assumed local datum. If later, the national grid coordinates of at least two of the surveyed points are known, the local coordinates of all the other surveyed points can be transformed to the national grid coordinate system using what is called similarity transformation.

To illustrate this transformation from one coordinate system to another, let us refer to Figure 7.8. Assume that the local coordinate system is represented by the $(y' - x')$ axes and the national grid coordinate system is represented by the $(y - x)$ axes. Assume also that the angle between the two coordinate systems is β (β is positive counterclockwise and negative clockwise). In order to bring the local $(y' - x')$ coordinate axes to coincide with the grid $y - x$ coordinate system, the following steps are to be taken:

1 - Rotate the $(y' - x')$ coordinate system by an angle β (see Figure 7.8) so that the resulting $(y'' - x'')$ axes will become parallel to the $y - x$ axes.

2 - If there is a scale difference between the objects in the $(y' - x')$ system and their counterparts in the $(y - x)$ system, the rotated coordinates in step 1 should be multiplied by a scale factor (s).

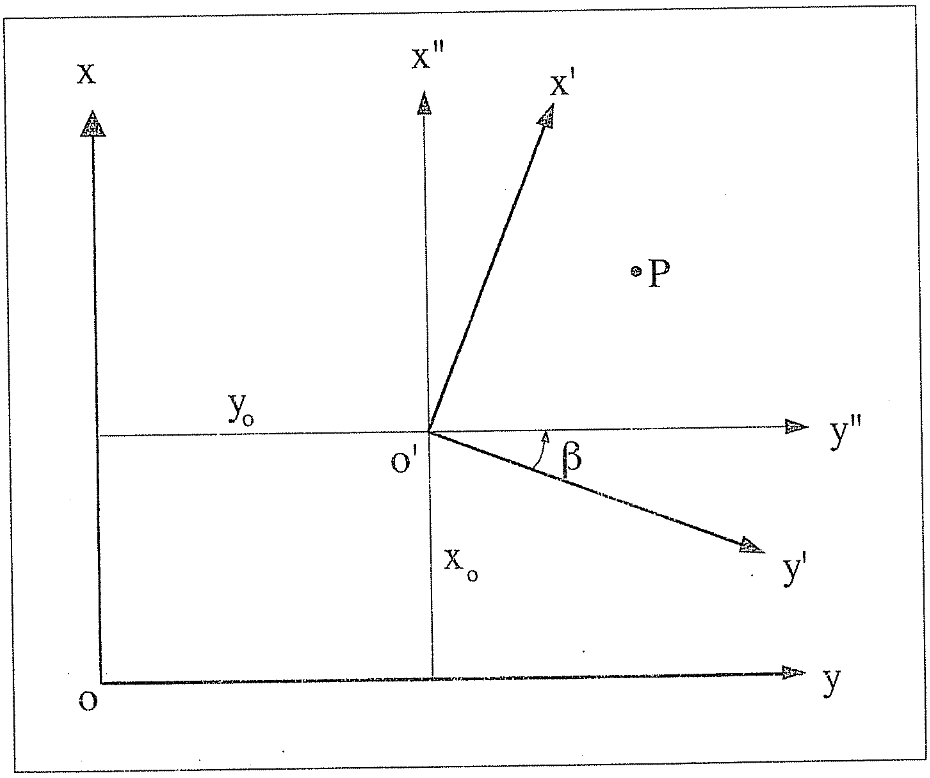


FIGURE 7.8: Relation between two-dimensional coordinate systems.

3 - Translate the o' to o by moving it a distance x_0 parallel to the x - axis and a distance y_0 parallel to the y - axis.

Mathematically, assume that the local coordinates of point P are $(y'_p\ ,\ x'_p)$, then the grid coordinates $(y_p\ ,\ x_p)$ are given by:

$$y_p = y_0 + s\left(y'_p \cdot \cos\beta + x'_p \cdot \sin\beta\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.23)$$

$$x_p = x_0 + s\left(-y'_p \cdot \sin\beta + x'_p \cdot \cos\beta\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.24)$$

Equations (7.23) and (7.24) have four unknowns which are: y_0, x_0, s and β. In order to solve for these unknowns, four equations are needed. This means that two points of known coordinates in both coordinate systems are needed. Each point will provide two equations. In most field surveying problems, the scale between the local and the national grid coordinate systems is the same (i.e., s = 1). The angle β between the two coordinate systems can be calculated as follows:

$$\beta = AZ_g - AZ_\ell \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.25)$$

Where AZ_g is the grid azimuth of the line joining the two known points, and
AZ_ℓ is the local azimuth of the line joining the two known points.

The remaining two unknowns (y_0 and x_0) are calculated from equations (7.23) and (7.24) as follows:

$$y_0 = y_p - \left(y'_p \cdot \cos\beta + x'_p \cdot \sin\beta\right) \quad \dots\dots\dots\dots\dots\dots (7.26)$$

$$x_0 = x_p - \left(-y'_p \cdot \sin\beta + x'_p \cdot \cos\beta\right) \quad \dots\dots\dots\dots\dots\dots (7.27)$$

Note: If the national grid coordinates are known for more than 2 points, then the 4 unknowns of the similarity transformation (s, β, x_0, y_0) are calculated using the principle of least squares adjustment (see Chapter 10).

EXAMPLE 7.8:

For the land parcel in Example 7.7, assume that the grid coordinates of points 1 & 2 are as follows: 1 (182790.00 , 174519.32),
2 (182811.51 , 174519.32).

Calculate the transformed coordinates of points 3, 4 and 5.

SOLUTION

From Equation (7.25), $\beta = AZ_g - AZ_\ell$

The grid azimuth

$$AZ_g = \tan^{-1}\left(\frac{182811.51 - 182790.00}{174519.32 - 174519.32}\right) = 90° \; 00' \; 00''$$

The local azimuth $AZ_\ell = \tan^{-1}\left(\frac{117.10 - 95.67}{85.72 - 83.83}\right) = 84° \, 57' \, 36''$

$\Rightarrow \beta = 90° \; 00' \; 00'' - 84° \; 57' \; 36'' = 5° \; 02' \; 24''$

From Equations (7.26) and (7.27),

$$y_0 = 182790.00 - \left(95.67 \cos(5°\ 02'\ 24") + 83.83 \sin(5°\ 02'\ 24")\right)$$
$$= 182687.34$$

$$x_0 = 174519.32 - \left(-95.67 \sin(5°\ 02'\ 24") + 83.83 \cos(5°\ 02'\ 24")\right)$$
$$= 174444.22$$

Now applying Equations (7.23) and (7.24) on the assumed coordinates of points 3, 4 and 5 (with s=1) gives:

$$y_3 = 182687.34 + \left(112.83 \cos(5°\,02'\,24") + 130.10 \sin(5°\,02'\,24")\right)$$
$$= 182811.16$$

$$x_3 = 174444.22 - \left(-112.83 \sin(5°\,02'\,24") + 130.10 \cos(5°\,02'\,24")\right)$$
$$= 174563.90$$

In the same manner, the coordinates of points 4 & 5 are:
4 (182782.07 , 174563.11) , 5 (182781.85 , 174529.57)

7.3 TRAVERSE SURVEYING

Traverse surveying is a measurement procedure used for determining the horizontal relative positions (y & x coordinates) of a number of survey points. While leveling is used to establish the elevations of points, traverse surveying is used to determine the horizontal coordinates of these points. Basically, it consists of repeated application of the method of locating by angle and distance (Section 7.2.2). By starting from a point of known horizontal coordinates and a line of known direction, the location of a new point is determined by measuring the distance and angle from the known point. Then, the location of another new point is determined by angle and distance measurement from the newly located point. This procedure is repeated from point to point. The resulting geometric figure is called a ***traverse***.

7.3.1 PURPOSE OF THE TRAVERSE

The traverse serves several purposes among which are:

1 - Property surveys to establish boundaries.

2 - Location and construction layout surveys for highways, railways and other works.

3 - Providing Ground control points for photogrammetric mapping.

7.3.2 TYPES OF TRAVERSE

There are two main types of traverse:

a) *Open traverse* (Figure 7.9). This originates at a point that could be of known or unknown position and terminates at a different point of unknown position. To minimize errors, distances can be measured twice, angles repeated, and magnetic azimuth observed on all lines. This type is used in certain works such as locating the centerline of a tunnel during construction. For projects requiring high accuracy, it is preferable not be used.

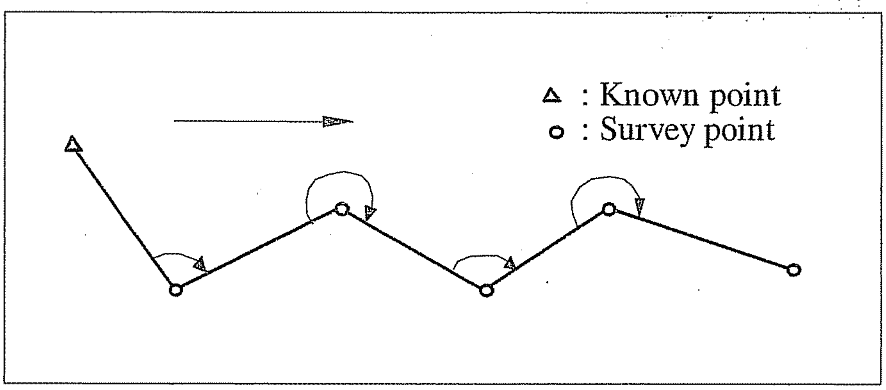


FIGURE 7.9: Open traverse.

b) *Closed traverse* (Figure 7.10). This type originates at a point (of known or assumed position) and terminates at the same point yielding a closed loop traverse (Figure 7.10a), or originates at a line of known coordinates and ends at another line of known coordinates (or a coordinate and direction) yielding a closed connecting traverse (Figure 7.10b). This is a requirement by the Survey Department in Palestine. The closed traverse is preferred to the open traverse because it provides a check on errors.

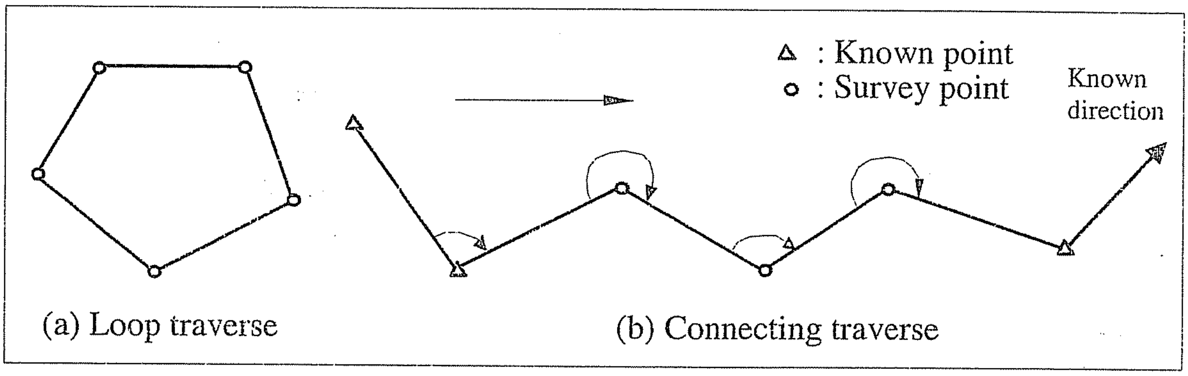


FIGURE 7.10: Closed traverse

7.3.3 CHOICE OF TRAVERSE STATIONS

Traverse stations should be located so that:

1 - Traverse lines should be as close as possible to the details to be surveyed.

2 - Distances between traverse stations should be approximately equal and the shortest line should be greater than one third of the longest line.

3 - Stations should be chosen on firm ground, or monumented in a way to make sure that they are not easily lost or damaged.

4 - When standing on one station, it should be easy to see the backsight and foresight stations.

7.3.4 TRAVERSE COMPUTATIONS AND CORRECTION OF ERRORS

The following computations and error correction are usually associated with traverse surveying.

A) Azimuth of a line:

The requirement here is to calculate the azimuth (α_2) of line BC from the known azimuth (α_1) of line AB and the clockwise measured angle (ϕ) between the two lines (see Figure 7.11). Two cases can be distinguished here:

1. When $(\alpha_1 + \phi) > 180°$. From Figure 7.11:

$$\alpha_2 = \phi - (180° - \alpha_1)$$

$$\Rightarrow \alpha_2 = \phi + \alpha_1 - 180° \qquad \text{(7.28)}$$

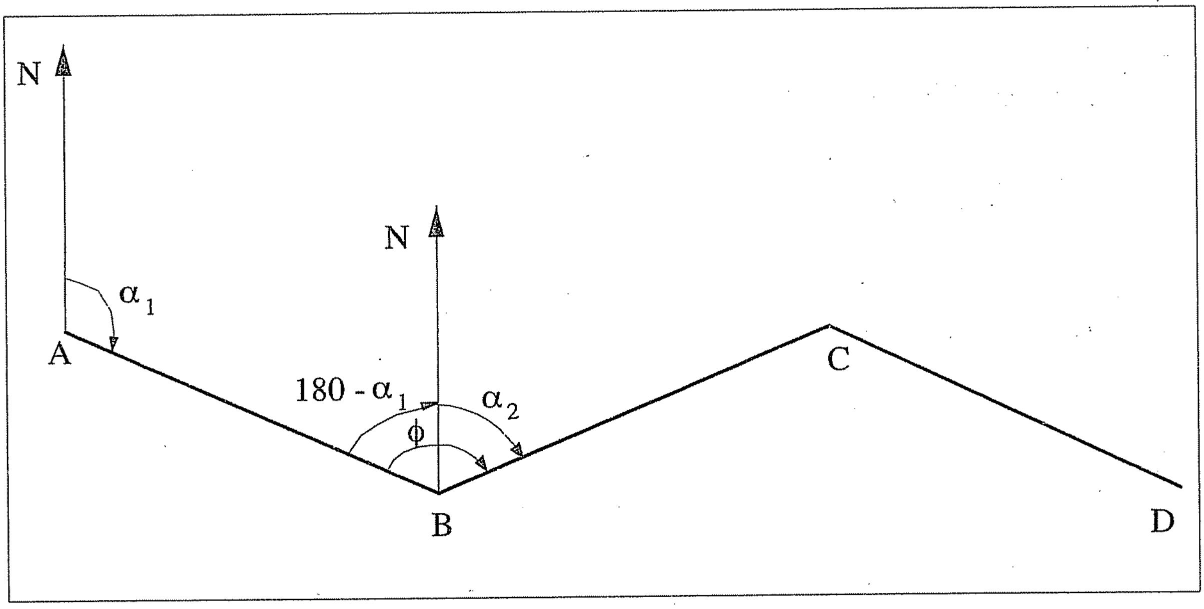


FIGURE 7.11: Case (1): $\alpha_1 + \phi > 180°$

2. When $(\alpha_1 + \phi) < 180°$. From Figure 7.12):

$$\alpha_2 = \phi + 180° + \alpha_1$$

$$\Rightarrow \alpha_2 = \phi + \alpha_1 + 180° \qquad \text{(7.29)}$$

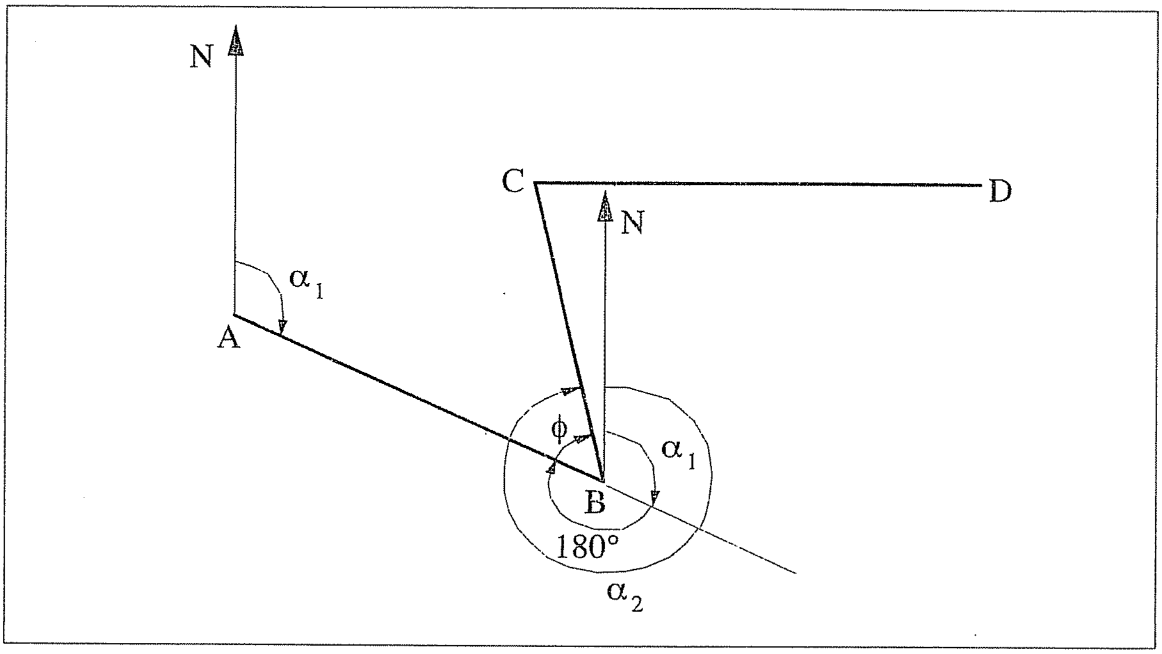


FIGURE 7.12: Case (2): $\alpha_1 + \phi < 180°$

In general,

$$\alpha_2 = \alpha_1 + \phi \pm 180° \qquad \text{(7.30)}$$

Where α_2 = the azimuth of the following line,

α_1 = the azimuth of the previous line, and

ϕ = the clockwise angle between the previous line and the following line.

If $\alpha_1 + \phi < 180°$ $\Rightarrow$ add 180° to get α_2

If $\alpha_1 + \phi > 180°$ $\Rightarrow$ subtract 180° to get α_2

Now, if the coordinates of point B are known, then the coordinates of point C are calculated as follows:

$$y_C = y_B + d_{BC} \cdot \sin \alpha_2$$

$$x_C = x_B + d_{BC} \cdot \cos \alpha_2$$

B) Checks and Correction of Errors:

When performing the traverse calculations, the following conditions must be satisfied for a closed traverse (Figure 7.13):

$$y_{\text{last point}} - y_{\text{first point}} = \Sigma\Delta y_{\text{all lines}} \quad \ldots\ldots\ldots\ldots (7.31)$$

$$x_{\text{last point}} - x_{\text{first point}} = \Sigma\Delta x_{\text{all lines}} \quad \ldots\ldots\ldots\ldots (7.32)$$

When the last point is the same as the first point, then: $\Sigma\Delta y = 0$ and $\Sigma\Delta x = 0$.

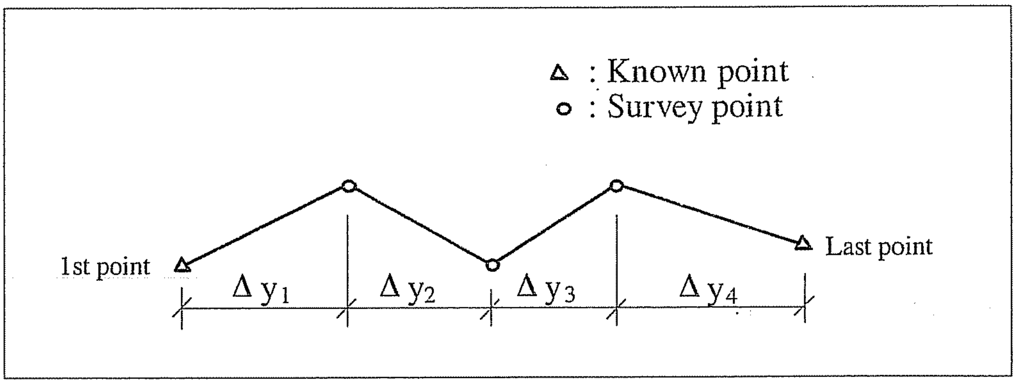


FIGURE 7.13: A closed connecting traverse.

In order to meet the previous two conditions (equations 7.31 and 7.32), the following checks and corrections are performed:

(1) ***Angle Correction.*** This can be done in two ways:

a) Closed loop traverse. For a closed loop traverse of *n* sides (Figure 7.14):

Sum of internal angles = (n - 2) x 180° . . . . (7.33)

For the traverse in Figure 7.14:
Σ Internal angles = (5 - 2) x 180° = 540°
If the sum is found to be 540° 00' 15", ⇒ Error = +15"

No. of internal angles = 5
⇒ Correction for each angle = -15"/5 = -3"
⇒ Subtract 3" from each angle

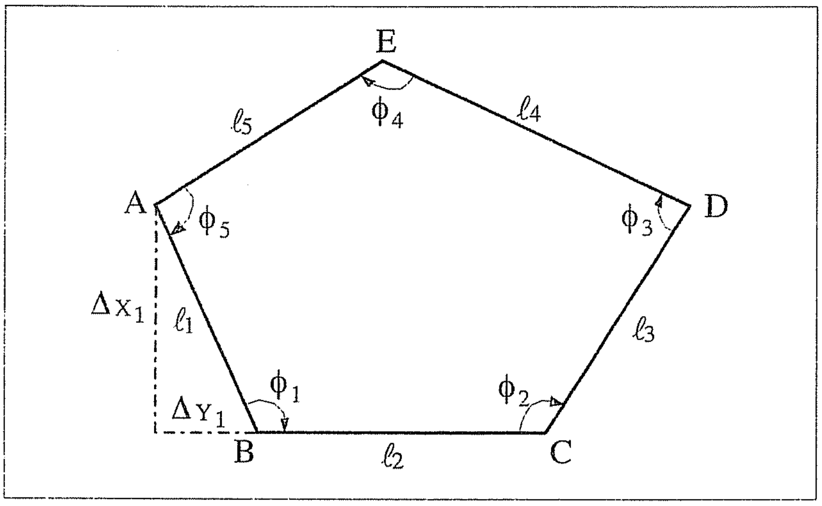


FIGURE 7.14: A closed loop traverse.

b) *For both loop and connecting closed traverses.* If the azimuth of the last line in the traverse is known, this azimuth is compared with the calculated azimuth, and the error is distributed between the angles. Assume that the known azimuth of last line is α_n , and the calculated one is α_c, then the error ε_α is:

$$\varepsilon_\alpha = \alpha_c - \alpha_n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.34)$$

For n measured angles:

$$\Rightarrow \text{Correction/angle} = -\frac{\varepsilon_\alpha}{n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.35)$$

This correction is added to each angle in the traverse and the azimuths of all lines are recalculated.

To save time and effort, the azimuth correction can be applied to the preliminary computed azimuths directly in order to get to the corrected ones. Let α'_i be the initially computed azimuth of the *i-th* line in the traverse, then the corrected azimuth α_i of this line is:

$$\alpha_i = \alpha'_i - i \cdot \left(\frac{\varepsilon_\alpha}{n}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.36)$$

(2) ***Position Correction.*** It always happens to have small errors in the values of $\Sigma\Delta y$ and $\Sigma\Delta x$ in such a way that they will not meet the condition of equations (7.31) and (7.32). This happens even after correcting the angles. The reason is that the errors in the measured distances have not yet been accounted for.

Assume that the calculated and known coordinates of the last point are (y_c, x_c) and (y_n, x_n) respectively. Then:

Closure error in the y-direction $(\varepsilon_y) = y_c - y_n$ .... (7.37)

Closure error in the x-direction $(\varepsilon_x) = x_c - x_n$ .... (7.38)

Closure error in the position of the last point:

$$\varepsilon = \sqrt{\varepsilon_y^2 + \varepsilon_x^2} \qquad (7.39)$$

This is called the linear error of closure and it is equal to the length of the error vector connecting the calculated point with the known point.

How to correct and distribute this error?

Different methods can be used for the correction of the position closure error. These include the compass rule and the transit rule. Only the compass rule will be discussed here.

Compass Rule (Bowditch Rule):

This rule distributes the correction on the departures (Δy) and latitudes (Δx) of the measured traverse lines as follows:

$$\text{Correction to } \Delta y \text{ of line ij} = -\frac{\text{length of line ij}}{\text{total traverse length}} \cdot \varepsilon_y \qquad (7.40)$$

$$\text{Correction to } \Delta x \text{ of line ij} = -\frac{\text{length of line ij}}{\text{total traverse length}} \cdot \varepsilon_x \qquad (7.41)$$

The computation of the coordinates is now repeated using the corrected Δy's and Δx's. Alternatively, the preliminary computed coordinates of the traverse points can be corrected directly in the same manner like the azimuths. This is performed as follows:

$$cy_i = -\frac{L_i}{D} \cdot \varepsilon_y \quad \text{(7.42)}$$

$$cx_i = -\frac{L_i}{D} \cdot \varepsilon_x \quad \text{(7.43)}$$

Where cy_i and cx_i are the corrections to be applied to the computed coordinates y_i and x_i of station *i*,

L_i = cumulative traverse distance up to station *i*

D = total length of the traverse

The corrected coordinates of station *i* (y_i' , x_i') are:

$$y_i' = y_i + cy_i \quad \text{(7.44)}$$

$$x_i' = x_i + cx_i \quad \text{(7.45)}$$

7.3.5 ALLOWABLE ERRORS IN TRAVERSE SURVEYING

The Department of Surveying in the West Bank allows the following errors in traverse surveying:

	Allowable error	
	Important areas (example: urban areas)	Less important areas (example: rural areas)
Measured distances Measured angles Closure error	$\Delta\ell = 0.0005\ell + 0.03$ m $\Delta = 60''\sqrt{n}$ $\varepsilon = 0.0006\ \Sigma\ell + 0.20$ m	$\Delta\ell = 0.0007\ell + 0.03$ m $\Delta = 90''\sqrt{n}$ $\varepsilon = 0.0009\ \Sigma\ell + 0.20$ m

Where ℓ = measured length, Δ = angle closure error in seconds

n = number of measured angles, $\varepsilon = \sqrt{\varepsilon_y^2 + \varepsilon_x^2}$ (Equation 7.39)

$\Delta\ell$ = allowable error in the measured distance

EXAMPLE 7.9:

Perform the following calculations for the traverse shown in Figure 7.15.

(a) Distribute the angular error of closure and compute the corrected azimuths of all lines.

(b) Calculate the linear and relative errors of closure.

(c) Balance the traverse by the Compass rule and compute the coordinates of the traverse stations.

(d) Compute the distance, azimuth and reduced bearing of each line using the final coordinates.

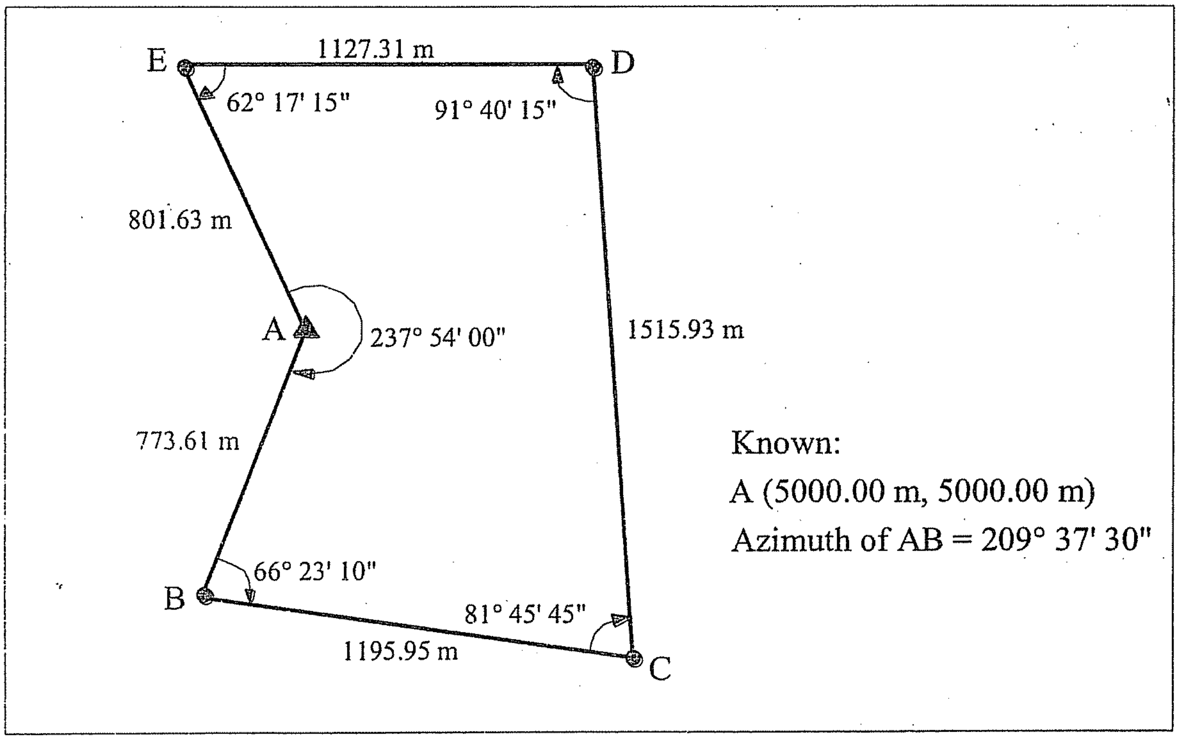


FIGURE 7.15: Traverse of Example 7.9.

SOLUTION:

Line	Preliminary Azimuth	Correction	Corrected Azimuth
AB	209° 37' 30"		
$+\hat{B}$	66 23 10		
	276 00 40		
	- 180		
BC	96 00 40	- 5"	96° 00' 35"
$+\hat{C}$	81 45 45		
	177 46 25		
	+180		
CD	357 46 25	- 10"	357° 46' 15"
$+\hat{D}$	91 40 15		
	449 26 40		
	-180		
DE	269 26 40	- 15"	269° 26' 25"
$+\hat{E}$	62 17 15		
	331 43 55		
	-180		
EA	151 43 55	- 20"	151° 43' 35"
$+\hat{A}$	237 54 00		
	389 37 55		
	-180		
AB	209 37 55	- 25"	209° 37' 30"
	209 37 30		

Closure error = + 25"

Correction per angle = -25"/5 = -5"

Note: It is also possible to correct the angles by comparing their sum to (n-2).180 = 540°, correcting the angles individually, and then recalculating the azimuths again.

TABLE 7.2: Preliminary coordinates.

Station	Corrected Azimuth (α_{ij})	Distance d_{ij} (m)	Departure $\Delta y_{ij} = d_{ij} \sin\alpha_{ij}$	Latitude $\Delta x_{ij} = d_{ij} \cos\alpha_{ij}$	Preliminary coordinates y	x
A					5000.00	5000.00
	209° 37′ 30″	773.61	-382.41	-672.48		
B					4617.59	4327.52
	96° 00′ 35″	1195.95	1189.38	-125.21		
C					5806.97	4202.31
	357° 46′ 15″	1515.93	-58.96	1514.78		
D					5748.01	5717.09
	269° 26′ 25″	1127.31	-1127.26	-11.01		
E					4620.75	5706.08
	151° 43′ 35″	801.63	379.72	-705.99		
A					5000.47	5000.09

$$\sum d_{ij} = 5414.43 \text{ m}$$

Closure error: $\varepsilon_y = +0.47$ m

$\varepsilon_x = +0.09$ m

$$\text{Linear error of closure} = \sqrt{(0.47)^2 + (0.09)^2} = 0.48 \text{ m}$$

$$\text{Relative error of closure} = \frac{1}{5414.43/0.48} \cong \frac{1}{11{,}000}$$

TABLE 7.3: Corrected coordinates.

Station	Cumulative Distance	y-coordinate Preliminary	y Correction	y Final	x-coordinate Preliminary	x Correction	x Final
A	0	5000.00	0	5000.00	5000.00	0	5000.00
B	773.61	4617.59	-0.07	4617.52	4327.52	-0.01	4327.51
C	1969.56	5806.97	-0.17	5806.80	4202.31	-0.03	4202.28
D	3485.49	5748.01	-0.30	5747.71	5717.09	-0.06	5717.03
E	4612.80	4620.75	-0.40	4620.35	5706.08	-0.08	5706.00
A	5414.43	5000.47	-0.47	5000.00	5000.09	-0.09	5000.00

TABLE 7.4: Final Results.

Line	Final Distance	Final Azimuth	Final Reduced Bearing
A – B	773.65	209° 37′ 45″	S 29° 37′ 45″ W
B – C	1195.86	96° 00′ 39″	S 83° 59′ 21″ E
C – D	1515.90	357° 45′ 57″	N 2° 14′ 03″ W
D – E	1127.41	269° 26′ 22″	S 89° 26′ 22″ W
E – A	801.60	151° 43′ 52″	S 28° 16′ 08″ E

EXAMPLE 7.10:

A traverse starts from the line AB and goes through points C, D and E. Given that:

- The coordinates of points A and B are:

	East or Y (m)	North or X (m)
A	8300.40	6310.30
B	8358.30	6031.73

- The angles measured in a clockwise direction are:

$A\hat{B}C = 92° \ 30' \ 10''$

$B\hat{C}D = 164° \ 18' \ 20''$

$C\hat{D}E = 94° \ 55' \ 48''$

- The length of BC = 60.00 m, and
- The reduced bearing of line DE = N 20° W

Calculate:

a) The corrected azimuth for all lines, and

b) The coordinates of point C.

SOLUTION:

a) The corrected azimuth of all lines:

The azimuth of line AB (α_{AB}):

$$\alpha_{AB} = \tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right) = \tan^{-1}\left(\frac{8358.30 - 8300.40}{6031.73 - 6310.30}\right)$$

$$= \tan^{-1}\left(\frac{57.90}{-278.57}\right) = -11° \, 44' \, 30'' + 180° = 168° \, 15' \, 30''$$

The 180° was added because line AB lies in the second quadrant.

The known azimuth of line DE = 360° - 20° = 340°

The calculation of the corrected azimuths is done as follows:

Line		Preliminary Azimuth	Correction	Corrected Azimuth
AB		168° 15' 30"		
	$+\hat{B}$	92 30 10		
		260 45 40		
		- 180		
BC		80 45 40	+ 4"	80° 45' 44"
	$+\hat{C}$	164 18 20		
		245 04 00		
		-180		
CD		65 04 00	+ 8"	65° 04' 08"
	$+\hat{D}$	94 55 48		
		159 59 48		
		+180		
DE		339 59 48	+ 12"	340° 00' 00"

Closure error = - 12"

Correction per angle = +12"/3 = +4"

b) The coordinates of point C are:

$$y_C = y_B + BC \sin AZ_{BC} = 8358.30 + 60.00 \times \sin(80°\,45'\,44")$$
$$= 8417.52 \text{ m}$$
$$x_C = x_B + BC \cos AZ_{BC} = 6031.73 + 60.00 \times \cos(80°\,45'\,44")$$
$$= 6041.36 \text{ m}$$

PROBLEMS

7.1 Figure 7.16 shows the locations of the beginning point 1 and end point 2 of a new tunnel with respect to four survey points: A, B, C and D . The coordinates of two control points A & D and the design coordinates of points 1 & 2 are given below :

POINT	Y-COORDINATE	X-COORDINATE
A	112598.38 m	211121.62 m
D	114985.01 m	209880.34 m
1	113525.91 m	210475.76 m
2	114624.23 m	210837.84 m

The following measurements were made to points B & C:
- Horizontal Distance AB = 1746.76 m,
- Horizontal Distance DB = 1572.87 m
- Horizontal Angle CDA = 29° 59' 35",
- Horizontal Angle DAC = 43° 41' 9.8"

Calculate:

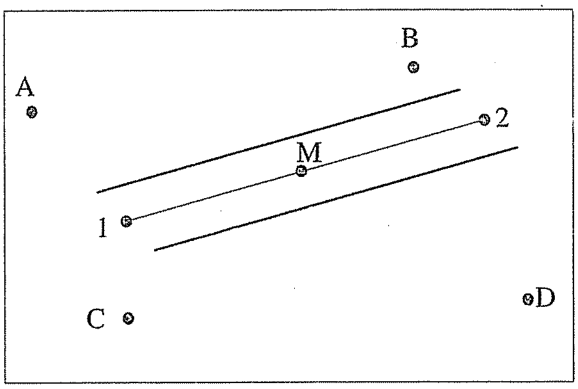


FIGURE 7.16

a. The coordinates of point B.
b. The coordinates of point C.
c. The azimuth and distance of the line from point D to point 2.
d. The coordinates of point M located on the centerline of the tunnel mid-way between points 1 & 2.
e. The coordinates of points E and F which are at distances 10.00 m and 15.00 m from point 1 along line 1-2 with offsets +3.00 m and -4.00 m respectively.
f. Calculate the distance and offset of point B from line 1-2.

7.2 Two circles intersect at points 1 & 2. The coordinates of the centers of these circles are (100.00,100.00) & (200.00,250.00) m, while the radii are 100.00 m and 120.00 m respectively. Calculate the coordinates of the points of intersection 1 & 2.

7.3 In a bridge construction project, the position of the pile-driving barge is determined by the method of resection. A theodolite is set up on the barge, and angles are measured to three control points: A, B and C (Figure 7.17). The coordinates of the control points are:

Control Point	Y-Coordinate	X-Coordinate
A	146298.36 m	92175.27 m
B	143721.45 m	89332.15 m
C	150036.79 m	91628.21 m

The following angles are measured at the barge position P:

BPA = 37° 23' 08"

APC = 33° 47' 36"

Compute the coordinates of the barge position P.

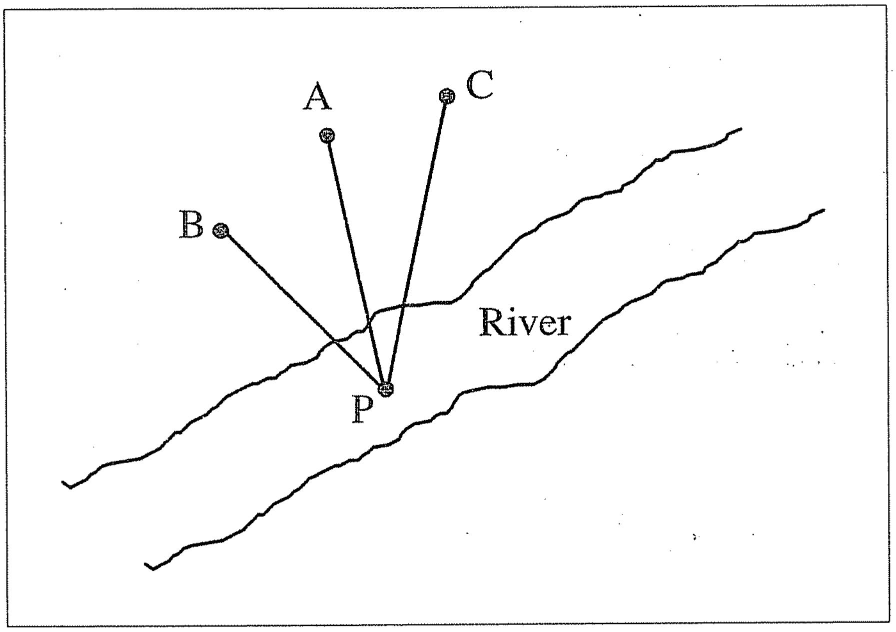


FIGURE 7.17

7.4 For a certain land parcel, the total station was stationed at point 100 and the following measurements were taken:

Station No. (محطة الجهاز): 100 H.I. (ارتفاع الجهاز): 1.60 (m)

التاريخ: 2003 / 7 /23 نضال صادق تميم اسم المساح :

رقم النقطة Point #	المسافة المائلة S (m)	الزاوية الأفقية (H.A.)			الزاوية السمتية (Z.A.)			ارتفاع العاكس HT (m)	ملاحظات Notes
		°	'	"	°	'	"		
1	116.85	0	00	00	93	20	55	1.55	+ (صليب)
2	112.29	12	51	07	90	33	25	2.75	سلسال
3	122.75	18	05	03	88	11	42	3.60	A.I.
4	145.22	10	58	47	89	55	11	1.55	سلسال
5	150.07	356	26	24	84	23	24	1.55	A.I.
6	130.19	15	00	00	95	17	00	3.00	ع.ك

Given that the elevation and coordinates of point 100 are (Y = 500.00 m, X = 500.00 m, H = 200.00) and that the azimuth of the line from 100 to 1 is 0° 00' 00", calculate the elevations and coordinates of all the points.

7.5 In problem 7.4, given that the grid coordinates of points 100 & 1 are as follows:
100 (170320.00 , 185814.50), 1 (170340.26 , 185929.38).

Calculate the transformed coordinates of points 2, 3, 4, 5 and 6.

7.6 The coordinates of points A & B are:

Point	Y-coordinate	X-coordinate
A	368120.30 m	172315.28 m
B	367813.20 m	172603.13 m

A traverse starting at B continues over the points C & D. Compute the coordinates of points C & D if:

- Clockwise angle at B = 95° 16' 30" ,
- Clockwise angle at C = 285° 32' 20"
- Distance BC = 212.50 m,
- Distance CD = 172.30 m

What should be available to check the calculated coordinates?

7.7 Compute the adjusted coordinates of the points of the following closed loop traverse (Figure 7.18):

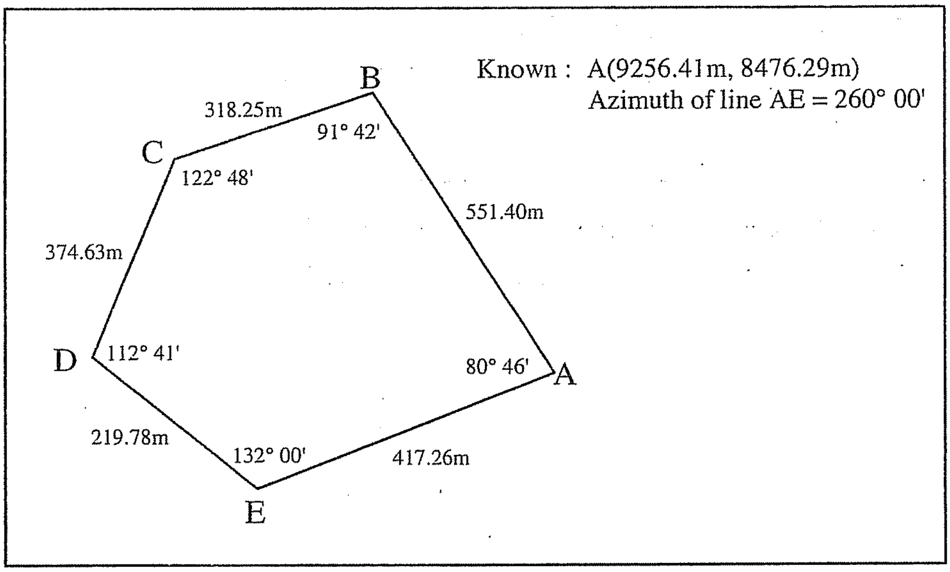


FIGURE 7.18

If one of the five angles in the above figure was wrong (i.e., has a blunder), explain how to find this erroneous angle.

7.8 A five-sided loop traverse has been computed giving the coordinate differences (departures and latitudes) shown in the next table:

Side	ΔY (m)	ΔX (m)
AB	-43.62	-61.39
BC	+70.45	-34.71
CD	+50.85	+48.10
DE	-23.01	+73.37
EA	-53.73	-25.86

(a) Determine the error in both Y and X directions as well as the linear misclosure of the traverse.

(b) The linear misclosure indicates a blunder of +1 m in the length of one of the sides of the traverse. Find which side contains the blunder and, after eliminating its effect, re-compute the requirements of part (a).

7.9 A closed connecting traverse starts at point 221B(y=166238.10m, x=180067.29m) with the total-station directed towards point 203M(y=176000.14m, x=178658.08m), and goes over points 100 and 200, and closes at point 693W(y=164095.24m, x=177510.91m) with the total-station directed towards point 679W (y=168816.43m, x=173371.62m) (Figure 7.19). Does this traverse comply with the specifications of the Department of Surveying in the West Bank? If yes, calculate the corrected coordinates of all points.

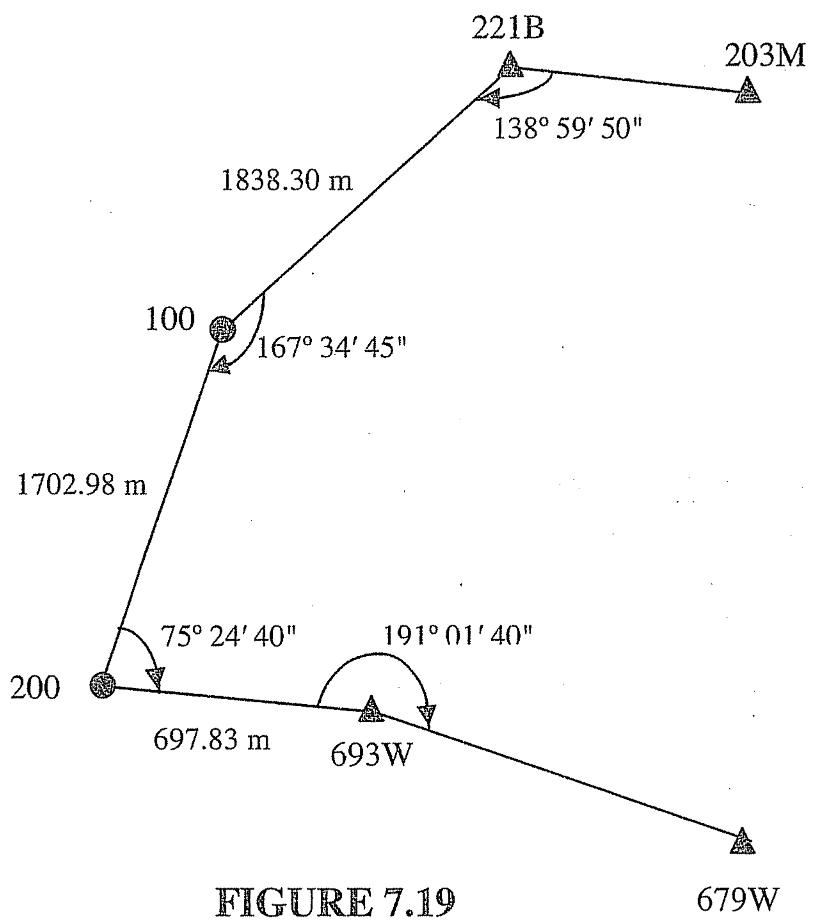


FIGURE 7.19

8 AREAS AND VOLUMES

8.1 INTRODUCTION

Knowing the area enclosed in a tract of land is one of the basic goals for conducting a surveying measurement process. As a result, this chapter will deal with the calculations connected with the measurement of areas of land etc., and of volumes and other quantities connected with engineering and building works. Areas are considered at the beginning, since the computation of areas is involved in the calculation of volumes.

In general, areas can be of regular or irregular shapes. Regular areas are those bounded by straight lines or curved lines with certain defined mathematical formula, such as a circle. Irregular ones are those where at least one part of the boundary cannot be defined by any mathematical formula. Whether being regular or irregular, areas can be computed using mathematical methods, as well as instrumental methods. The instrumental method through the use of a device called planimeter will be considered first.

8.2 THE POLAR PLANIMETER

The planimeter is an instrument by means of which the area of a plotted, closed figure (regular or irregular) may be determined directly by tracing the perimeter and reading the result from the scale. This instrument is illustrated in Figure 8.1. Its main components are:

1. Anchor point or pole P
2. Tracing point T
3. Roller R
4. Tracing arm, and
5. Pole arm

Depending on the size of the plotted area to be measured, the polar planimeter can be used in two ways:

(1) With the pole outside the figure to be measured, and
(2) With the pole inside the figure to be measured.

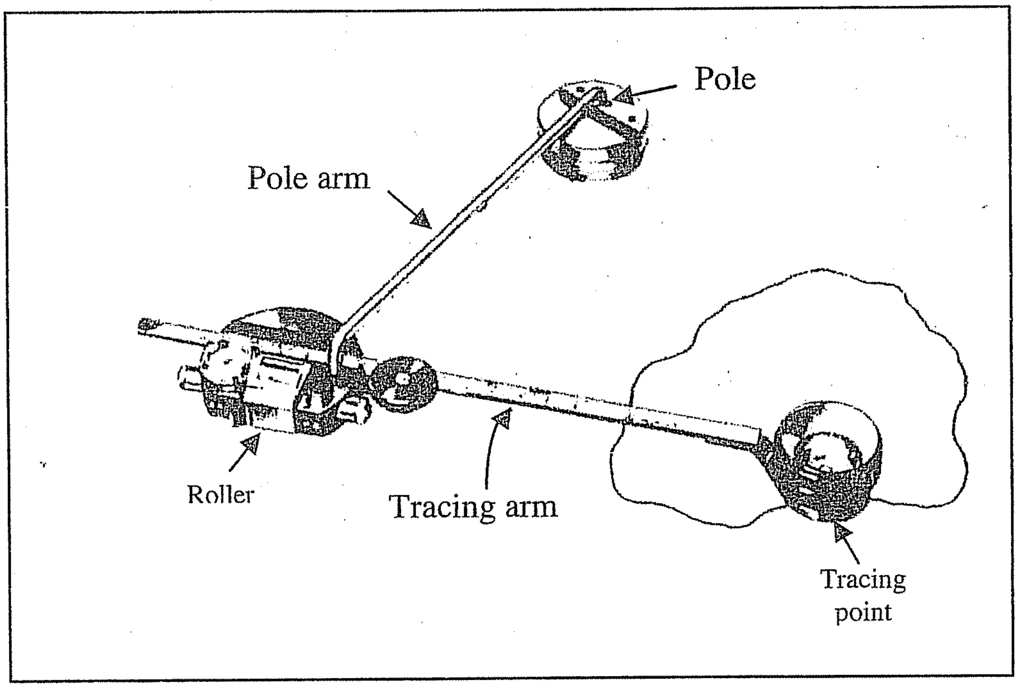


FIGURE 8.1: Optical polar planimeter.

1) The pole outside the figure:

Figure 8.2 illustrates the situation. The planimeter is fixed outside the figure by the anchor pole P. When the tracing point T moves from C to G along the perimeter of the figure, point A moves to E through a circular arc with radius equal to PA = r.

The movement from C to G can be divided into two separate movements: translation from AC to EF and a rotation from EF to EG.

The area ACFGEA = Area ACFE + Area EFG

Let d = length of tracing arm
δs = distance between parallel lines AC & EF
$\delta\alpha$ = Angle FEG

For infinitesimal area δA:
Area ACFE $\approx \delta s \,.\, d$, EF $\approx$ AC = d
FG $\approx$ EF . $\delta\alpha = d \,.\, \delta\alpha$

$$\Rightarrow \text{Area EFG} = \frac{1}{2}(FG \cdot EF) = \frac{1}{2}(d \cdot \delta\alpha \cdot d) = \frac{1}{2}d^2 \cdot \delta\alpha$$

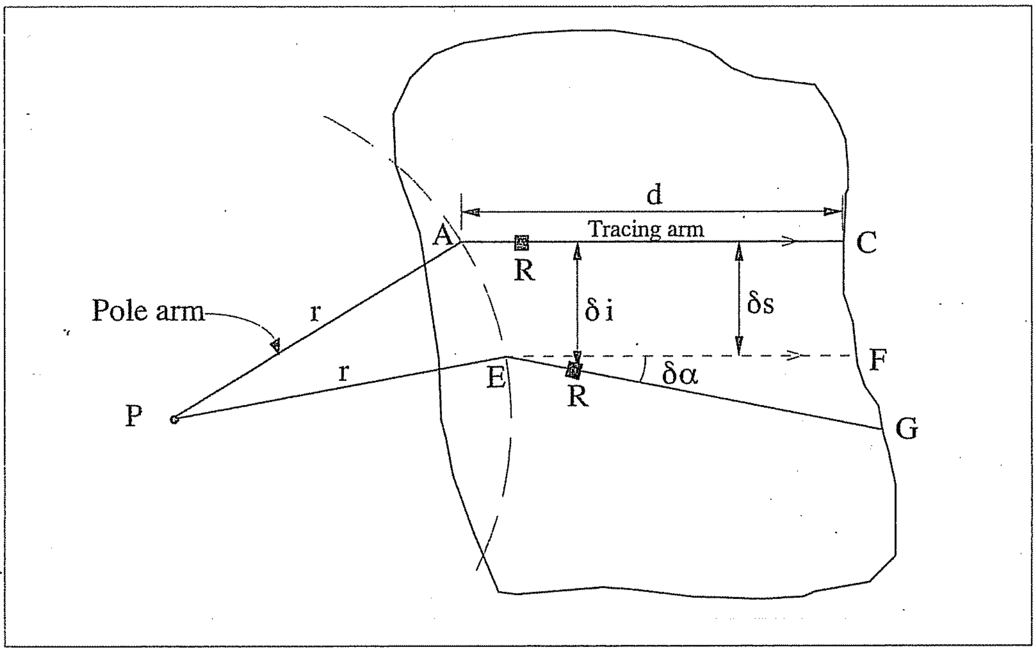


FIGURE 8.2: The planimeter is outside the area to be measured.

$$\Rightarrow \qquad \delta A = d \cdot \delta s + \frac{1}{2} d^2 \cdot \delta\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.1)$$

During the movement of the tracing point from C to G, the roller R, which is at a distance = kd from A, where 0 < k < 1, moves a distance = δi perpendicular to AC.

$$\delta i = \delta s + AR \cdot \delta\alpha = \delta s + k \cdot d \cdot \delta\alpha$$

$$\Rightarrow \quad \delta s = \delta i - k \cdot d \cdot \delta\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.2)$$

Substitute Equation (8.2) into (8.1)

$$\delta A = d\left(\delta i - k \cdot d \cdot \delta\alpha\right) + \frac{1}{2} d^2 \cdot \delta\alpha$$

$$\Rightarrow \quad \delta A = d \cdot \delta i + d^2 \cdot \delta\alpha\left(\frac{1}{2} - k\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.3)$$

If the tracing point is allowed to go through the whole perimeter of the figure, then the enclosed area:

$$A = \int dA = \int\left[d \cdot \delta i + d^2 \cdot \delta\alpha\left(\frac{1}{2} - k\right)\right]$$

$$A = d \int \delta i + d^2\left(\frac{1}{2} - k\right) \int \delta\alpha$$

$$\Rightarrow \quad A = d \cdot i + d^2 \cdot \alpha\left(\frac{1}{2} - k\right) + C \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.4)$$

In this situation, the tracing arm goes back to its start position without making a circle around P, $\Rightarrow \quad \alpha = 0$
$C = 0$

$$\Rightarrow \quad A = d \cdot i \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.5)$$

Where i = algebraic sum of the rotational motion of the roller R. By knowing the number of rotations of R by a counter, and the radius of the roller, i can be calculated.

Since d is constant, and for simplicity, it is replaced by another constant J that takes into account the units of measurement and the scale of the drawing, and its value can be taken from tables prepared for this purpose. Equation (8.5) can be written as:

$$A = J \cdot i \qquad \text{...(8.6)}$$

2) The pole inside the figure:

In the case of the pole being inside the figure, the pole arm traces a circle inside the figure of radius equal to r = length of PA. Figure 8.3 illustrates this case. Therefore, the values of α and C in Equation (8.4) become here 2π and πr^2 (area of the circle generated by the pole arm). Substituting these values into Equation (8.4):

$$A = d \cdot i + d^2 \cdot 2\pi\left(\frac{1}{2} - k\right) + \pi r^2$$

$$\Rightarrow \quad A = d \cdot i + \pi\left(d^2 - 2d^2k + r^2\right) \qquad \text{..........................(8.7)}$$

Or $A = di + $ (the area of a circle of radius $= \sqrt{d^2 - 2d^2k + r^2}$)

This circle is called the zero circle or circle of correction.

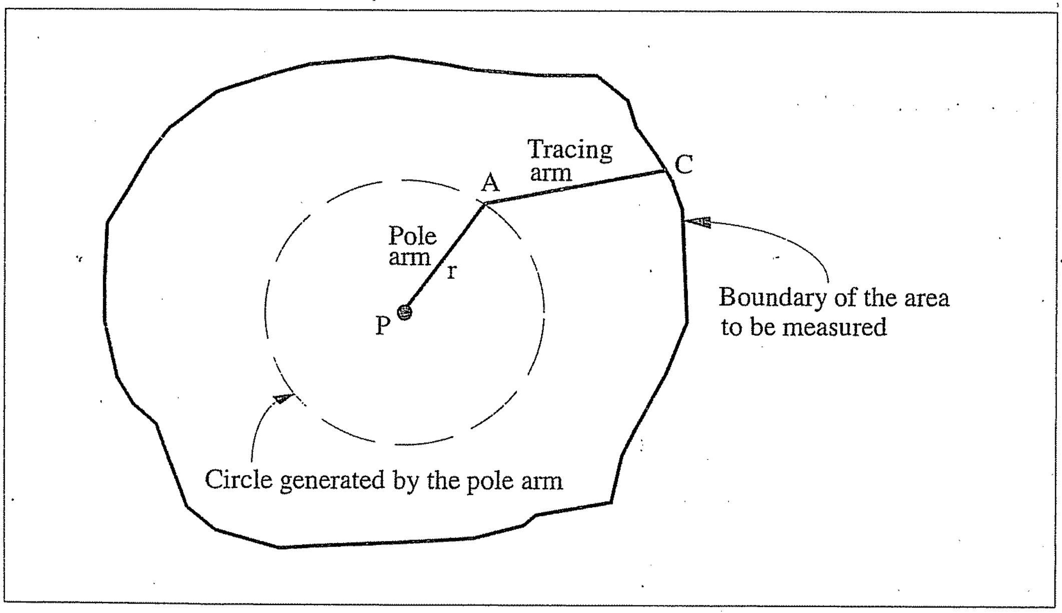


FIGURE 8.3: The planimeter is inside the area to be measured.

If the roller lies between points A and C (Figures 8.2 and 8.3), then $2d^2k$ in Equation (8.7) should be negative, otherwise, it should be positive. Therefore, Equation (8.7) becomes:

$$A = d \cdot i + \pi\left(d^2 \pm 2d^2k + r^2\right) \quad \text{.......................} (8.8)$$

Since d, k and r are all constants for a given planimeter, the area of the circle of correction is constant and it is usually provided by the manufacturers.

The tables of the planimeter constants and conversion factors usually look like:

Map Scale	Length of Tracing Arm	Vernier Unit Equivalent
1:500	17.85 cm	4 m^2
1:1000	.	.
1:2000	.	.
1:5000	.	.
.	.	.
.	.	.

Note 1: If the map scale is not available in the planimeter table, an arm length that belongs to a scale that is close to the map scale is chosen from the table and used and the resulting false area is corrected to the actual map scale as follows:

$$\text{Actual area} = \text{Measured false area} \times \left(\frac{\text{False Scale}}{\text{Actual Scale}}\right)^2$$

Or:

$$A_t = A_f \cdot \left(\frac{S_f}{S_t}\right)^2 \quad \text{.............................} (8.9)$$

Where f : false
t : true

Note 2: The accuracy of the results obtained from using the planimeter in the measurement of areas depends mainly on the original accuracy of the drawn map, as well as on the experience of the operator when tracing the boundary of the figure.

EXAMPLE 8.1:

An area drawn on a map of scale 1:25000 was measured by a planimeter that has a tracing arm equivalent to a map scale of 1:10,000 and was found to be 250,017 m^2. Calculate the actual area.

SOLUTION:

From Equation (8.9):

$$A_t = A_f \cdot \left(\frac{S_f}{S_t}\right)^2, \quad S_f = \frac{1}{10{,}000}, \quad S_t = \frac{1}{25000}$$

$$A_f = 250{,}017 \text{ m}^2$$

$$\Rightarrow \quad A_t = 250{,}017 \times \left(\frac{\frac{1}{10{,}000}}{\frac{1}{25000}}\right)^2 = 1{,}562{,}606.25 \text{ m}^2$$

$$= 156.26 \text{ ha}$$

$$= 1.5626 \text{ km}^2$$

EXAMPLE 8.2:

A planimeter was used to measure an area on a plan of scale 1:2500. The planimeter pole was outside the area, and the measurement was repeated three times with different pole positions. The readings were as follows:

Measurement No.	Initial Reading (Cycles)	Final Reading (Cycles)	Difference (Cycles)
1	0.000	3.127	3.127
2	1.187	4.316	3.129
3	3.295	6.423	3.128

If each cycle is equivalent to 1000 cm^2 on the map for the above scale, calculate the area on the ground in km^2.

SOLUTION:

$$\text{Average No. of cycles} = \frac{3.127 + 3.129 + 3.128}{3} = 3.128 \text{ cycles}$$

Area on the map = 3.128 x 1000 = 3128 cm^2

According to the map scale, 1 cm on the map = 2500 cm on the ground.

Or 1 cm on map = 25 m on ground

1 cm^2 on map = 25^2 m^2 on ground = 625 m^2

$\Rightarrow$ Area on ground = 3128 x 25^2

= 1955000 m^2 = 195.5 ha

= 1.955 km^2

8.3 THE DIGITAL PLANIMETER

This is an advanced and more accurate version of the polar planimeter where the area can be directly read from a screen. It consists mainly of a roller, display, tracing arm and tracing magnifier (Figure 8.4). The information about the figure for which the area is to be measured, such as the scale and units of measurements, are fed into the instrument and the perimeter of this figure is traced in the same way as for the polar manual planimeter. The amount of the area is then read from the display (Figure 8.5). More information about using the digital planimeter can be found in the operation manual of the instrument.

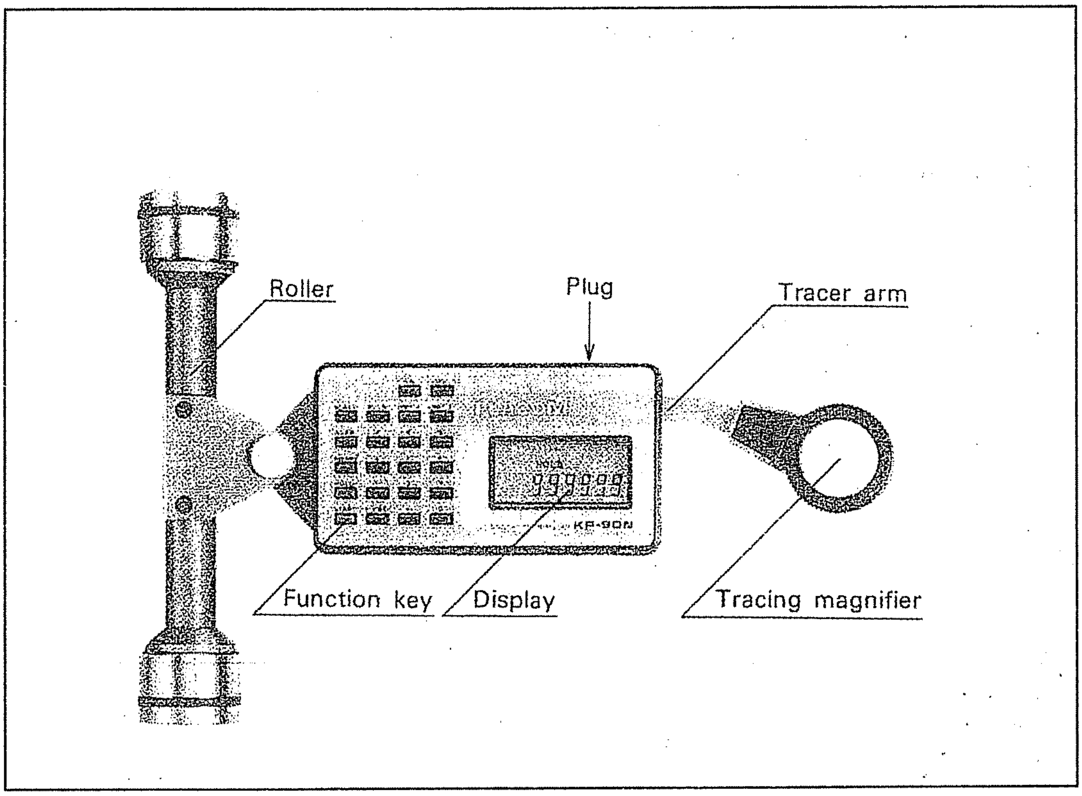


FIGURE 8.4: The digital planimeter.

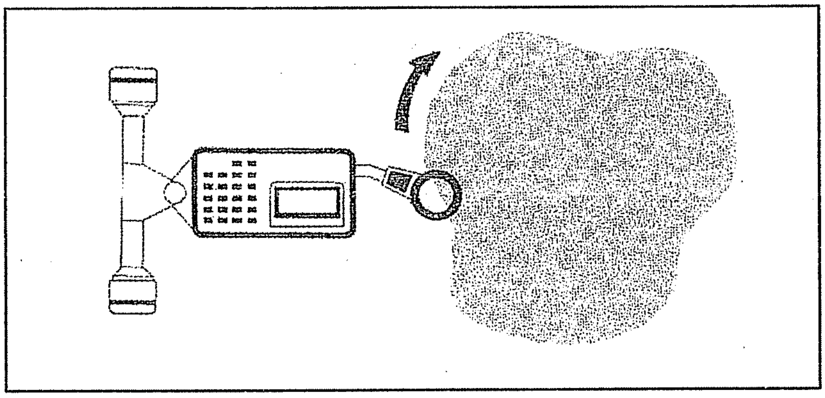


FIGURE 8.5: Using the digital planimeter.

8.4 AREAS OF REGULAR FIGURES

Regular figures are those ones that are surrounded by straight lines or by some well-defined geometric shape such as a circle or circular segment. The areas of these figures are calculated either by some mathematical formulae or by the method of coordinates.

8.4.1 MATHEMATICAL FORMULAE FOR AREA CALCULATION

Some tracts of land have well defined geometric shapes for which mathematical formulae are used for area calculation. These include (Figure 8.6):

1) *The triangle* (Figure 8.6a). The area of a triangle can be calculated as follows:

$$\text{Area} = \frac{1}{2} b \cdot h \qquad \text{(8.10)}$$

Or,

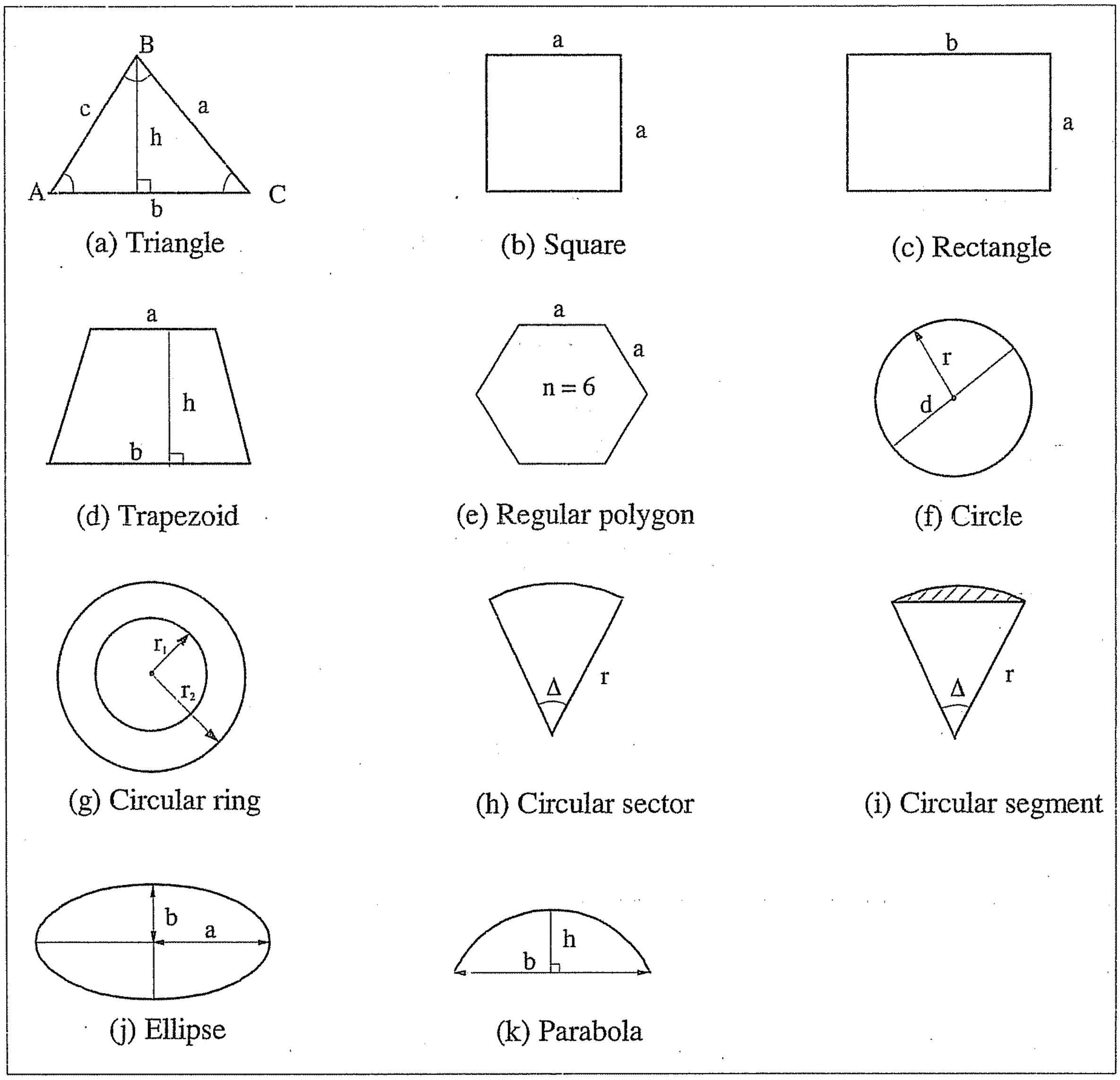


FIGURE 8.6: Some of the regular geometric shapes.

$$\text{Area} = \frac{1}{2} a \cdot b \cdot \sin(C)$$

$$\text{Area} = \frac{1}{2} b \cdot c \cdot \sin(A) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.11)$$

$$\text{Area} = \frac{1}{2} a \cdot c \cdot \sin(B)$$

Or,

$$\text{Area} = \sqrt{S(S - a)(S - b)(S - c)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.12)$$

Where $S = \frac{1}{2}(a + b + c)$ = half the perimeter of the triangle

Note: The area of a figure which is surrounded by more than three sides of known lengths can be calculated by dividing the figure into triangles, calculating the areas of the single triangles and then adding the areas of these triangles to get the whole area of the figure.

2) *The square* (Figure 8.6b). The area of the square is:

$$\text{Area} = a^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.13)$$

3) *The rectangle* (Figure 8.6c). The area of the rectangle is:

$$\text{Area} = a \cdot b \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.14)$$

4) *The trapezoid* (Figure 8.6d). The area of the trapezoid is:

$$\text{Area} = \frac{1}{2}(h)(a + b) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.15)$$

5) *The regular polygon* (Figure 8.6e). Let (n) represent the number of sides of the regular polygon, and (a) represent the side length, the area of the regular polygon is:

$$\text{Area} = \frac{1}{4} n \cdot a^2 \cdot \cot\left(\frac{180°}{n}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.16)$$

Problem: Prove that the above relation is correct.

6) *The circle* (Figure 8.6f). The area of the circle is:

$$\text{Area} = \pi \cdot r^2 = \frac{1}{4}\pi \cdot d^2 \quad \ldots\ldots\ldots\ldots (8.17)$$

Where r is the radius and d is the diameter of the circle.

7) *The circular ring* (Figure 8.6g). The area of the circular ring is:

$$\text{Area} = \pi\left(r_2^{\ 2} - r_1^{\ 2}\right) \quad \ldots\ldots\ldots\ldots (8.18)$$

8) *The circular sector* (Figure 8.6h). The area of the circular sector is:

$$\text{Area} = \frac{1}{360} \cdot \pi \cdot \Delta \cdot r^2 \quad \ldots\ldots\ldots\ldots (8.19)$$

9) *The circular segment* (Figure 8.6i). The area of the circular segment with $\Delta < 180°$ is:

$$\text{Area} = \frac{1}{2} r^2 \left(\frac{\pi\Delta}{180} - \sin \Delta\right) \quad \ldots\ldots\ldots\ldots (8.20)$$

10) *The ellipse* (Figure 8.6j). The area of the ellipse is:

$$\text{Area} = \pi \cdot a \cdot b \quad \ldots\ldots\ldots\ldots (8.21)$$

11) *The parabola* (Figure 8.6k). The area of the parabola is:

$$\text{Area} = \frac{2}{3} b \cdot h \quad \ldots\ldots\ldots\ldots (8.22)$$

8.4.2 AREAS BY THE METHOD OF COORDINATES

One of the principal purposes of conducting a boundary survey is to acquire the data needed to determine the area of a tract of land. Usually, a loop traverse is run along the perimeter of the tract, and the traverse is balanced according to the procedure described in Chapter 7. Since the coordinates of the corner points are already available, it is particularly convenient to use these traverse point coordinates to compute the area enclosed in a traverse. Figure 8.7 illustrates the method for a closed loop traverse.

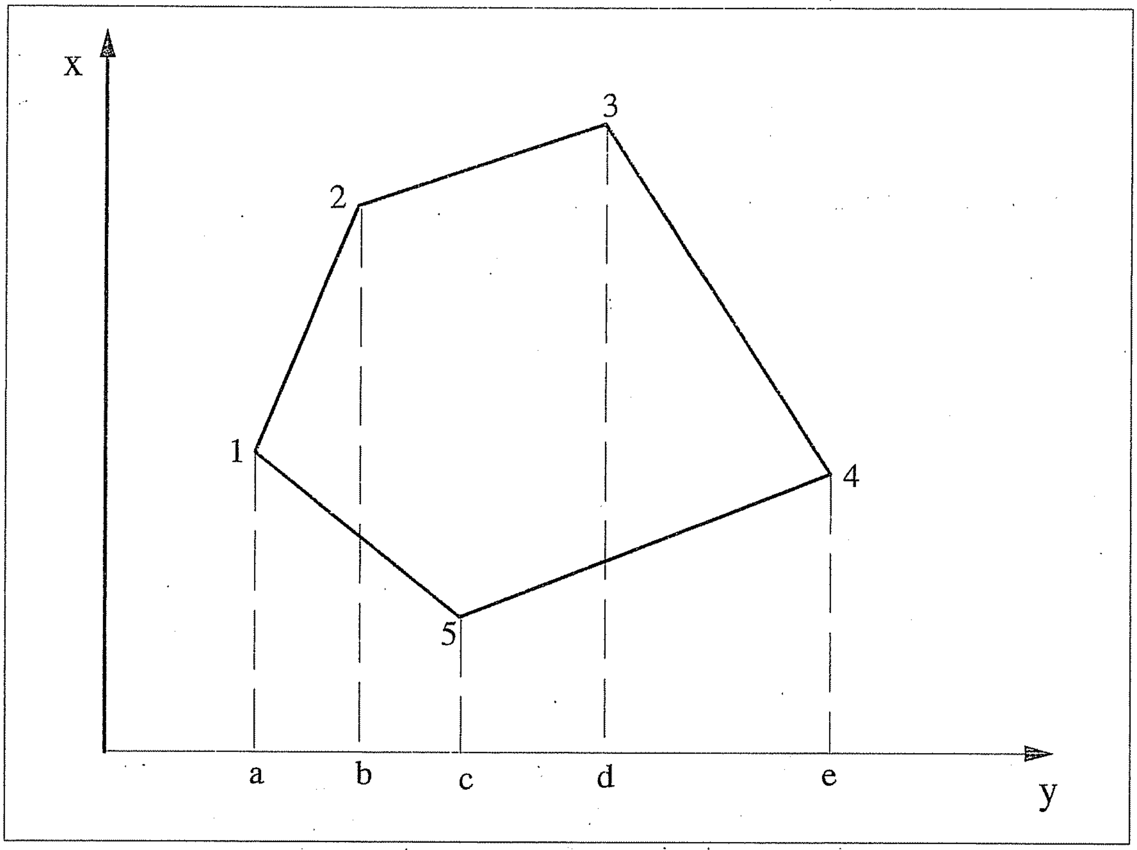


FIGURE 8.7: Area enclosed in a traverse.

Area enclosed in the traverse = Sum of trapezoidal areas a12b, b23d and d34e
- Sum of trapezoidal areas e45c and c51a

Area of any trapezoid can be expressed in terms of the known point coordinates. For example:

$$\text{Area of trapezoid a12b} = \frac{1}{2}\left(y_2 - y_1\right)\left(x_2 + x_1\right)$$

$\Rightarrow$ Area enclosed in the traverse (A):

$$A = \frac{1}{2}[(y_2 - y_1)(x_2 + x_1) + (y_3 - y_2)(x_3 + x_2) + (y_4 - y_3)(x_4 + x_3)$$
$$- (y_4 - y_5)(x_4 + x_5) - (y_5 - y_1)(x_5 + x_1)] \qquad \text{.............} (8.23)$$

Rearranging the terms:

$$A = \frac{1}{2}[x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_5 + x_5 y_1$$
$$- y_1 x_2 - y_2 x_3 - y_3 x_4 - y_4 x_5 - y_5 x_1]$$
$$= \frac{1}{2}\left[y_1(x_5 - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_4)\right.$$
$$\left. + y_4(x_3 - x_5) + y_5(x_4 - x_1)\right] \qquad \text{..................} (8.24)$$

The computations can be tabulated as follows:

Point	y	x	Positive product (Solid product)	Negative product (Dashed product)
1	y_1	x_1		
2	y_2	x_2	$x_1\ y_2$	$y_1\ x_2$
3	y_3	x_3	$x_2\ y_3$	$y_2\ x_3$
4	y_4	x_4	$x_3\ y_4$	$y_3\ x_4$
5	y_5	x_5	$x_4\ y_5$	$y_4\ x_5$
1	y_1	x_1	$x_5\ y_1$	$y_5\ x_1$
			Sum 1	Sum 2
Area $= \frac{1}{2}(\lvert \text{Sum1} - \text{Sum 2} \rvert)$				

The only requirement in the preceding table is to arrange the points in a clockwise or counterclockwise order and to tabulate the first point again at the end.

EXAMPLE 8.3:

Find the area of the following closed loop traverse (ABCDEA):

Station	y (ft)	x (ft)
A	-57.41	-231.66
B	-311.26	-79.49
C	-31.66	123.48
D	62.35	309.11
E	172.76	-19.44

SOLUTION:

Point	y	x	Positive product (Solid product)	Negative product (Dashed product)
1	-57.41	-231.66		
2	-311.26	-79.49	72,106.49	4,563.52
3	-31.66	123.48	2,516.65	-38,434.38
4	62.35	309.11	7,698.98	-9,786.42
5	172.76	-19.44	53,401.84	-1,212.08
1	-57.41	-231.66	1,116.05	-40,021.58
			136,840.01	-84,890.94

$$\Rightarrow \quad \text{Area} = \frac{1}{2}\left[\left|136840.01 - \left(-84890.94\right)\right|\right]$$

$$= 110865.48 \text{ ft}^2 = \frac{110865.48}{43560} = 2.545 \text{ acres}$$

By rearranging the terms in Equation (8.24), the following variation of the area equation can be derived:

$$\text{Area} = \frac{1}{2}\left[x_1\left(y_2 - y_5\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_4 - y_2\right) + x_4\left(y_5 - y_3\right) + x_5\left(y_1 - y_4\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (8.25)$$

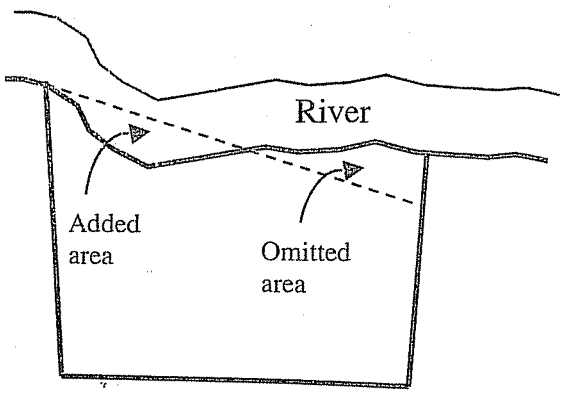
This equation is also commonly used.

8.5 AREAS OF IRREGULAR FIGURES

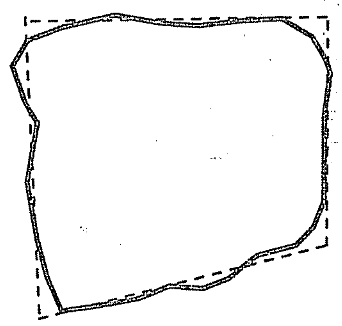
The following methods may be used for determining the area of irregular curvilinear figures such as ponds, lakes and the areas enclosed between chain lines and natural boundaries:

8.5.1 GIVE AND TAKE METHOD

In this method, the irregular figure is replaced by a regular one of approximately equal area that can be calculated by one of the methods explained earlier. The irregularly curved boundary is transferred to straight lines in a process that equates between the omitted parts of the figure and those that are added. Figure 8.8 shows two examples. In the first, the boundary consists of straight lines except from one side where a sinuous river borders the land parcel. For the purpose of area calculation, this side is replaced by the dashed line that takes from the area as much as it adds to it. In the second example, the figure is totally irregular and is replaced by a regular one of approximately equal area.



(a) Partially irregular figure.



(b) Totally irregular figure.

FIGURE 8.8: Give and take method.

8.5.2 GRAPHICAL METHOD (COUNTING SQUARES)

An overlay of squared tracing paper is laid on the drawing (Figure 8.9). The number of squares and parts of squares that are enclosed by the figure under consideration is counted, and knowing the scale of the drawing and the size of the squares on the overlay, the total area of the figure can be computed.

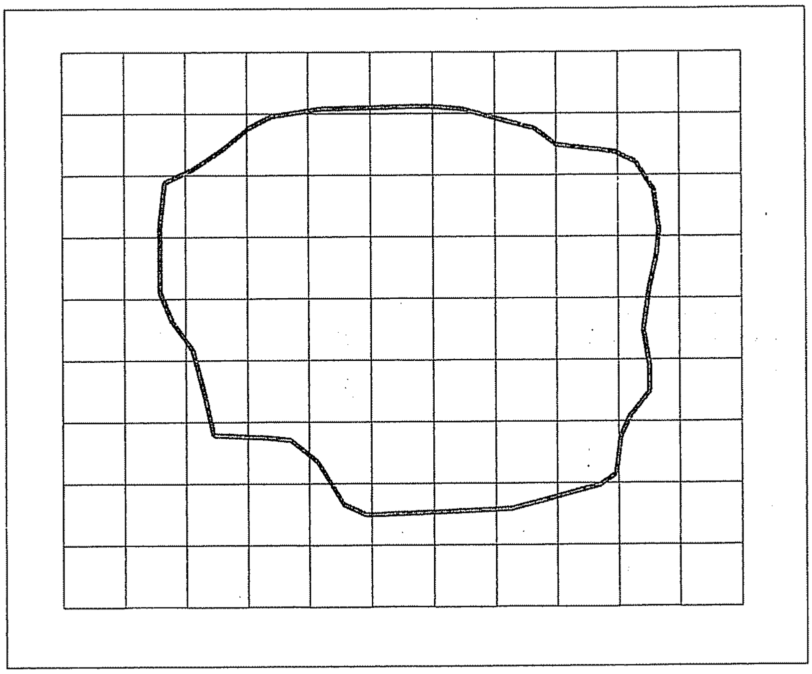


FIGURE 8.9: The method of counting squares.

For example:

If the number of squares ≈ 42.5 (Figure 8.9), area of each square = $1 cm^2$

and the scale = $\frac{1}{1000}$

$$\Rightarrow \quad \text{Area} = 42.5 \times 1 \times (1000)^2 = 4.25 \times 10^7 \text{ cm}^2$$
$$= 4250 \text{ m}^2$$

The accuracy of the calculated area here depends on the original accuracy of the drawn figure, the accuracy of the surveyor in counting and approximating the number of squares contained in the figure, and the size (area) of the squares. The smaller the area of the square, the higher is the accuracy that can be achieved.

8.5.3 TRAPEZOIDAL RULE

Figure 8.10 shows an area bounded by a survey line and a land boundary. The survey line is divided into a number of small equal intercepts of length b, and the offsets h_1 , h_2 , ... etc., measured, either directly on the ground or scaled from the plan. The area is divided into a series of trapezoids.

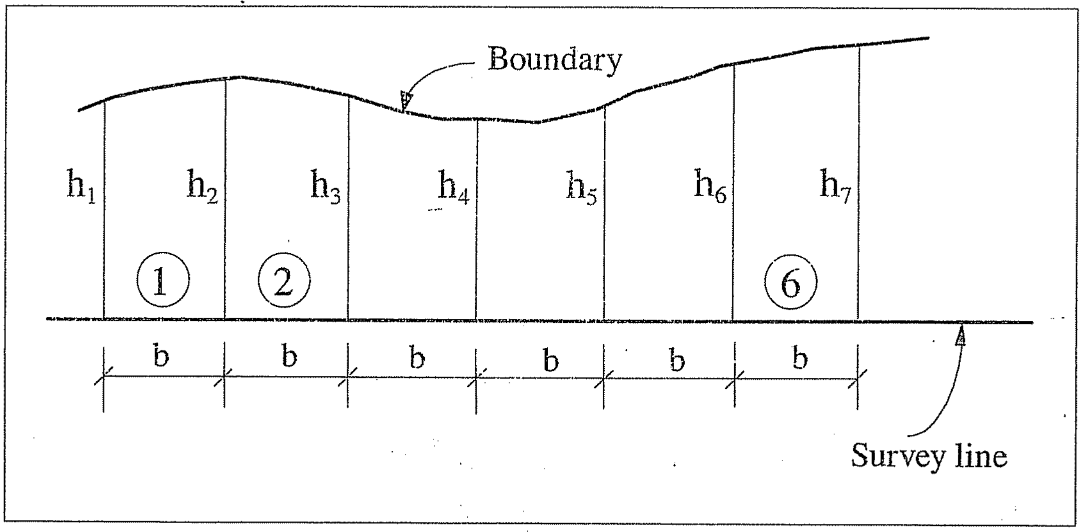


FIGURE 8.10: Trapezoidal rule.

$$\text{Area of trapezoid (1)} = \frac{h_1 + h_2}{2} \cdot b$$

$$\text{Area of trapezoid (2)} = \frac{h_2 + h_3}{2} \cdot b$$

. .

. .

$$\text{Area of trapezoid (6)} = \frac{h_6 + h_7}{2} \cdot b$$

Summing up, we get for *n* offsets:

$$\text{Area} = \frac{b}{2}\left[h_1 + h_n + 2\left(h_2 + h_3 + \ldots + h_{n-1}\right)\right] \quad \ldots\ldots\ldots\ldots(8.26)$$

If the area of a narrow strip of ground is required, this method may be used by running a straight line down the strip as shown in Figure 8.11, and then measuring offsets at equal intercepts along it.

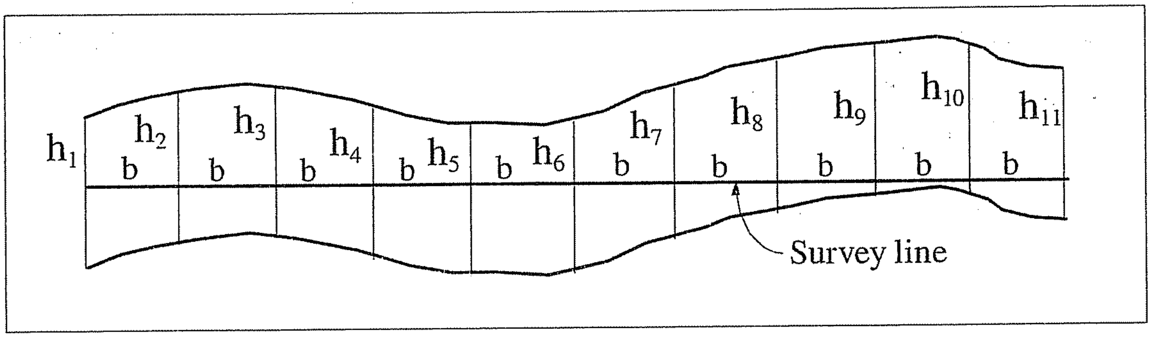


FIGURE 8.11: Area of a narrow strip by the Trapezoidal rule.

EXAMPLE 8.4:

Calculate the area between offsets 1 and 10 in Figure 8.11 where these offsets, scaled from the plan at intervals of 10 m, are:

Offset	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
Length (m)	16.76	19.81	20.42	18.59	16.76	17.68	17.68	17.37	16.76	17.68

SOLUTION:

Using Equation (8.26),

$$\text{Area} = \frac{10}{2}[16.76 + 17.68 + 2(19.81 + 20.42 + 18.59 + 16.76 + 17.68 + 17.68 + 17.37 + 16.76)]$$

$$= 1622.9 \text{ m}^2 = 0.1623 \text{ ha}$$

8.5.4 SIMPSON'S ONE-THIRD RULE

This method, which gives greater accuracy than other methods, assumes that the irregular boundary is composed of a series of parabolic arcs. It is essential that the figure under consideration be divided into an even number of equal strips.

Referring to Figure 8.10, consider the first three offsets, which are shown enlarged in Figure 8.12.

The portion of the area contained between offsets h_1 and h_3

$$= \text{ABGCDA}$$

$$= \text{trapezoid ABFCDA} + \text{area BGCFB}$$

$$= \frac{h_1 + h_3}{2} \cdot 2b + \frac{2}{3} \cdot 2b\left(h_2 - \frac{h_1 + h_3}{2}\right)$$

$$= \frac{b}{3}\left(h_1 + 4h_2 + h_3\right)$$

For the next pair of intercepts, area contained between offsets h_3 and h_5

$$= \frac{b}{3}\left(h_3 + 4h_4 + h_5\right)$$

For the final pair of intercepts, area contained between offsets h_5 and h_7

$$= \frac{b}{3}\left(h_5 + 4h_6 + h_7\right)$$

Summing up, we get:

$$\text{Area} = \frac{b}{3}\left(h_1 + h_7 + 4\left(h_2 + h_4 + h_6\right) + 2\left(h_3 + h_5\right)\right)$$

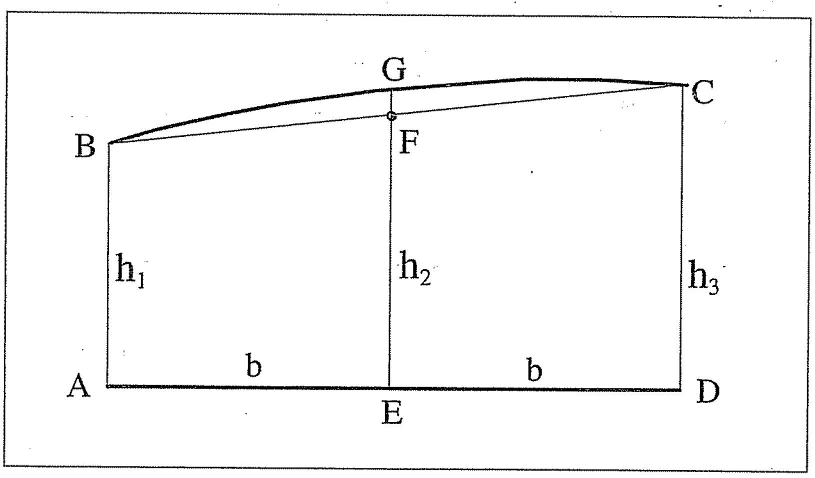


FIGURE 8.12: Simpson's one-third rule.

In the general case,

$$\text{Area} = \frac{b}{3}\left(X + 2O + 4E\right) \quad \text{.......................... (8.27)}$$

Where X = sum of first and last offsets
O = sum of the remaining odd offsets
E = sum of the even offsets

EXAMPLE 8.5:

In a chain survey, the following offsets were taken to a fence from a chain line:

Chainage (m)	0	20	40	60	80	100	120	140	160	180
Offset (m)	0	5.49	9.14	8.53	10.67	12.50	9.75	4.57	1.83	0

Compute the area enclosed between the fence and the chain line.

SOLUTION:

Here, we have 10 offsets and 9 intercepts of length = 20 m, and since Simpson's one-third rule applies to an odd number of offsets (even number of intercepts), it will be used here to calculate the area contained between the 1st and 9th offsets. The residual triangular area between the 9th and 10th offsets is calculated separately.

Offset No.	Offset	Simpson's multiplier	Product
1	0	1	0
2	5.49	4	21.96
3	9.14	2	18.28
4	8.53	4	34.12
5	10.67	2	21.34
6	12.50	4	50.00
7	9.75	2	19.50
8	4.57	4	18.28
9	1.83	1	1.83
			Σ = 185.31

$$\text{Area}\left(h_1 - h_9\right) = \frac{20}{3} \times 185.31 \qquad = 1235.40 \text{ m}^2$$

$$\text{Area}\left(h_9 - h_{10}\right) = \frac{20}{2} \times 1.83 \quad = 18.30 \text{ m}^2$$

$$\text{Total area} \qquad = 1253.70 \text{ m}^2 \qquad = 0.1254 \text{ ha}$$

8.6 CALCULATION OF VOLUMES

Depending on the nature of the engineering project for which the volume of earthwork is to be calculated, the following methods are used:

8.6.1 VOLUME BY AVERAGE - END - AREA (AEA) METHOD

All highway and railroad construction projects involve a considerable amount of earthwork, usually indicated by the terms cut (excavation) and fill (embankment). The quantity of earthwork on a particular job reflects some of the project design features especially the total cost.

To ascertain before construction the preliminary cut and fill amounts, it is necessary to fix the longitudinal grade-line, select a roadway design cross-section, and perform cross sectioning as described before (Chapter 4). The various cross-sections are then plotted (see Figure 8.13). The areas of cut and fill are then calculated (or planimetered) and the volumes computed by the average - end - area (AEA) method or by the prismoidal formula. The AEA method is generally used by the engineering and construction community. The AEA formula is (see Figure 8.14):

$$V = \left(\frac{A_1 + A_2}{2}\right) \cdot L \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8.28)$$

Where V = volume

A_1 and A_2 are the areas of the end sections

L = distance between sections

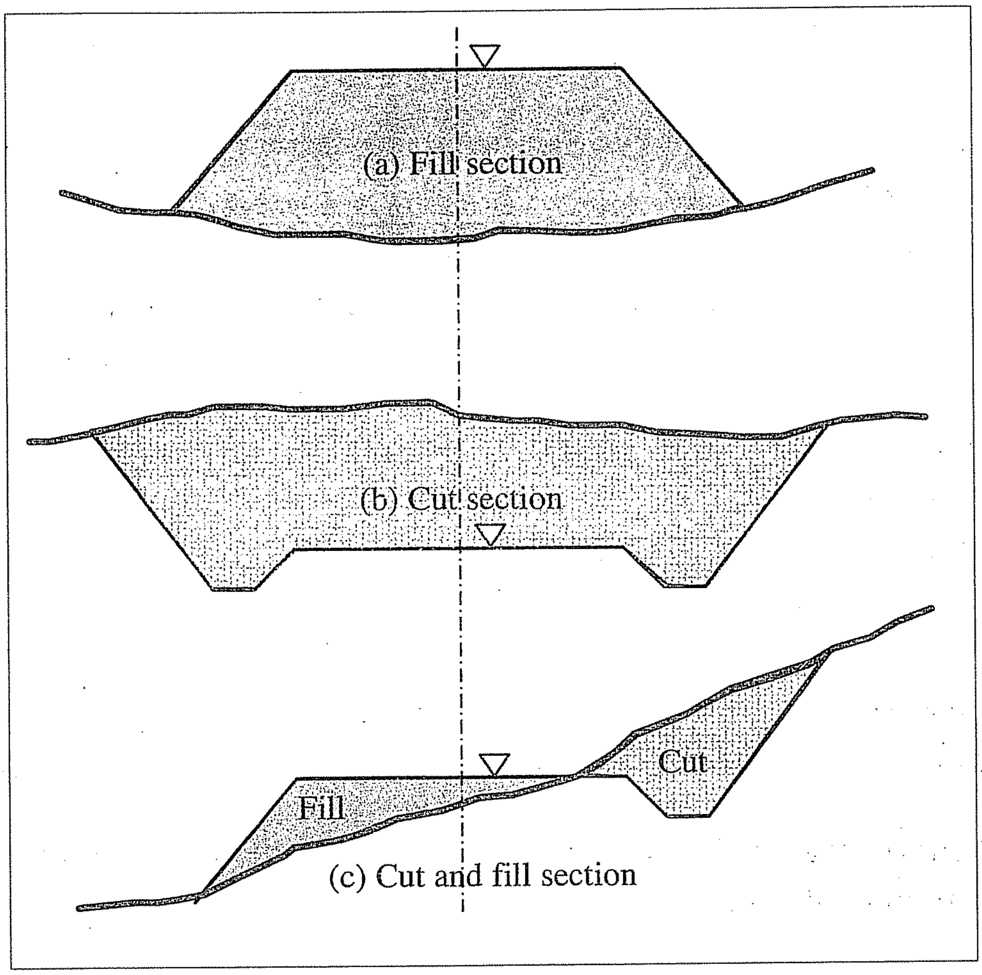


FIGURE 8.13: Types of highway cross-sections.

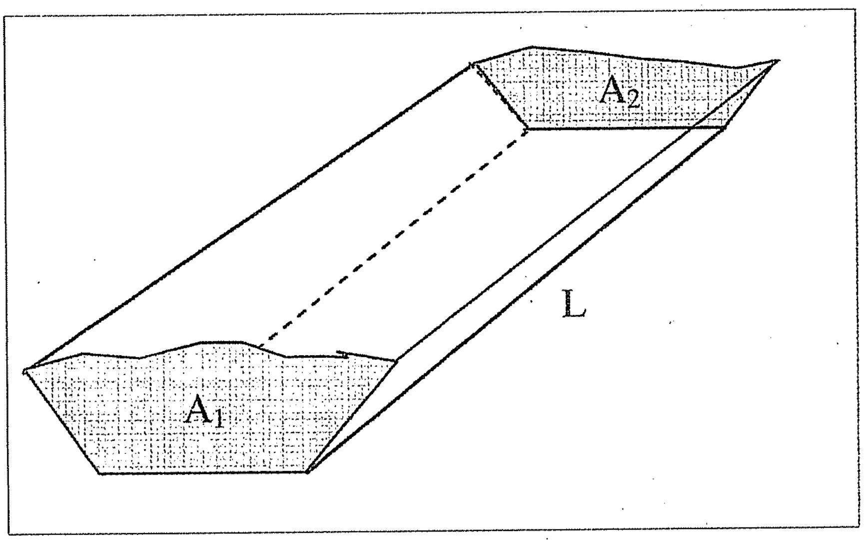


FIGURE 8.14: Average-End-Area method.

For n sections, distance L apart,

$$V = \frac{L}{2}\left(A_1 + A_n + 2\left(A_2 + A_3 + \ldots + A_{n-1}\right)\right) \qquad \text{.........(8.29)}$$

If the ground is uneven or changes slope abruptly, intermediate sections are taken such that the resulting errors in volumes will not be serious. Theoretically, this method is not exact, but the resulting errors are insignificant.

In the case of side-hill sections like that in Figure 8.13c, it is necessary to calculate the cut and fill separately because payment for earthwork is based on excavated quantities only. Fill quantities are needed for studies of balancing excavation and embankment.

EXAMPLE 8.6:

The planimetered areas in m^2 of two side-hill cross-sections of a proposed highway are as follows:

Chainage	4200.0 m	C82	F112
Chainage	4250.0 m	C214	F78

C denotes cut, and F denotes fill. Calculate the quantities of earthwork

SOLUTION:

Distance L = 4250.0 - 4200 = 50.0 m

$$\text{Cut Volume} = \left(\frac{82 + 214}{2}\right) \times 50 = 7400 \text{ m}^3$$

$$\text{Fill Volume} = \left(\frac{112 + 78}{2}\right) \times 50 = 4750 \text{ m}^3$$

8.6.2 VOLUME BY PRISMOIDAL FORMULA

The prismoidal formula is equivalent to Simpson's one-third rule for areas. It works only for an even number of equal strips between cross sections.

For n cross-sectional areas separated by equal strips of length L, the prismoidal formula is:

$$V = \frac{L}{3}\left[A_1 + A_n + 2\left(A_3 + A_5 + \ldots + A_{n-2}\right) + 4\left(A_2 + A_4 + \ldots + A_{n-1}\right)\right] \qquad \ldots\ldots\ldots\ldots (8.30)$$

Where n = odd number

For n = 3

$$\Rightarrow \quad V = \frac{L}{3}\left(A_1 + 4A_2 + A_3\right) \qquad \ldots\ldots\ldots\ldots (8.31)$$

8.6.3 VOLUME FROM CONTOUR MAPS

Since a contour map is a representation of the earth's surface in its three dimensions, it is commonly used in engineering to determine volume of earthwork and water storage capacity of proposed reservoir sites.

The contour interval will determine the distance L in the average - end - area or prismoidal formula methods, and for accuracy this should be as small as possible, preferably 1 or 2 meters. The areas enclosed by individual contour lines are best taken off the plan by means of a planimeter.

EXAMPLE 8.7:

Figure 8.15 represents the underwater contour lines behind a dam. The areas enclosed by the shown contour lines were measured by a planimeter and found to be:

Contour AMSL	800	810	820	830	840	850
Area (m^2)	20365	41375	117120	160340	211210	298140

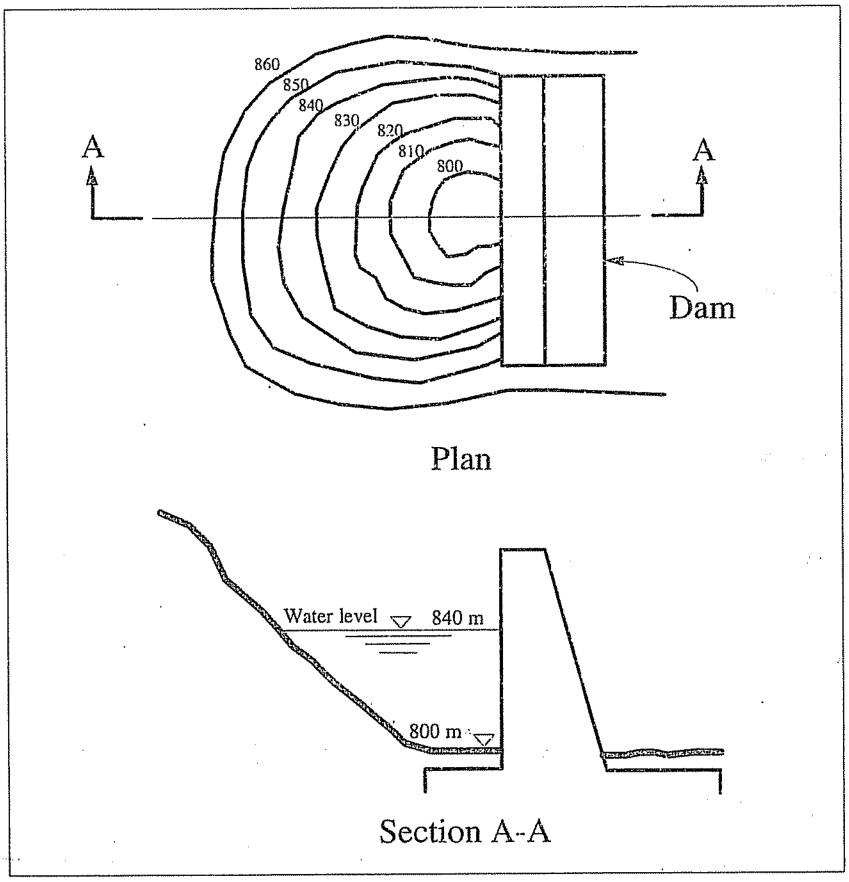


FIGURE 8.15: Volume calculation from contour lines.

Knowing that the bottom of the reservoir is flat with a level of 800 m AMSL, calculate the volume of water in the reservoir when it reaches the maximum allowed level that is 840 m AMSL. Use both A-E-A and prismoidal methods.

SOLUTION:

(1) A-E-A Method:

Contour interval = L = 10 m

$$V = \frac{L}{2}\left[A_{800} + A_{840} + 2\left(A_{810} + A_{820} + A_{830}\right)\right]$$

$$= \frac{10}{2}\left[20365 + 211210 + 2(41375 + 117120 + 160340)\right]$$

$$= 4346225 \text{ m}^3$$

(2) By Prismoidal Formula

$$V = \frac{L}{3}\left[A_{800} + A_{840} + 2\left(A_{820}\right) + 4\left(A_{810} + A_{830}\right)\right]$$

$$= \frac{10}{3}\left[20365 + 211210 + 2 \times 117120 + 4\left(41375 + 160340\right)\right]$$

$$= 4242250 \text{ m}^3$$

8.6.4 VOLUME FROM SPOT LEVELS

This is the method by means of which the earthworks involved in the construction of large tanks, basements, borrow pits, etc., and similar works with vertical sides may be calculated. The volume of excavation may be determined by taking conventional cross-sections before and after removal of the materials. However, the computation can be simplified by first dividing the area into squares or rectangles of suitable size, and then taking level readings at the corners both before and after the excavation.

The volume removed from any rectangle is computed as the average of the heights of the four corners times the area. Some corners are common to more than one rectangle; thus A1, in Figure 8.16, is common to one, A2 is common to two, B3 is common to three, and B2 is common to four rectangles. By summing up the volumes within all the rectangles, the following equation results:

$$V = \frac{A}{4}\left(\Sigma h_1 + \Sigma 2h_2 + \Sigma 3h_3 + \Sigma 4h_4\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8.32)$$

Where V is the volume

A is the area of one rectangle or square

h_1, h_2, h_3 and h_4 are the corner heights common to one, two, three and four rectangles respectively.

The following example illustrates this procedure:

EXAMPLE 8.8:

Figure 8.16 shows a borrow pit in the shape of a grid with all the dimensions and reduced levels. If this area is to be leveled to a constant level of 200.0 m for all the grid corners, calculate the volume of cut.

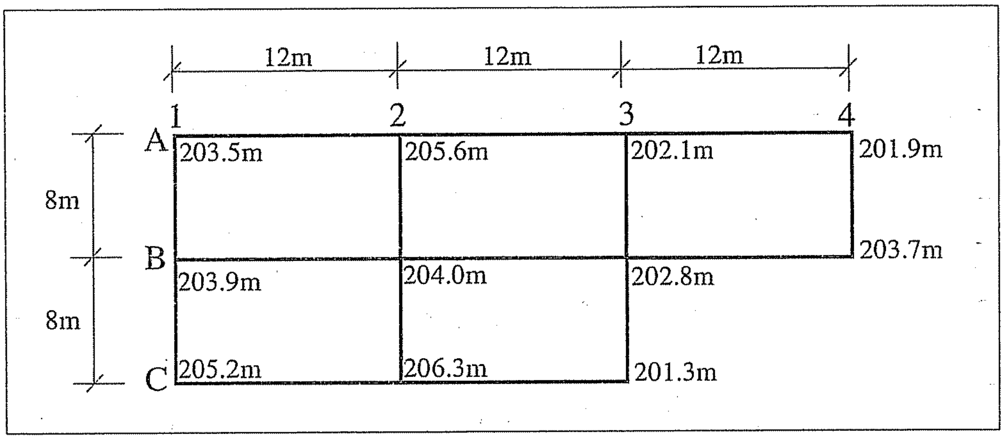


FIGURE 8.16

SOLUTION:

Point	Old Elev. (m)	New Elev. (m)	Cut (*c*)	No. of Rectangles (*n*)	*c . n*
A1	203.5	200.0	3.5	1	3.5
A2	205.6	200.0	5.6	2	11.2
A3	202.1	200.0	2.1	2	4.2
A4	201.9	200.0	1.9	1	1.9
B1	203.9	200.0	3.9	2	7.8
B2	204.0	200.0	4.0	4	16.0
B3	202.8	200.0	2.8	3	8.4
B4	203.7	200.0	3.7	1	3.7
C1	205.2	200.0	5.2	1	5.2
C2	206.3	200.0	6.3	2	12.6
C3	201.3	200.0	1.3	1	1.3
				Sum =	75.8

$$\text{Volume of cut} = 8 \times 12 \times \frac{75.8}{4} = 1819.2\ \text{m}^3$$

8.7 THE MASS-HAUL DIAGRAM

For some engineering projects, especially in highway design, it is not sufficient to merely calculate the amounts of cut and fill at certain locations. Other factors need to be considered which include the balance between the volumes of cut and fill, as well as the transportation needs of the excavated materials. This means that in the same project, the fill sites should be adjacent to the excavation sites so that the transportation costs of excavated material can be minimized. As a result, a system is needed to plan the earthwork for the project. For this purpose, the mass-haul diagram comes to serve several goals among which:

1) To establish the extent to which there will be a surplus of excavated material or a deficiency of fill material, and
2) To assist in the economic evaluation of borrow pits or tip sites.

Materials that have been excavated and that are to be used for fill cannot be assumed to occupy the same volume (space) after compaction as they did before excavation. They will be subject to either shrinkage or bulking. For example, sand and gravel materials will occupy up to 10% less volume after compaction than they did before excavation, while others such as solid rocks will occupy up to 40% more volume than the original one. These factors should be taken into consideration when planning the earthwork movement and costs on large construction schemes.

To understand the mass-haul diagram, let us refer to the longitudinal section in Figure 8.17a. Cross sections have been taken at 30m intervals, and the cumulative volumes are calculated using the appropriate shrinkage or bulking factors (Table 8.1). A positive volume indicates a surplus of excavated materials, while a negative volume indicates a deficiency of fill. A graph is then

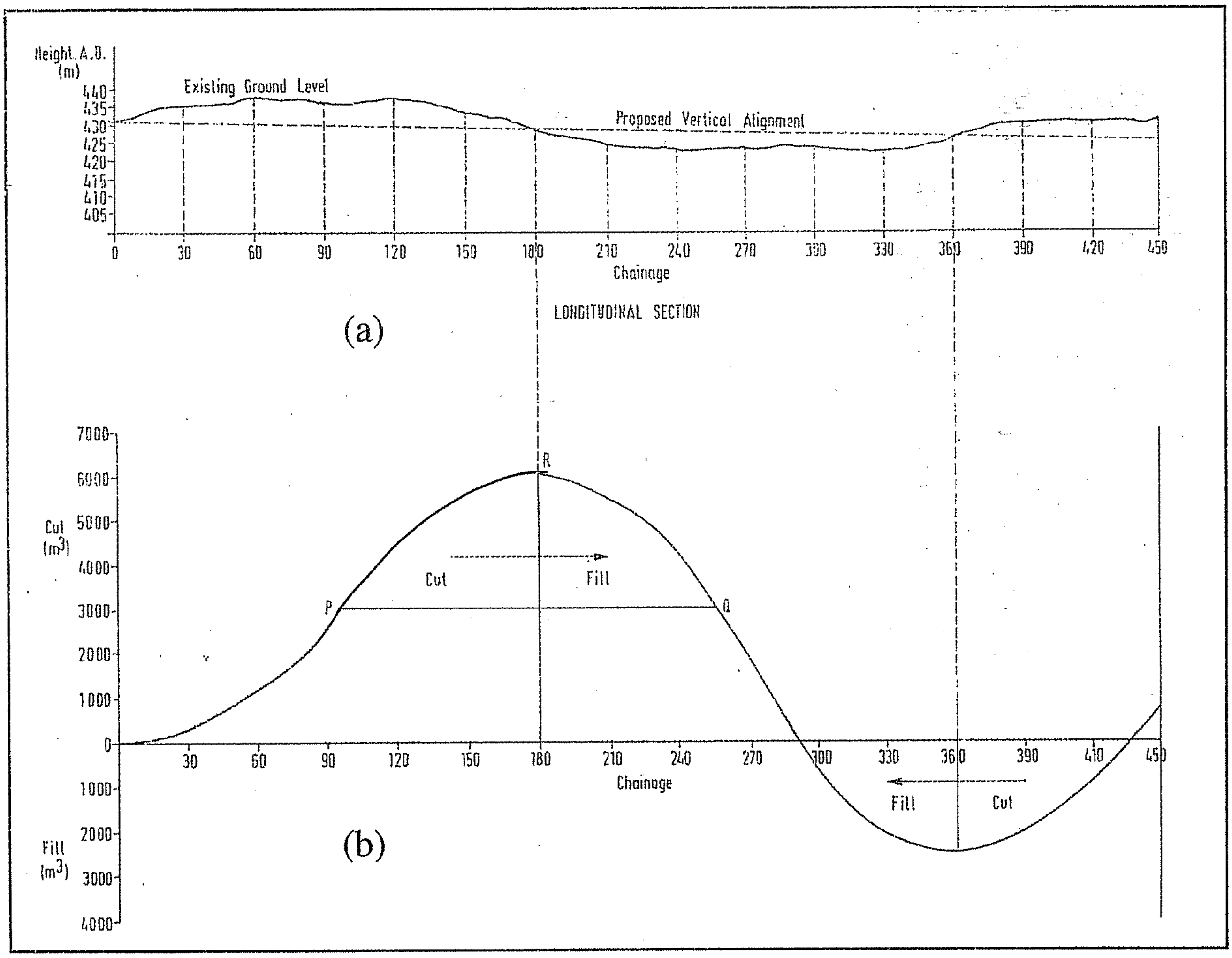


FIGURE 8.17: Mass-Haul Diagram.

TABLE 8.1

Chainage (m)	Volume (m^3)		Bulking or shrinkage factor	Corrected volume (m^3)	Cumulative volume (m^3)
	Cut	Fill			
0	----	----	----	----	----
30	330	----	0.9	+300	+300
60	1020	----	0.9	+920	+1220
90	1540	----	0.9	+1390	+2610
120	2620	----	0.9	+2360	+4970
150	1180	----	0.9	+1060	+6030
180	200	----	0.9	+180	+6210
210	----	660	----	-660	+5550
240	----	1240	----	-1240	+4310
270	----	2460	----	-2460	+1850
300	----	2530	----	-2530	-680
330	----	1420	----	-1420	-2100
360	----	390	----	-390	-2490
390	385	----	1.3	+500	-1990
420	860	----	1.3	+1120	-870
450	1180	----	1.3	+1530	+660

plotted with the chainage (distance) being as the horizontal axis, and the cumulative volume as the vertical axis. The resulting figure is termed the mass-haul diagram (Figure 8.17b). From this figure, the following important notes can be made:

1) Where the slope of the curve is positive, excavation is taking place. When a maximum point is reached, the excavation ceases and the fill starts.

2) Where the slope of the curve is negative, fill is taking place. When a minimum point is reached, the fill ceases and the excavation starts.

3) The ordinate at any point on the curve represents the volume of cut or fill in hand at that point.

4) The difference between the ordinates at two different points represents the amount of cut or fill that exists between these two points, given that there is no maximum or minimum curve value between the two points.

5) A horizontal line drawn on the curve is known as a balancing line, such as line PQ on Figure 8.17b. The excavation that takes place between P and R is given by the difference between the ordinates of P and R. This will be exactly equal to the amount of fill required between points R and Q. The arrows indicate the direction in which the excavated material must be transported.

The horizontal axis of the mass-haul diagram will, of course, be a balancing line, but not necessarily the best one. A good balancing line (could be the selected or designed centerline of a road) is the one that will satisfy the following:

a) Balance the amounts of cut and fill. This means that the cumulative volume at the end of the curve should be zero or close to zero.

b) Produce a curve similar to that of the sine function with minimum distances between the consecutive maximum and minimum points. This means that the surplus of excavated materials will be transported for a short distance to be used as fill in the adjacent sites.

PROBLEMS

8.1 A planimeter was used to determine the area of tract of land drawn to scale 1 inch = 400 ft. If the initial reading was 3781 and the final reading was 6359, and given that each vernier unit is equivalent to 0.01 square inch on the map, calculate the area of the tract on the ground to the nearest acre.

8.2 The area enclosed between the chain line AB and the boundary line of a tract of land is to be determined by the method of coordinates. The field record of offsets from the chain line AB is as follows:

Distance from point A (m)	Offset (m)
0	4.0
5	7.5
12	4.7
20	8.2
32	9.3
40	3.6
52	1.8

Compute the area.

8.3 A tract of land is triangular in shape. If you know that the coordinates of the boundary points are A(1,1), B(20,10) and C(11,25), calculate the area of the tract of land by: (a) the method of coordinates, and (b) Equation 8.12. Compare the two answers.

8.4 Calculate the area of the traverse of problem 7.7.

8.5 Calculate the area of the tract of land shown in Figure 8.18 given that the radius of the circular arc is 160.0 m.

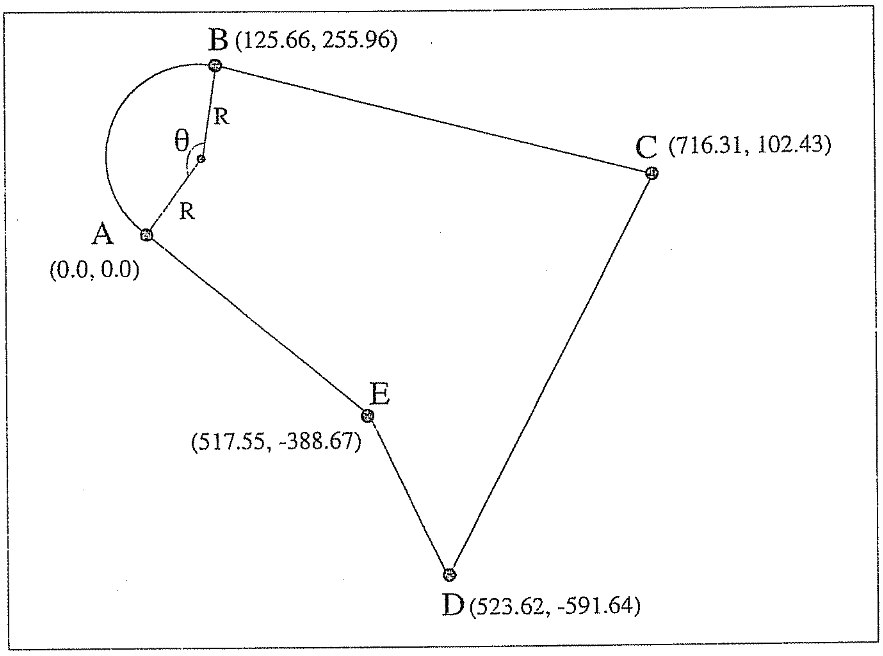


FIGURE 8.18

8.6 To determine the area of a tract of land bordered by the high water line of a sinuous stream, offsets from a chain line AB were measured at regular intervals of 10 m as follows: 7.65, 5.32, 7.86, 9.50, 10.36, 8.15, 4.22, 6.45, 7.50 and 8.25 .

Find the area by: (a) Trapezoidal Rule. (b) Simpson's Rule.

8.7 Calculate the area of a circular tract of land whose radius is 20 m using:
(a) Mathematical formula ($A=\pi r^2$) (b) Trapezoidal Rule
(c) Simpson's Rule.
Use b=5 m for Trapezoidal and Simpson's rules. Compare the results from the three methods.

8.8 The coordinates (in m) of the corners of a triangular plot of land ABC are as follows:

Point	y	x
A	1110	1226
B	1468	1612
C	1752	1340

The area is to be divided into two equal parts by a line MN, where M is a point on AB and 25 m from A and N is a point on BC. Calculate the coordinates of point N.

8.9 Calculate the shaded area ABC in Figure 8.19.

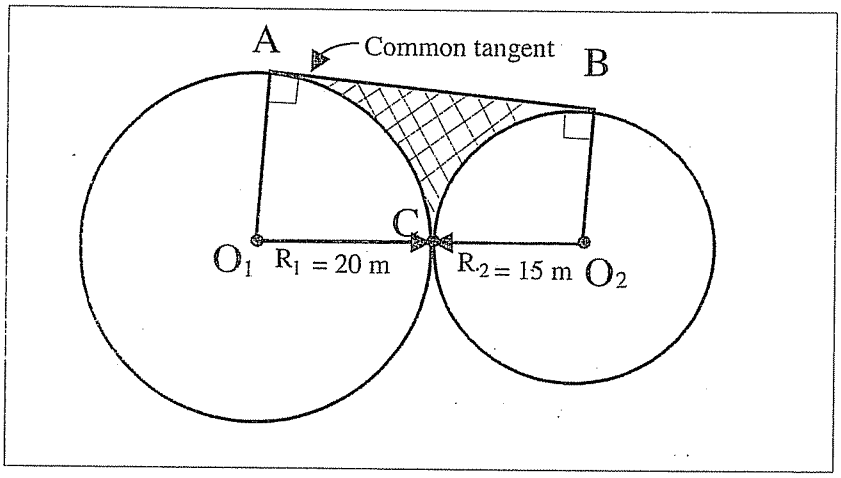


FIGURE 8.19

8.10 The planimetered areas, in square meters, of cross-sections along a short portion of a highway improvement project are as follows:

Chainage (m)	Cut (m²)	Fill (m²)
2550.0	12.0	0.0
2575.0	9.0	0.0
2585.0	3.5	6.0
2600.0	0.0	8.0
2625.0	0.0	5.5

Calculate the net amount of earth needed or available over the portion of the highway.

8.11 A regular grid consisting of a (10m x 10m) squares has the following spot elevations:

Point #	Elevation (m)
A1	17.06
A2	17.48
A3	17.63
B1	17.37
B2	17.70
B3	17.96
C1	17.58
C2	18.01
C3	18.25
D1	17.83
D2	18.19
D3	18.42

a) If the land covered by the above grid is to be excavated to a new design level that is 12.00 m, calculate the resulting cut volume.

b) Calculate the design level that will make the amount of cut and fill equal.

8.12 A 10m by 10m square land parcel has the following heights for its corners respectively: 106m, 104m, 99m and 96m.

1. If this land is to be leveled to a 100m height, calculate the two resulting volumes of cut and fill.

2. If this land is to be transformed to a regular box, calculate the height of the top of the box.

8.13 What is meant by Mass-Haul diagram? What are the characteristics of an ideal diagram? Give a reasonable sketch for an ideal diagram.

9 ROUTE SURVEYING

9.1 INTRODUCTION

Route Surveying is that branch of surveying which deals with the planning, design, and construction of any route of transportation. Routes, such as those for railroads and highways, are generally designed to satisfy specific geometric criteria with respect to horizontal and vertical alignment.

The horizontal component of such routes consists of sequence of straight lines, termed tangents, connected by curves. These curves are usually arcs of circles. However, easement or transition curves in the form of spirals are frequently used to provide easy and gradual transitions between tangents and circular curves along high-speed routes.

The vertical alignment consists of a sequence of straight sections of grade lines connected by vertical curves. These curves are usually parabolic in shape because certain characteristics of this form of curves make it more convenient than circular and elliptical curves.

This chapter will first deal with the calculation and field layout of horizontal curves, which includes both simple circular and transition curves. Later, it will deal with the computation and layout of vertical parabolic curves.

9.2 HORIZONTAL CURVES

Different horizontal curves are used to connect the straight lines. These curves are:

a - Circular curves

b - Spiral or easement curves.

9.2.1 CIRCULAR CURVES

There are four categories of circular curves:

1 - Simple circular curves (Figure 9.1a)

2 - Compound circular curves (Figure 9.1b)

3 - Broken - back circular curves (Figure 9.1c), and

4 - Reversed circular curves (Figure 9.1d)

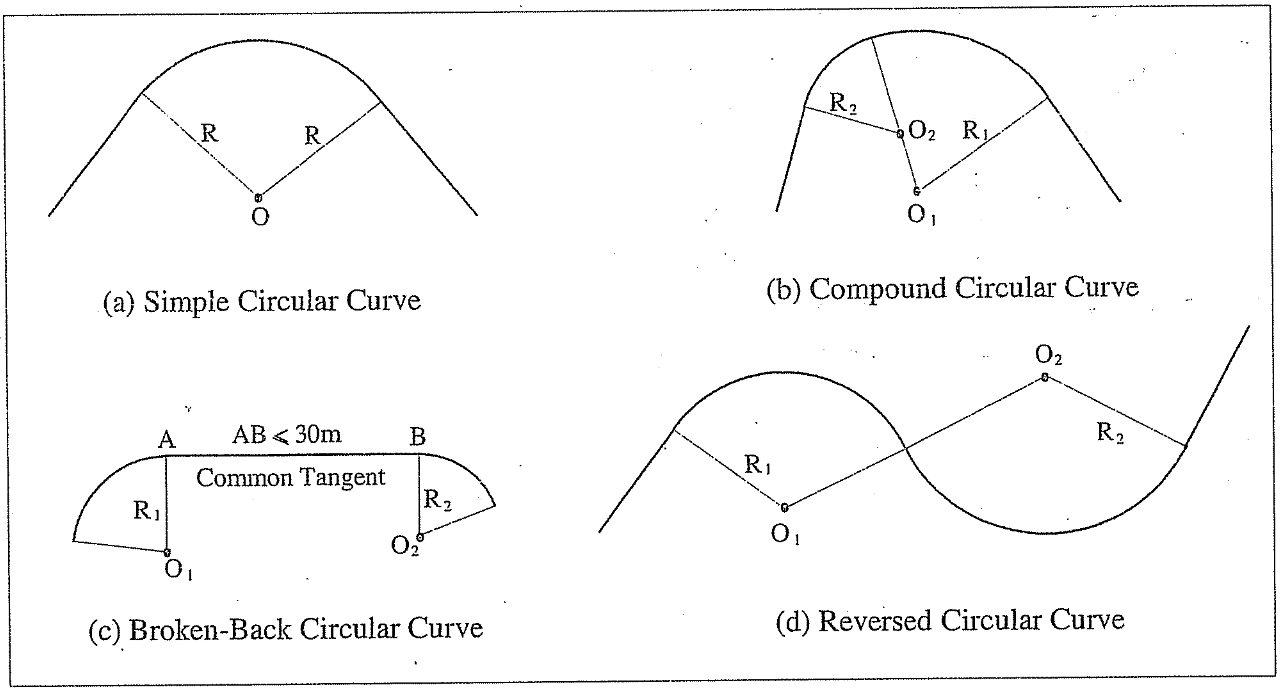


FIGURE 9.1: Types of circular curves.

9.2.1.1 GEOMETRY OF SIMPLE CIRCULAR CURVES

The geometry along the centerline of a circular curve is illustrated in Figure 9.2. The back tangent (AB) intersects the forward tangent (BC) at point B, which is referred to as the point of intersection (PI). The angle of intersection (Δ) is the angle formed by the intersection of the two tangents at the PI. The central angle of the curve is the angle subtended by the curve at the center of the circle (Point O). The central angle is numerically equal to the intersection angle Δ. The point of curvature (PC) is the point at which the curve departs from the back tangent as one proceeds along the curve in the direction in which the chainage increases. The point of tangency (PT), on the other hand, marks the end of the curve and the beginning of the forward tangent.

Figure 9.2 shows the seven elements of circular curves. These are the radius (R), the angle of intersection (Δ), The tangent distance (T), the length of circular curve (L), the long chord (LC), the mid-ordinate (M) and the external distance (E). If any two of these seven elements are known, the other five elements can be calculated, even though sometimes not straight forward. For the case when Δ and R are known, the rest of the elements are computed as follows:

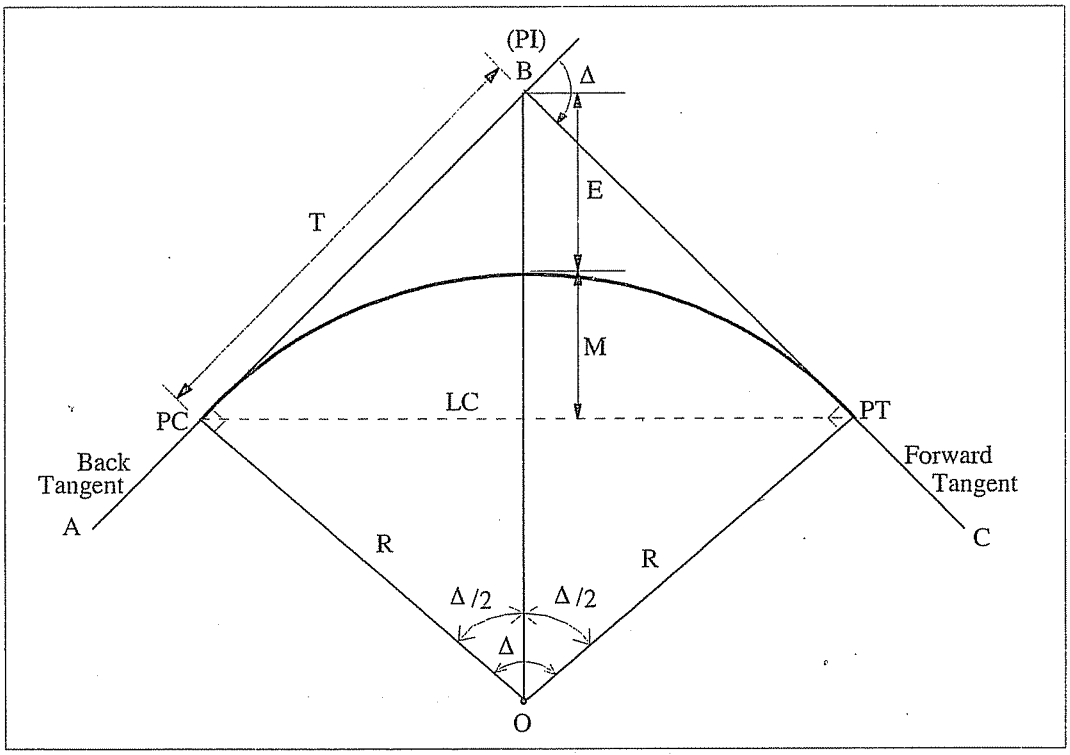


FIGURE 9.2: Geometry of a circular curve.

1) The tangent distance (T) is the distance along the tangent from the PC or PT to the PI. From Figure 9.2, it is evident that:

$$T = R \tan\frac{\Delta}{2} \qquad \text{(9.1)}$$

2) The length of the circular curve L is:

$$L = \frac{\Delta}{180^\circ}\pi R \qquad \text{(9.2)}$$

3) The long chord (LC) is the chord joining the PC and the PT:

$$\frac{LC}{2} = R\sin\frac{\Delta}{2}$$

$$\Rightarrow LC = 2R\sin\frac{\Delta}{2} \qquad \text{(9.3)}$$

4) The mid-ordinate (M) is the perpendicular distance from the mid-point of the curve to the long chord. Then:

$$\frac{R - M}{R} = \cos\frac{\Delta}{2} \text{ or } M = R - R\cos\frac{\Delta}{2}$$

$$\Rightarrow M = R\left(1 - \cos\frac{\Delta}{2}\right) \qquad \text{(9.4)}$$

5) The external (E) is the distance from the mid-point of the curve to the PI. Evidently,

$$\frac{R}{E+R} = \cos\frac{\Delta}{2}, \quad \text{or}$$

$$E = R\left(\frac{1}{\cos\frac{\Delta}{2}} - 1\right) \qquad \text{(9.5)}$$

9.2.1.2 DEGREE OF CURVATURE - ARC DEFINITION

According to the Jordanian Ministry of Public Works, the degree of curvature of a curve may be defined as the central angle subtended by an arc distance of 30 m along the curve (See Figure 9.3). It will be denoted by D_a. When $D_a = 1^\circ$, the curve is a one-degree curve; when $D_a = 2^\circ$, the curve is a two-degree curve, and so on. The arc definition is most commonly used in highway applications. The degree of curvature on modern high-speed highways is usually less than 4°.

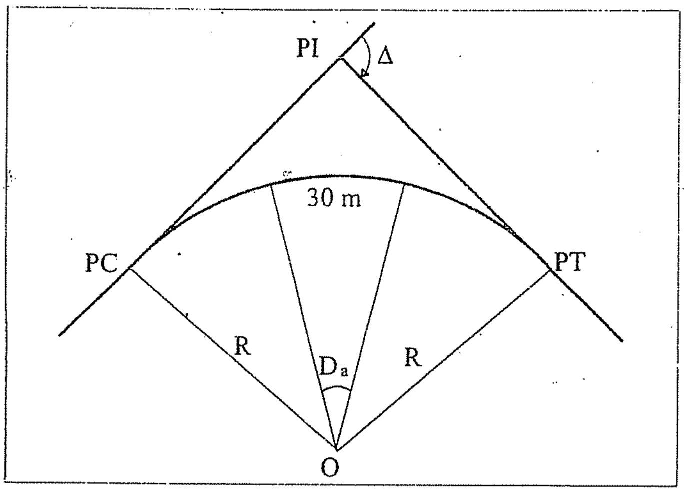


FIGURE 9. 3: Degree of curvature - Arc definition.

From Figure 9.3, $\dfrac{30}{L} = \dfrac{D_a}{\Delta}$

Substitute Equation (9.2) for L:

$$\Rightarrow \frac{30}{\dfrac{\Delta \pi R}{180^\circ}} = \frac{D_a}{\Delta}$$

$$\Rightarrow D_a = \frac{1718.873}{R} \quad \text{(R in m)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.6)$$

If D_a is known, then the length of curve, L (in m) is

$$L = 30\,\frac{\Delta}{D_a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.7)$$

According to the British Standards, the degree of curvature is the central angle subtended by an arc distance of 100 ft along the curve. Equations (9.6) and (9.7) become:

$$D_a = \frac{5729.58}{R} \quad \text{(R in ft)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.8)$$

$$L = 100\,\frac{\Delta}{D_a} \quad \text{(in ft)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.9)$$

9.2.1.3 DEGREE OF CURVATURE - CHORD DEFINITION

In railroad applications, especially in the countries which use the English system of units, the degree of curvature is usually defined as the central angle subtended by a chord distance of 100 ft, as shown in Figure 9.4. It is called the chord definition of the degree of curvature and will be denoted by D_c.

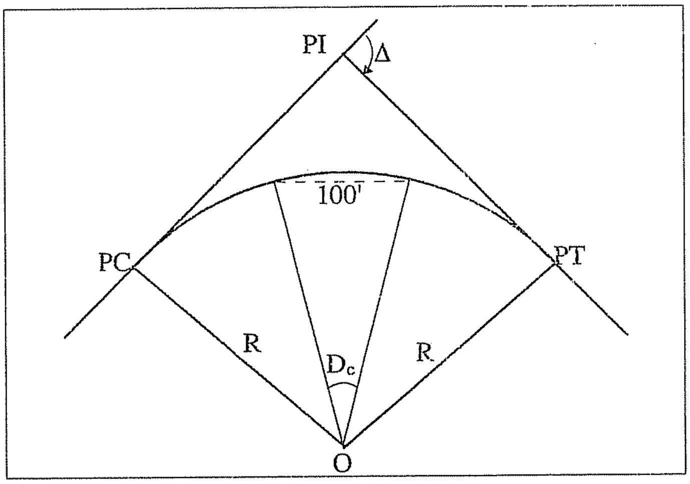


FIGURE 9. 4: Degree of curvature - Chord definition.

The following relationship exists between the radius, R, and the degree of curvature, D_c:

$$100 = 2R \sin\frac{D_c}{2}$$

Therefore,

$$R = \frac{50}{\sin\frac{D_c}{2}} \quad \text{(R in ft)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.10)$$

EXAMPLE 9.1:

The angle of intersection of two tangents of a road (Δ) is equal to 44° 18'. If these tangents are to be connected by a circular curve which has a D_a of 1° 48'. Calculate: R, T, L, E and M.

SOLUTION:

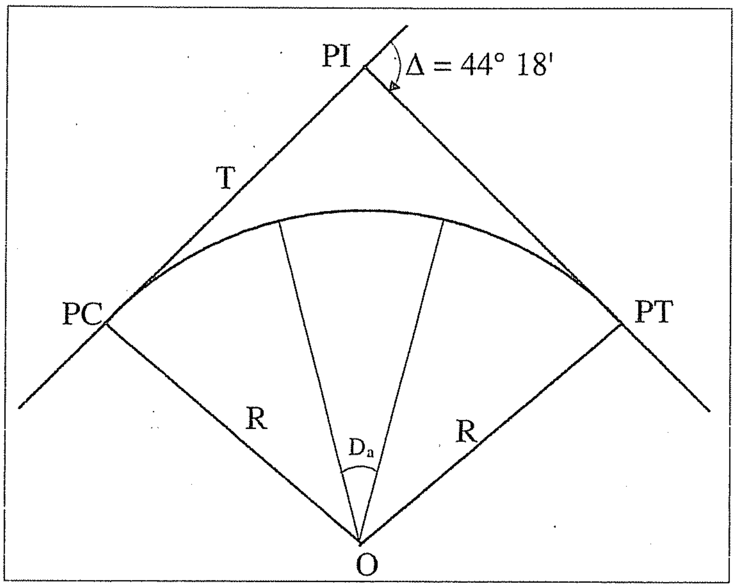


FIGURE 9.5: Circular curve of Example 9.1.

$$R = \frac{1718.873}{D_a} = \frac{1718.873}{1.8} = 954.93 \text{ m}$$

$$T = R \tan\frac{\Delta}{2} = 954.93 \tan\frac{44.3}{2} = 388.73 \text{ m}$$

$$L = 30 \times \frac{\Delta}{D_a} = 30 \times \frac{44.3}{1.8} = 738.33 \text{ m}$$

$$E = R\left(\frac{1}{\cos\frac{\Delta}{2}} - 1\right) = 954.93\left(\frac{1}{\cos\frac{44.3}{2}} - 1\right) = 76.09 \text{ m}$$

$$M = R\left(1 - \cos\frac{\Delta}{2}\right) = 954.93\left(1 - \cos\frac{44.3}{2}\right) = 70.47 \text{ m}$$

9.2.1.4 LINEAR METHODS FOR SETTING OUT SIMPLE CIRCULAR CURVES

These methods use only the chain surveying principles and do not need angle measuring equipment such as theodolites. They are used to lay out short curves which do not require high accuracy, as well as curves at street intersections where vision is clear, and it is possible to extend the tape in a straight and horizontal line with a length equal to the radius of the circular curve R. These methods are:

1 - BY ORDINATES FROM THE LONG CHORD

Bisect the long chord at B. Measure a distance x to the left or right of point B on the long chord, and then erect a perpendicular of length y. The resulting point G lies on the curve. From Figure 9.6:

$$y = \sqrt{R^2 - x^2} - \sqrt{R^2 - \left(\frac{LC}{2}\right)^2} \qquad (9.11)$$

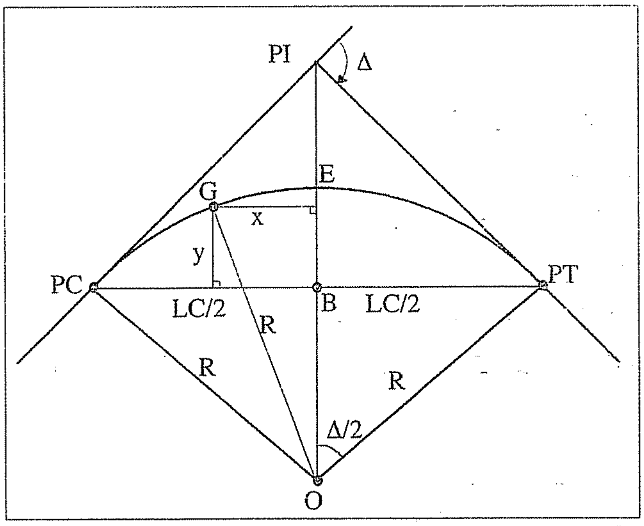


FIGURE 9.6: Setting out circular curves by ordinates from the long chord.

2 - BY OFFSETS FROM THE TANGENTS.

Measure a distance x from PC on the back tangent (or from PT on the forward tangent), and then erect a perpendicular of length y. The resulting point G lies on the curve. From Figure 9.7:

$$y = R - \sqrt{R^2 - x^2} \qquad (9.12)$$

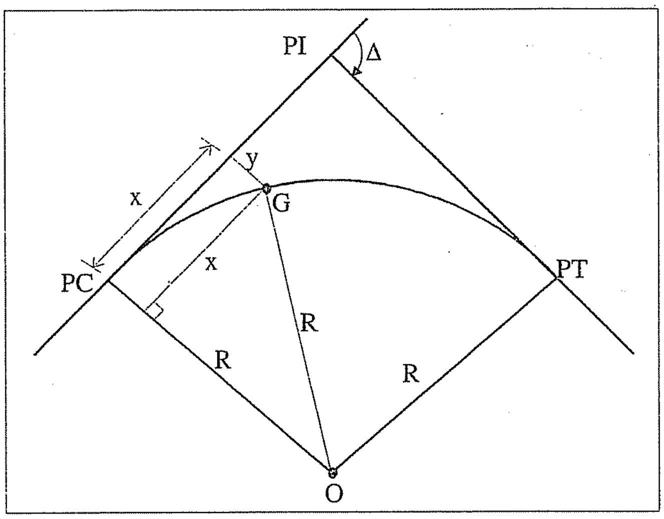


FIGURE 9.7: Setting out circular curves by offsets from the tangents.

3 - BY RADIAL OFFSETS

Measure a distance x from PC on the back tangent (or from PT on the forward tangent), and then measure the distance y along the radial line AO. The resulting point G lies on the curve. From Figure 9.8:

$$y = \sqrt{R^2 + x^2} - R \qquad \text{(9.13)}$$

This method is mainly used when it is possible to locate the center O and to extend the tape between the center and the tangents.

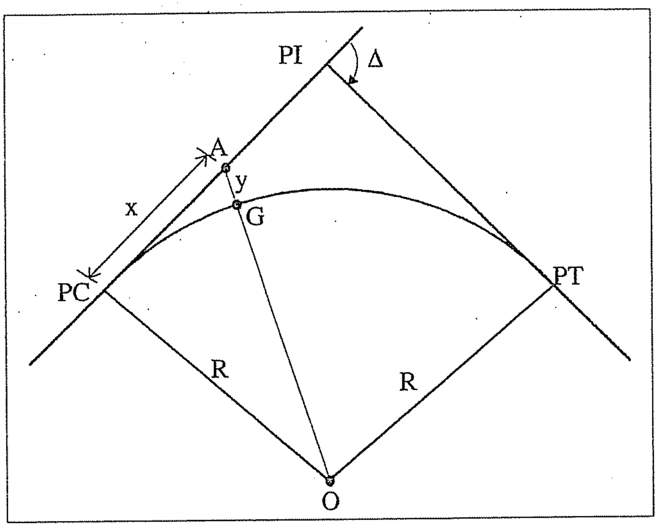


FIGURE 9.8: Setting out circular curves by radial offsets

4 - BY OFFSETS FROM THE MIDDLE POINTS OF CHORDS

Bisect the long chord at B, and then erect a perpendicular of length equal to $AB = R - R\cos(\Delta/2)$. After that bisect A-PT and A-PC at F and G respectively and erect two offsets of length equal to FK or $GJ = R - R\cos(\Delta/4)$. By repeating the above procedure, the resulting points A, J, K, etc. will lie on the curve (Figure 9.9).

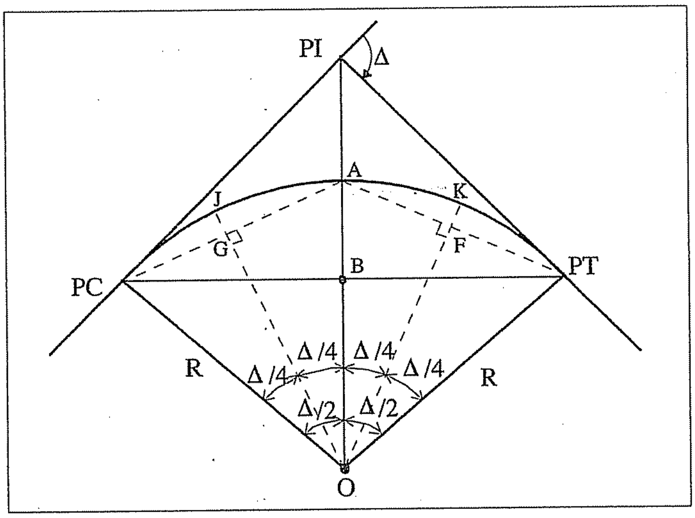


FIGURE 9.9: Setting out circular curves by offsets from the middle points of chords

9.2.1.5 CURVE LAYOUT BY DEFLECTION ANGLES USING ONE THEODOLITE

Stations along a circular curve are most frequently located in the field by measuring deflection angles from the PC. The method is based on the geometric principle that the angle between a tangent and a chord at a point on a circular curve is equal to one-half the angle subtended by the chord. Figure 9.10 illustrates this geometric principle.

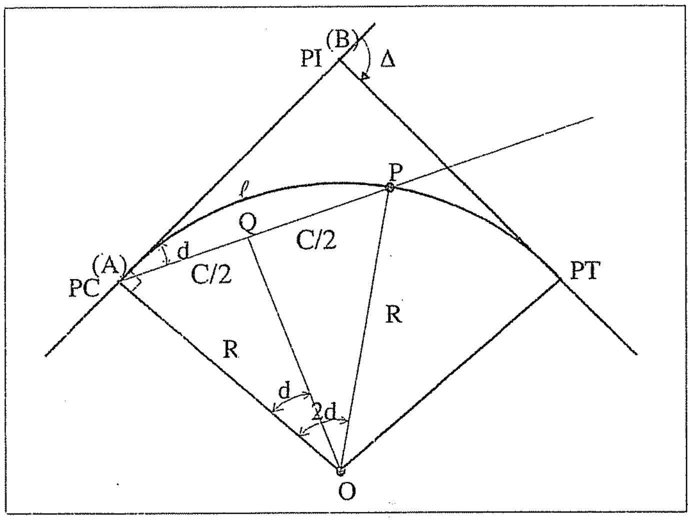


FIGURE 9.10: Principle of deflection angles.

Let angle $A\hat{O}P = 2d$, and let point Q be the intersection of the chord AP with the perpendicular from O. The angle $A\hat{O}Q = P\hat{O}Q = d$. Since $B\hat{A}P + P\hat{A}O = 90°$, and $P\hat{A}O + A\hat{O}Q = 90°$; then $B\hat{A}P = A\hat{O}Q = d$.

If D_a is the degree of curvature (arc definition), then $\dfrac{2d}{D_a} = \dfrac{\ell}{30}$

$$\Rightarrow d = \frac{\ell}{60} \cdot D_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.14)$$

Where ℓ is the arc length subtending the central angle 2d.

Let C = chord length AP. Then from Figure 9.10:

$$C = 2R \sin(d) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.15)$$

Now, let us take $C \leq \dfrac{R}{20}$, then $C \cong \ell$. To prove this, take C to be a maximum value of R/20. Then:

$$C = \frac{R}{20} = 2R \sin(d) \Rightarrow \sin(d) = \frac{1}{40} \quad \Rightarrow d = 1.432544°$$

$$\ell = \frac{2d}{180}\pi R, \quad \text{but} \quad R = 20C \ (\text{because } C = \frac{R}{20})$$

$$\Rightarrow \quad \ell = \frac{2 \times 1.432544}{180}\pi(20C) \cong 1.0001C$$

$$\Rightarrow \quad \ell \cong C$$

GENERAL PROCEDURE:

The following procedure is used for laying out simple circular curves (see Figure 9.11). Equipment required include: one theodolite, tapes, pins and ranging rods.

A. Preliminary work and calculations:

1) Locate the PI by extending the tangents xx_1 & yy_1. Measure Δ and then calculate the tangent distance $T = R\tan\frac{\Delta}{2}$.
2) From the PI, locate PC and PT by measuring a distance equal to T along the two tangents.
3) Calculate the circular curve length (L) from Equations (9.2) or (9.7).
4) Divide L into a number of partial arcs each $\leq R/20$. By doing this, the partial chord C becomes approximately equal to the partial arc ℓ.
5) Choose the first partial chord length C_1 so that the station (chainage) of the first point on the curve becomes an even integer divisible by 5 or 10 (*the station or chainage of a point along a route is its distance from the beginning point of the route to this point measured along the centerline of the route – see section 9.2.1.9*).
6) Choose C for the intermediate chords so that $C \leq R/20$ and is a multiple of 5 or 10. This will allow all intermediate points on the curve to have even stations.
7) Choose the last partial chord C_2 so that $C_2 \leq R/20$ and $C_2 = L - (C_1 + nC)$, where n is the number of intermediate partial chords. In Figure 9.11, $n = 2$.
8) Calculate the partial deflection angles from:

$$d = \frac{D_a \cdot C}{60} \quad \text{.......................................(9.16)}$$

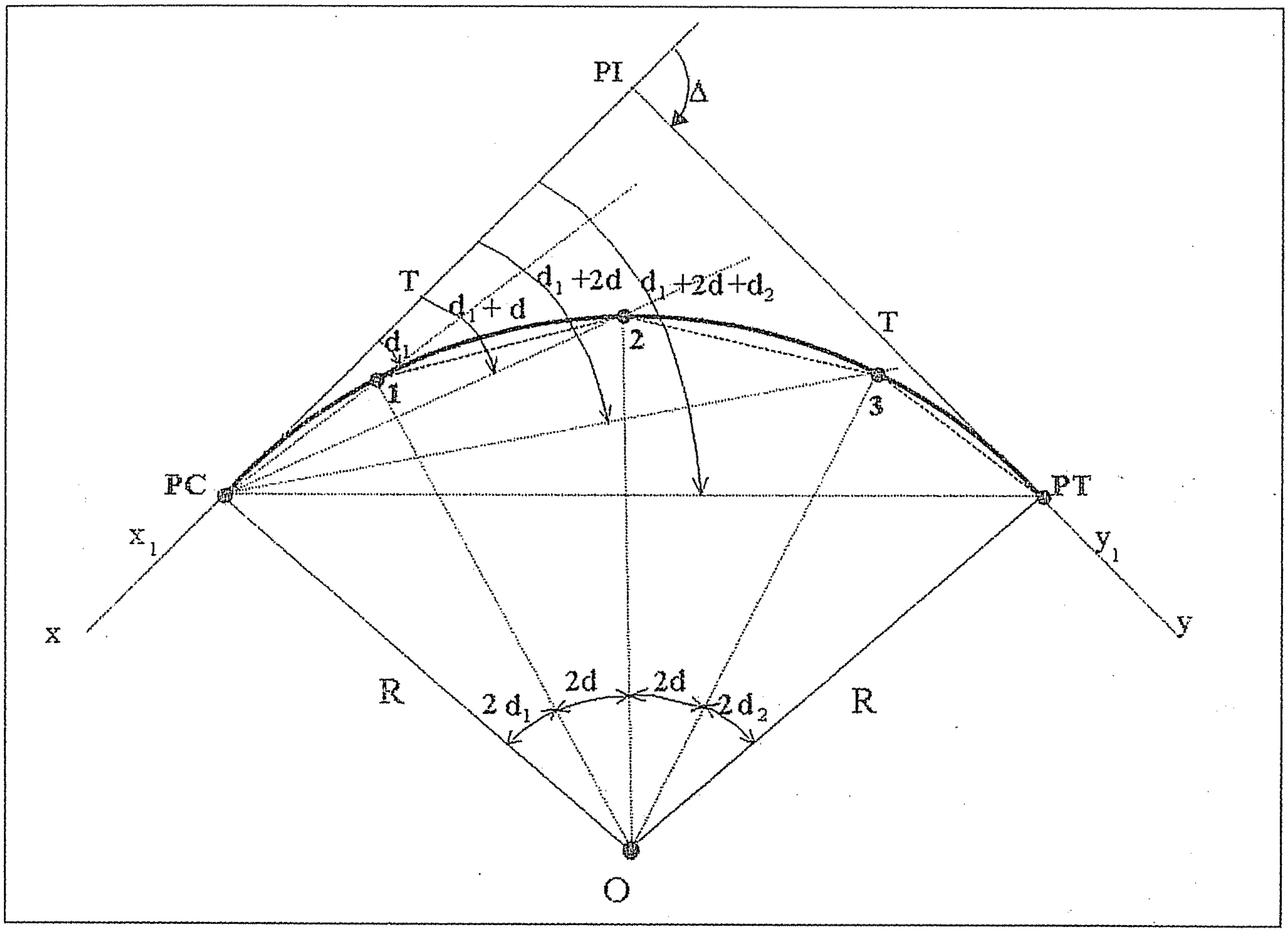


FIGURE 9.11: Layout of circular curves by deflection angles using one theodolite.

$$\text{Since } D_a = \frac{1718.873}{R},$$

$$d = \frac{28.648 \cdot C}{R}$$

$$\Rightarrow d_1 = \frac{28.648 \cdot C_1}{R} \qquad \text{.............................(9.17)}$$

$$d_2 = \frac{28.648 \cdot C_2}{R}$$

Problem: show that $d_1 + nd + d_2 = \Delta/2$.

B. Setting out the circular curve:

1) Set up the theodolite at the PC and bring it to read zero while sighting the PI.
2) Rotate the theodolite an angle equal to d_1 in the clockwise direction. The rear chain-man holds the zero of the tape over PC and the leader holds the tape at a length of C_1 , and begins moving left and right until the theodolite man sees him. At this point he drives an arrow into the ground marking the first point on the curve (point 1 in Figure 9.11).
3) Rotate the theodolite an angle equal to d so that the reading becomes d_1 + d. The rear man moves to point 1 holding the zero of the tape, while the leader is moving left and right with the tape extended horizontally a distance equal to C. When the leader is seen by the theodolite man, he drives another arrow into the ground marking point 2.
4) Continue on the above procedure until you reach the PT.

Notes:

1) When locating the last point before the PT (point 3 in Figure 9.11), we measure the distance from this point to the PT. If it is equal to C_2, then the work is perfect. If the error is 5 - 10 cm, it is distributed over all of the points on the curve by adjusting the pegs or arrows. If the error is large, the work should be repeated.

2) If the PT was not located before the laying out of the curve (i.e. by measuring a distance T from the PI on the forward tangent), then we must make sure that the PT located according to the previous procedure lies on the forward tangent (Y-Y_1). If it does not lie on the tangent, corrections and adjustments should be made.

EXAMPLE 9.2:

Two tangents from an axis of a road intersect at PI whose chainage (station) = 2140.00 m and make a deflection angle of 10° 35' 12". Required is to arrange all the information needed to set out a simple circular curve whose D_a = 4° by the deflection angle method and according to the Jordanian Ministry of Public Works System.

SOLUTION:

- $R = \dfrac{1718.87}{D_a} = \dfrac{1718.87}{4} = 429.72$ m

- $T = R \tan\dfrac{\Delta}{2} = 429.72 \tan\dfrac{10°\ 35'\ 12''}{2} = 39.81$ m

- $L = \dfrac{30\ \Delta}{D_a} = \dfrac{30 \times 10°\ 35'\ 12''}{4} = 79.40$ m

- Chainage of PC = Chainage of PI - T = 2140.00 - 39.81 = 2100.19 m
- Chainage of PT = Chainage of PC + L = 2100.19 + 79.40 = 2179.59 m

- Lengths of partial chords C:

$$C \le \frac{R}{20} = \frac{429.72}{20} = 21.49 \text{ m}, \Rightarrow \text{choose } C = 20.00 \text{ m}$$

Choose the station of point 1 to be 2120.00 m

$C_1 = 2120.00$ - Chainage of PC $= 2120.00 - 2100.19 = 19.81$ m

No. of intermediate partial arcs = 2

$C_2 = L - (C_1 + 2C) = 79.40 - (19.81 + 2 \times 20.00) = 19.59$ m

- Partial deflection angles:

$$d_1 = \frac{D_a \cdot C_1}{60} = \frac{4 \times 19.81}{60} = 1°\ 19'\ 14.4''$$

$$d = \frac{D_a \cdot C}{60} = \frac{4 \times 20.00}{60} = 1°\ 20'\ 00.0''$$

$$d_2 = \frac{D_a \cdot C_2}{60} = \frac{4 \times 19.59}{60} = 1°\ 18'\ 21.6''$$

All the required layout data are arranged in Table 9.1, and graphically represented in Figure 9.12.

TABLE 9.1: Layout data

Point #	Chord (m)	Chainage (m)	Deflection Angle ° ' "	Total Deflection Angle ° ' "
PC	0.00	2100.19	0 00 00.0	0 00 00.0
1	19.81	2120.00	1 19 14.4	1 19 14.4
2	20.00	2140.00	1 20 00.0	2 39 14.4
3	20.00	2160.00	1 20 00.0	3 59 14.4
PT	19.59	2179.59	1 18 21.6	5 17 36.0

Check: $d_1 + 2d + d_2 = 5° \ 17' \ 36" = \Delta/2 \Rightarrow$ O.K.

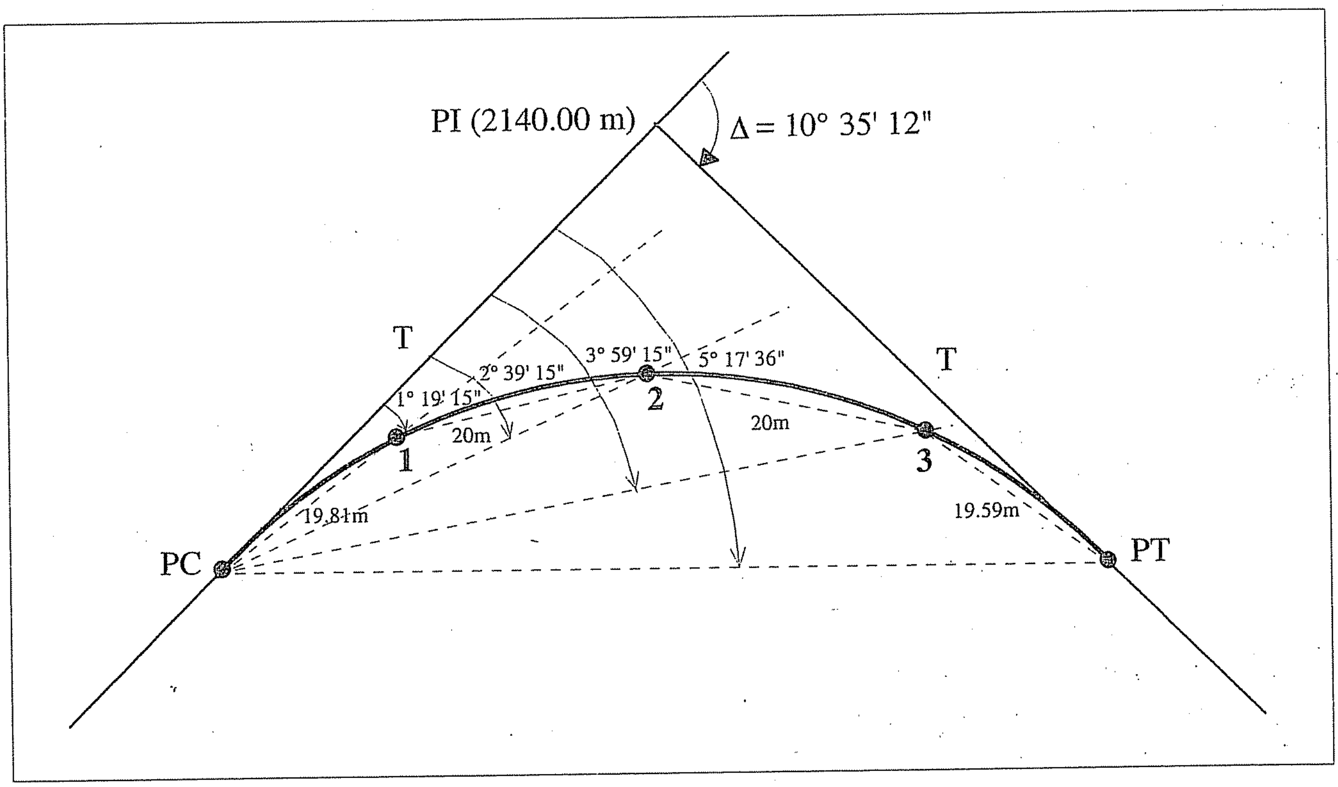


FIGURE 9.12: Curve layout of Example 9.2.(Note: the figure is not drawn to scale)

9.2.1.6 CURVE LAYOUT USING ELECTRONIC TOTAL STATIONS

With the advent of electronic total stations, circular curves can now be laid out much easier than ever before. This method is similar to the method in the previous section, except that long chords from the PC to the points to be located are measured using the electronic total station instead of the partial chords between consecutive points, which are measured using tapes. It has three advantages over the previous method:

1. The errors are not accumulative, which means that if an error happens when locating one of the curve points, this error will not affect the following points,
2. Can be used for all types of terrain as long as vision is not obscured, and gives more accurate results, and
3. Needs only two people to do the work.

The procedure can be easily described as follows:

1) Set up the total station over the PC and let the horizontal circle read zero when sighting the PI.

2) Rotate the alidade in a clockwise direction until the horizontal circle reads an angle equal (d_1).

3) The surveyor standing at the total station will order his assistant to move along the line of sight while carrying the reflector. He will then read the horizontal distance from the PC to the reflector and start signaling the assistant to move closer or farther until the horizontal distance becomes ($2R \sin d_1$). At this point, the assistant will drive an arrow into the ground locating the first point.

4) To locate point 2, the horizontal circle of the total station is brought to read ($d_1 + d$). Step 3 is then repeated until the horizontal distance between the PC and the reflector becomes [$2R \sin (d_1 + d)$].

5) This procedure is repeated until the whole circular curve is laid out. Table 9.2 shows the data needed to locate the circular curve in Example 9.2

TABLE 9.2: Layout data using a total station.

Point #	Chord (m)	Chainage (m)	Deflection Angle ° ' "	Total Def. Angle ° ' "	Long Chord from PC to the point
PC	0.00	2100.19	0 00 00.0	0 00 00.0	0.00
1	19.81	2120.00	1 19 14.4	1 19 14.4	19.81
2	20.00	2140.00	1 20 00.0	2 39 14.4	39.80
3	20.00	2160.00	1 20 00.0	3 59 14.4	59.77
PT	19.59	2179.59	1 18 21.6	5 17 36.0	79.29

9.2.1.7 CURVE LAYOUT BY DEFLECTION ANGLES USING TWO THEODOLITES

In this method, there is no need for measuring tapes or any linear measurement equipment. Therefore, it is preferred to be used for hilly uneven terrain where linear measurements using tapes are hard to make. Moreover, the errors in this method are not accumulative. However, two theodolites are required for this method of laying out simple circular curves, which makes it not feasible. The following steps are followed:

1) Calculate the lengths of partial chords, the partial deflection angles and the length of the tangent (T) as before.
2) Locate the points of curvature and tangency (PC & PT) by measuring a distance T from the PI on both the back and forward tangents.
3) Set up a theodolite at the PC and bring the horizontal circle to read zero while sighting the PI (i.e. in the direction of the back tangent). Set up another theodolite at the PT and bring the horizontal circle to read zero while sighting the PC (i.e. in the direction of the long chord).
4) Rotate the telescopes of both theodolites an angle equal to d_1 in a clockwise direction (see Figure 9.13).
5) A person carrying a ranging rod starts moving somewhere between the two theodolites while receiving signals from both theodolite-men standing at the PC and the PT until the ranging rod becomes at the intersection point of the lines of sight of both theodolites. An arrow is then driven at this point, which constitutes point 1 of the curve.

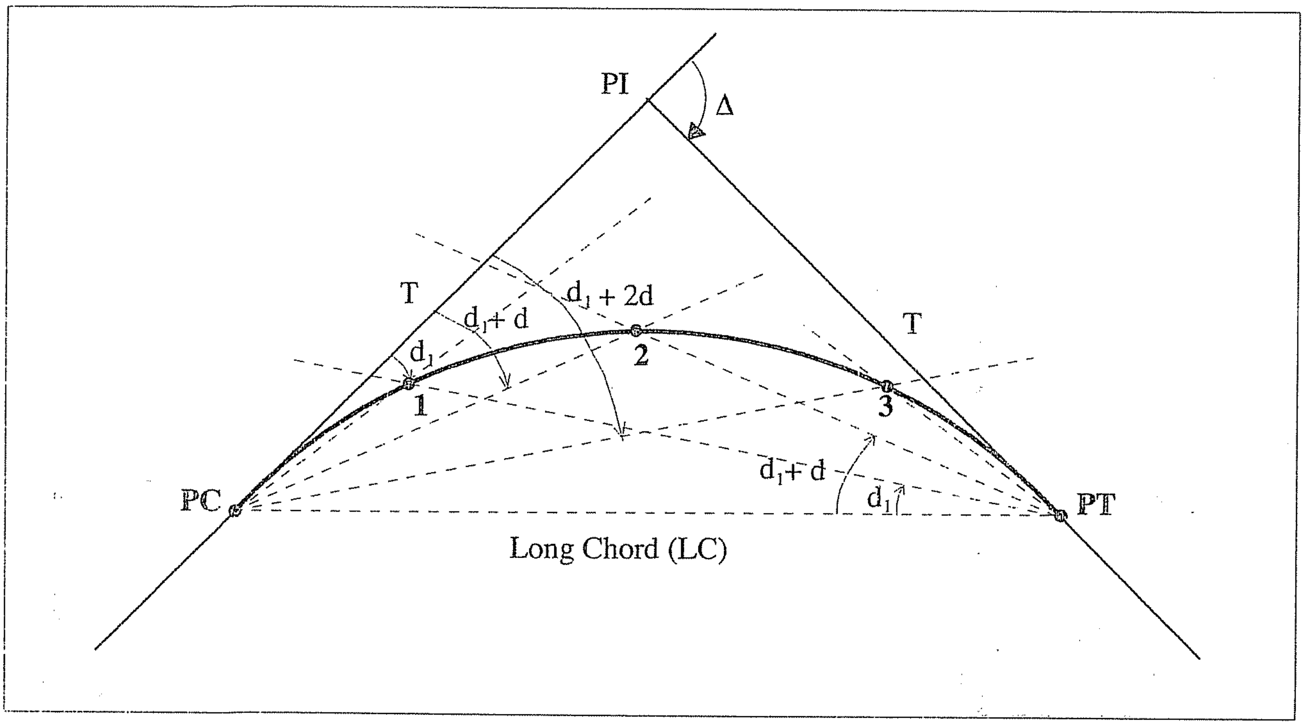


FIGURE 9.13: Layout of circular curves by deflection angles using two theodolites.

6) To locate point 2 of the curve, the telescopes of both theodolites are rotated in a clockwise direction to read an angle (d_1 + d) on the horizontal circle. The person carrying the ranging rod moves according to the signals received from the theodolite-men until the rod becomes at the intersection point of the lines of sight of the two theodolites. An arrow is driven at this point, which constitutes point 2 of the curve.

7) Repeat the preceding procedure to locate the rest of the curve points while making the theodolites read the right corresponding angles.

9.2.1.8 OBSTACLES AND DIFFICULTIES IN SETTING OUT CIRCULAR CURVES

These can be divided into three main categories:

(1) Inaccessible Point of Intersection:

Sometimes, it is not easy to reach the point of intersection due to the existence of a natural obstacle such as a river, a lake, a residential area, etc., and therefore the direct measurement of Δ and T from PI becomes difficult. Figure

9.14 illustrates such an obstacle, where a river crossing the two tangents makes it difficult to cross and reach P. In order to overcome this obstacle, choose two points J and K near the shore and measure the distance JK. Set up the theodolite at J and K and measure angles $\hat{2}$ and $\hat{1}$ respectively.

$$\text{Angle } \hat{3} = 180° - \hat{1} - \hat{2}$$

$$\Delta = 180° - \hat{3} = \hat{1} + \hat{2}$$

From the sine law for triangle PKJ:

$$JP = JK\frac{\sin\hat{1}}{\sin\hat{3}} \quad , \quad \text{and } PK = JK\frac{\sin\hat{2}}{\sin\hat{3}}$$

To locate PC and PT, measure the distance (T - JP) and (T - KP) respectively along the two tangents from points J & K (where T = R tan Δ/2). The first located point will be the PC and the second will be the PT.

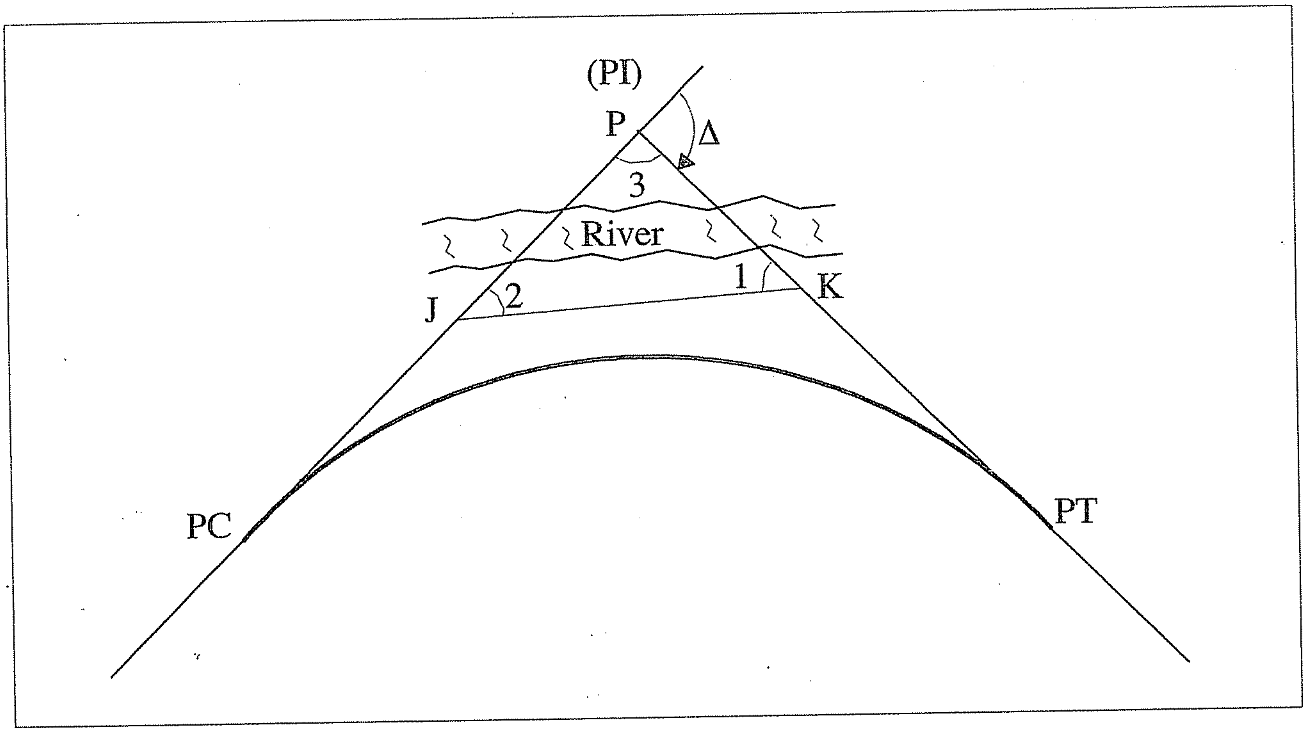


FIGURE 9.14: Inaccessible point of intersection.

(2) Vision Obstacle:

Occasionally, because of obstructions or the great length of the curve, it may be necessary to make an intermediate setup. In Figure 9.15, points 1 and 2 were located with the theodolite at PC. However, due to the existence of obstacle G, it is not possible to locate point 3 with the theodolite at PC. To solve this problem:

1 - Set up the theodolite at point 2, direct the telescope towards PC, and with the horizontal circle reading zero, invert the telescope to point in the direction 2-2'.

2 - Rotate the theodolite in a clockwise direction an angle = (d_1 + 2d) so that the telescope will point in the direction 2-2".

3 - With the rear chain-man holding the zero of the tape at point 2, the leader holds the tape at a length equal to the partial chord length (C) and moves left and right until he is seen by the theodolite man (i.e. in the direction of the line of sight). The leader drives a peg in the ground marking point 3.

4 - To locate the remaining points, continue with the calculated angles and chords exactly as if the theodolite is located at PC.

Problem: Prove that this method as described above is correct.

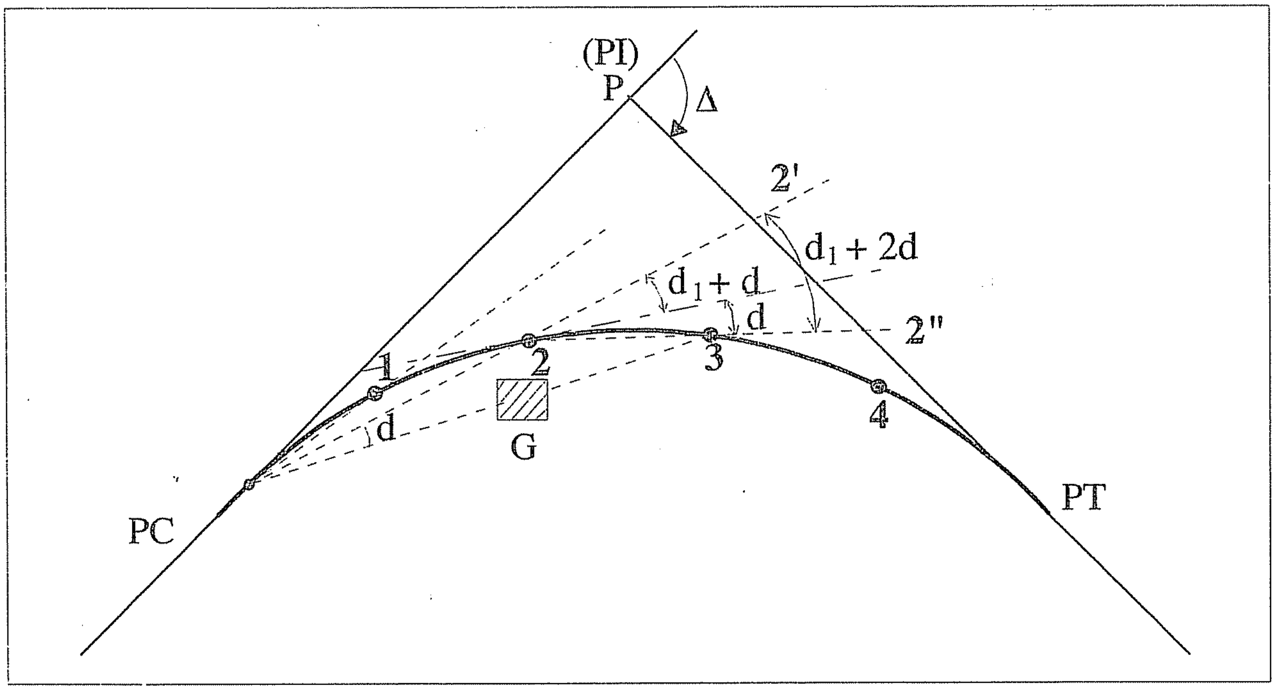


FIGURE 9.15: Vision obstacle

(3) Chaining Obstacle:

In Figure 9.16, assume that points 1 and 2 were located with the theodolite located at PC and according to the procedure in section 9.2.1.5. Due to the existence of the obstacle G between points 2 and 3, point 3 cannot be located by direct measurement of the partial chord length (C) between 2 and 3. Instead of that, point 3 is located by direct measurement of the distance from PC to 3. This distance is equal to $2R \sin(d_1 + 2d)$.

After locating point 3, the procedure is continued according to section 9.2.1.5.

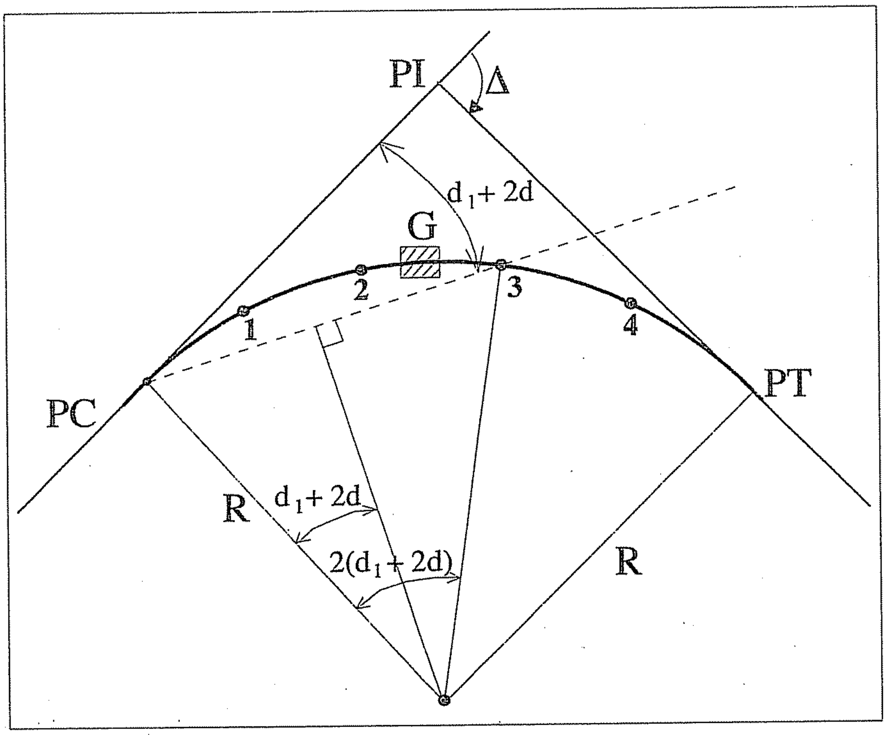
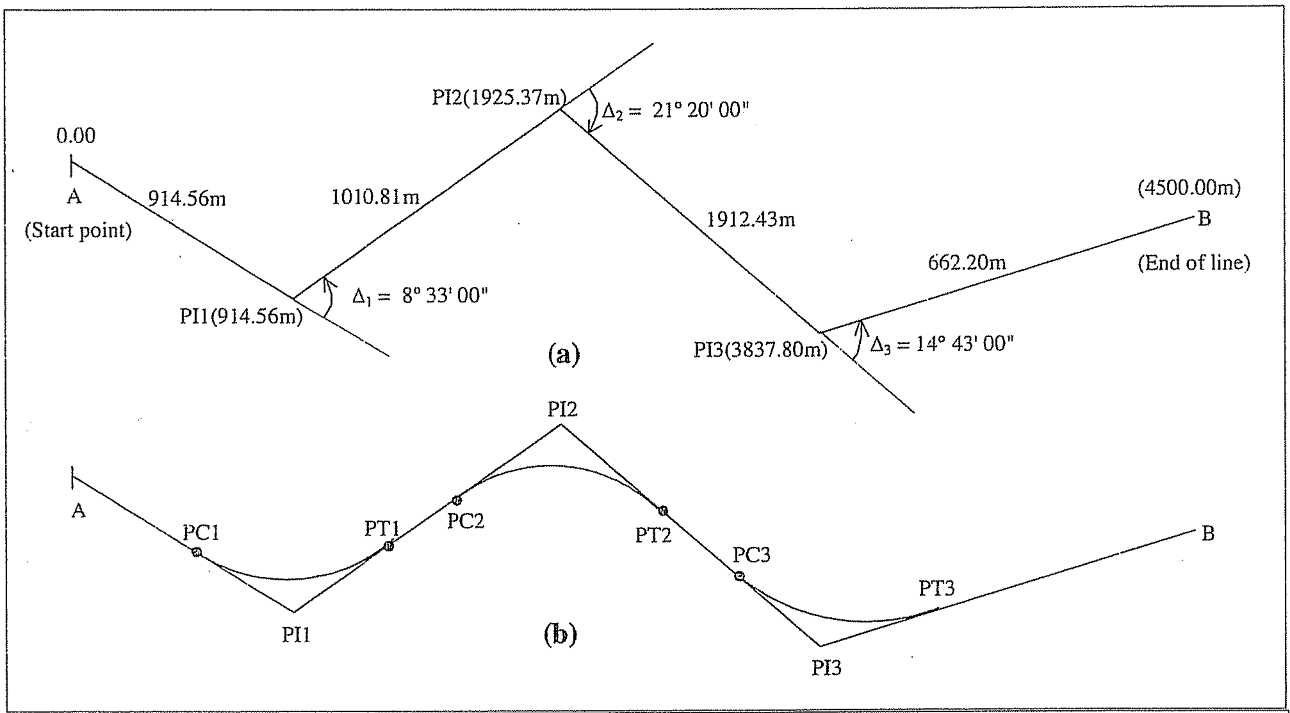


FIGURE 9.16: Chaining obstacle

9.2.1.9 STATIONING

The station or chainage of a point along a route is its distance from the beginning point of the route to this point measured along the centerline of the route. The procedure for computing the stations along the centerline of a route can be illustrated by the example shown in Figure 9.17.



$D_{a_1} = 2°$	$D_{a_2} = 3°$	$D_{a_3} = 3°$
$R_1 = 859.44$ m	$R_2 = 572.96$ m	$R_3 = 572.96$ m
$T_1 = 64.24$ m	$T_2 = 107.92$ m	$T_3 = 73.99$ m
$L_1 = 128.25$ m	$L_2 = 213.33$ m	$L_3 = 147.17$ m

FIGURE 9.17: Stationing

The route consists of four tangents and three points of intersection. The lengths of the tangents and the magnitude of the intersection angles are shown in Figure 9.17a. The values of D_a, L, R and T for each of the three curves are shown in Figure 9.17b. The stations of the PCs, PIs and PTs are computed as follows:

Beginning station (A) =	0.00 m	
	+ 914.56	
Station PI_1	914.56	
$-T_1$	64.24	
Station PC_1	850.32	
$+L_1$	128.25	
Station PT_1	978.57	
	+1010.81	Station PT_1 + distance from PT_1 to PI_2
	1989.38	
$-T_1$	64.24	
Station PI_2	1925.14	
$-T_2$	107.92	
Station PC_2	1817.22	
$+L_2$	213.33	
Station PT_2	2030.55	
	+1912.43	Station PT_2 + distance from PT_2 to PI_3
	3942.98	
$-T_2$	107.92	
Station PI_3	3835.06	
$-T_3$	73.99	
Station PC_3	3761.07	
$+L_3$	147.17	
Station PT_3	3908.24	
	+ 662.20	Station PT_3+distance from PT_3 to the ending station
	4570.44	
$-T_3$	73.99	
Ending station (B)	4496.45 m	

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In this manner, the stations along the centerline of the route are numbered through the curves and not through the PIs. The stationing provides the actual centerline distance from the beginning point of the project.

9.2.2 TRANSITION OR EASEMENT CURVES

9.2.2.1 INTRODUCTION

As a vehicle traverses from the tangent at the point of curvature (PC) on to the circular curve, it comes under the side effect of a centrifugal force. The value of both the centrifugal force (P) and centrifugal ratio (e) are given by:

$$P = \frac{WV^2}{gR}$$

And $$e = \frac{V^2}{gR}$$

Where: W = weight of vehicle
V = velocity
R = radius of curve
g = acceleration due to gravity

It can be seen from both formulae that P and e increase as R decreases. Therefore, since the straight line is theoretically considered to have a radius equal to ∞, the centrifugal force P will increase instantaneously from zero to its maximum value (assuming no change in V) as the vehicle moves from the tangent to the curve (Figure 9.18a). The passengers in the vehicle will experience a lateral shock as the tangent point is passed. In order to build the centrifugal force in a gradual and uniform manner, a curve of uniformly decreasing radius is inserted between the tangent and the circular curve. In this way, the lateral shock will be minimized (Figures 9.18b). This curve is called a transition or easement curve and it is the beginning section of a spiral curve (Figure 9.19).

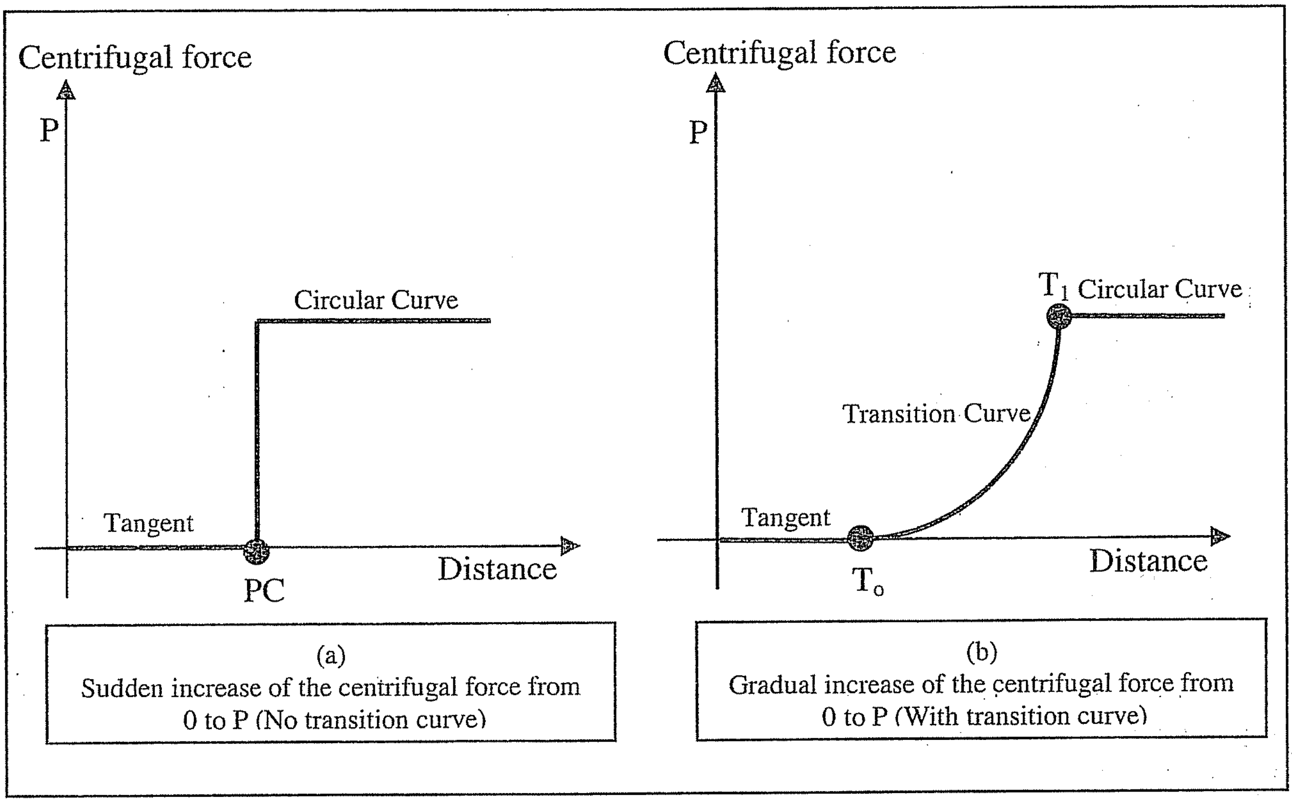

FIGURE 9.18: Relationship between the centrifugal force and distance as we move from the tangent to the circular curve (with and without a transition curve).

As Figure 9.19 shows, the radius of a transition curve decreases uniformly from infinity at its tangent point with the tangent (named T_o), to a minimum value (equal to the circular curve radius – R) at its tangent point with the circular curve (named T_1). As will be seen in the following sections, a circular curve will be joined by transition curves from both sides to ensure smooth and safe traffic flow (Figure 9.24). In some cases, the circular curve joining the two transitions is of zero length so that the single circular curve is replaced by two transition curves having one common tangent point.

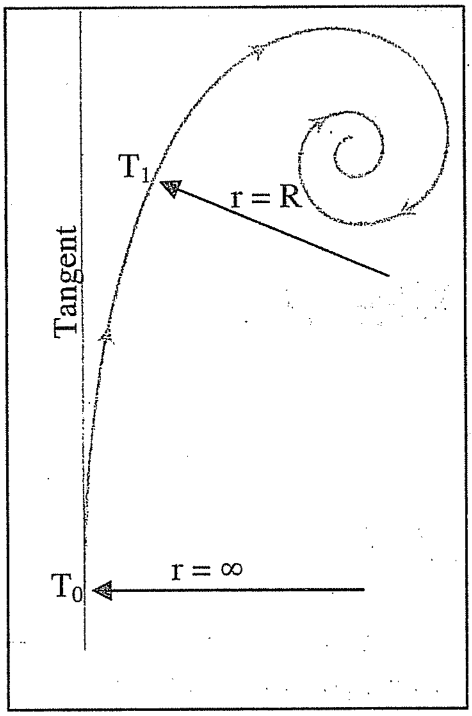

FIGURE 9.19: Spiral curve.

In addition to using a transition curve to connect the tangent with the circular curve, two other measures can be used to minimize the lateral effect of the centrifugal force, and thus, protect the vehicle and the passengers. These are:

1. Use special road signs on the road side to ask the drivers to slow down because they will be entering a curve.

2. Raise the outer edge of the road at the curve. This is called super-elevation, and is discussed in the next section.

9.2.2.2 SUPER-ELEVATION

When a vehicle or train crosses the point of curvature of a curve, it becomes under the effect of two forces: a side thrust equal to the centrifugal force, and its own weight (see Figure 9.20). By lifting the outer edge of the road or rails, i.e. by applying super-elevation to the curve, the resultant N of these two forces can be made to lie along the normal to the road surface or rails for a given speed.

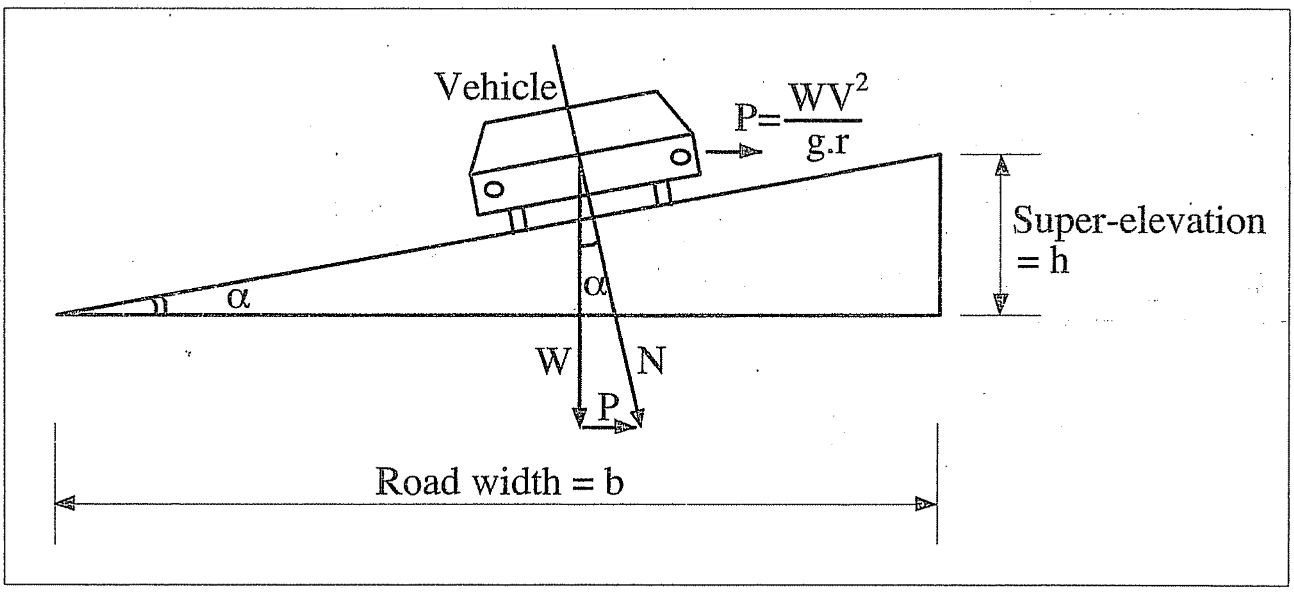

FIGURE 9.20: Super-elevation.

From the triangle of forces:

$$\tan \alpha_i = \frac{WV^2 / g\,r_i}{W} = \frac{V^2}{g\,r_i} \quad \text{.........................(9.18)}$$

Where r_i is the radius at any point i.

On the circular curve: $r_i = R$

$$\Rightarrow e_i = e = \frac{V^2}{gR} \quad \text{...........................(9.19)}$$

Divide Equation (9.18) by (9.19):

$$\Rightarrow \frac{e_i}{e} = \frac{R}{r_i} \quad \Rightarrow \quad e_i = e \cdot \frac{R}{r_i} \quad \text{..............................(9.20)}$$

This means that the value of $e_i = \tan \alpha_i$ at any point on the transition curve can be calculated from Equation (9.20) if e is known for the circular curve, which has a radius = R.

The amount of super-elevation (h) can be calculated from the following formula:

Super-elevation = $h = b.\tan \alpha_i = b.e_i$ .... (9.21)

Where b = width of road or railway track.

The maximum super-elevation occurs when e_i is maximum, which occurs at the circular curve. The transition curve allows this super-elevation to be introduced in a gradual manner so that it varies from zero on the tangent to its maximum value at the tangent point where the transition curve meets the circular curve.

To take side friction into consideration, normally super-elevation is determined from the following expression:

$$e_i = \frac{V^2}{314 r_i} \quad \text{...................................(9.22)}$$

Where: V = design speed in km/hr

r_i = radius of the curve in meters.

From the triangle of forces:

$$\tan \alpha_i = \frac{WV^2 / g\, r_i}{W} = \frac{V^2}{g\, r_i} \qquad \text{.......................(9.18)}$$

Where r_i is the radius at any point i.

On the circular curve: $r_i = R$

$$\Rightarrow e_i = e = \frac{V^2}{gR} \qquad \text{..........................(9.19)}$$

Divide Equation (9.18) by (9.19):

$$\Rightarrow \frac{e_i}{e} = \frac{R}{r_i} \Rightarrow e_i = e \cdot \frac{R}{r_i} \qquad \text{...........................(9.20)}$$

This means that the value of $e_i = \tan \alpha_i$ at any point on the transition curve can be calculated from Equation (9.20) if e is known for the circular curve, which has a radius = R.

The amount of super-elevation (h) can be calculated from the following formula:

$$\text{Super-elevation} = h = b.\tan \alpha_i = b.e_i \qquad \text{.................(9.21)}$$

Where b = width of road or railway track.

The maximum super-elevation occurs when e_i is maximum, which occurs at the circular curve. The transition curve allows this super-elevation to be introduced in a gradual manner so that it varies from zero on the tangent to its maximum value at the tangent point where the transition curve meets the circular curve.

To take side friction into consideration, normally super-elevation is determined from the following expression:

$$e_i = \frac{V^2}{314 r_i} \qquad \text{..................................(9.22)}$$

Where: V = design speed in km/hr

r_i = radius of the curve in meters.

EXAMPLE 9.3:

Calculate the super-elevation for a curved section of a road given that:
R = 275 m
V = 60 km/hr, and
b = 7.5 m

SOLUTION:

$$e = \frac{V^2}{314R} = \frac{(60)^2}{314 \times 275} = 0.042 = 4.2\%$$

Super-elevation = h = b x e
= 7.5 x 0.042 = 0.315 m

9.2.2.3 DERIVATION OF THE TRANSITION CURVE EQUATIONS

Summarizing the points that have been made so far, we have:

1) P increases uniformly with the distance ℓ from the beginning of the transition curve, i.e. $P \propto \ell$.

2) At a given point on the transition curve where the radius = r, the centrifugal force (P) is:

$$P = \frac{WV^2}{gr}$$

$\Rightarrow P \propto 1/r$ for constant velocity

Now, since ($P \propto \ell$) and ($P \propto 1/r$), $\Rightarrow \ell \propto 1/r$ (i.e. the length of the transition curve at a point is inversely proportional to the radius of the curve at that point. Thus, $\ell \cdot r = K$ (where K is some constant), and if the transition curve has a length L and the radius of the circular curve entered is R (see Figure 9.21), then:

$$L \cdot R = K$$

$$\text{or } \ell \cdot r = L \cdot R = K \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.23)$$

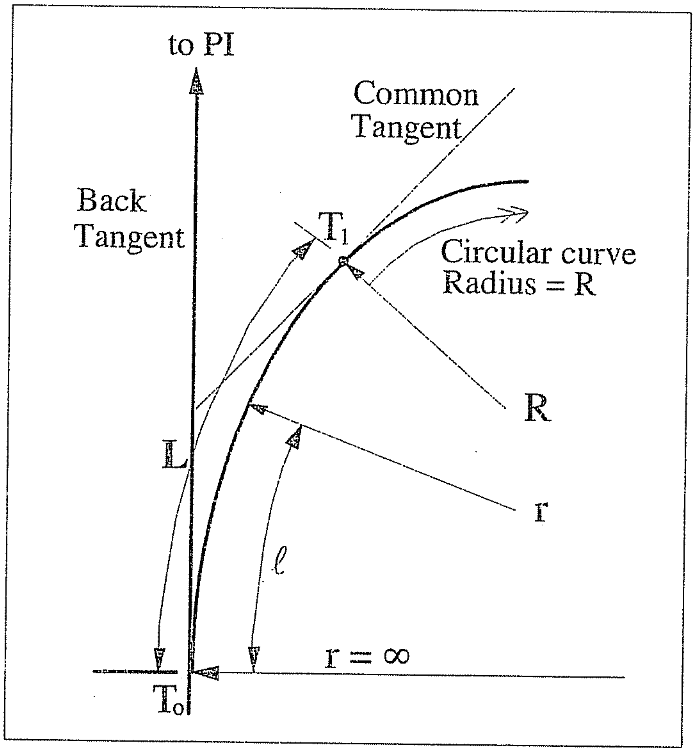

FIGURE 9.21: Transition curve.

T_0T_1 is the transition curve along which the radius varies from infinity at T_o to R at T_1. The tangent at T_1 is common to both transition and circular curves.

LENGTH OF TRANSITION CURVES:

The length of a transition curve may be taken:

1) As an arbitrary value from past experience, say 70 m,

2) Such that the super-elevation is applied at a uniform rate, say 0.10 m in 100 m, or

3) Such that the rate of change of radial acceleration equals a certain chosen value. Radial acceleration at any point = V^2/r. In Figure 9.21:

Radial acceleration at $T_0 = V^2/\infty = 0$ $(r = \infty)$
Radial acceleration at $T_1 = V^2/R$ $(r = R)$

For a uniform velocity V:

Time required to travel the transition curve (t) = L/V

Rate of change of radial acceleration (a) $= \dfrac{V^2/R - 0}{L/V} = \dfrac{V^3}{RL}$

$$\Rightarrow L = \frac{V^3}{aR} \qquad \text{.............................(9.24)}$$

Where a is usually taken to lie between 0.3 & 0.5 m/sec³.

THE TRANSITION CURVE LAYOUT EQUATIONS:

Consider two points A and B on the transition curve (Figure 9.22) separated by a distance $d\ell$, with A being ℓ from T_0, then:

$d\ell = r.d\phi$

But $r\ell = K$ (Equation 9.23)

$$\Rightarrow d\ell = \frac{K}{\ell} d\phi \;, \quad \text{or} \quad d\phi = \frac{\ell}{K} d\ell$$

By integration:

$$\Rightarrow \phi = \frac{\ell^2}{2K} + C$$

To find the constant C, substitute $\phi = 0$ when

$\ell = 0$,

$\Rightarrow 0 = 0^2/2K + C \qquad \Rightarrow C = 0$,

And since $K = \ell r = LR$

$$\Rightarrow \phi = \frac{\ell^2}{2RL} \qquad \text{.............................(9.25)}$$

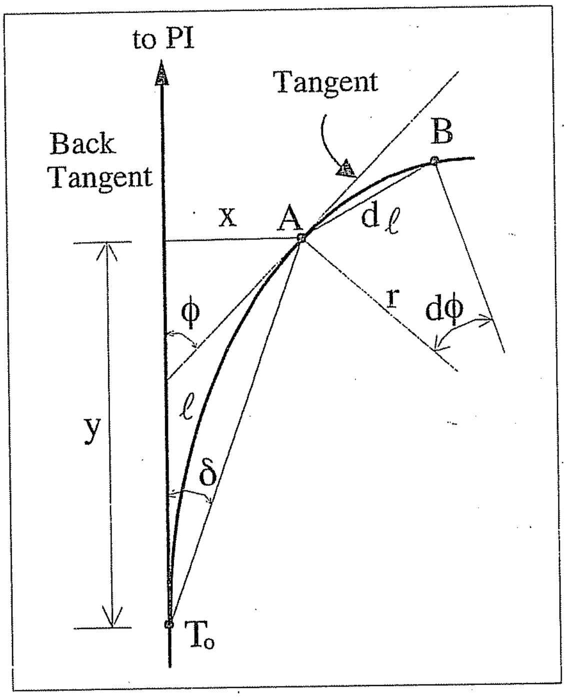

FIGURE 9.22: Geometry of a transition curve.

Equation (9.25) is the equation for the ideal transition spiral.

Now consider Figure 9.23:

$$\beta = \phi + \frac{d\phi}{2}$$

$$dx = d\ell \sin\beta = d\ell \sin\left(\phi + \frac{d\phi}{2}\right)$$

$$dy = d\ell \cos\beta = d\ell \cos\left(\phi + \frac{d\phi}{2}\right)$$

Since $\frac{d\phi}{2}$ is small compared to ϕ, it can be ignored:

$$\Rightarrow dx = d\ell \sin\phi$$

$$dy = d\ell \cos\phi$$

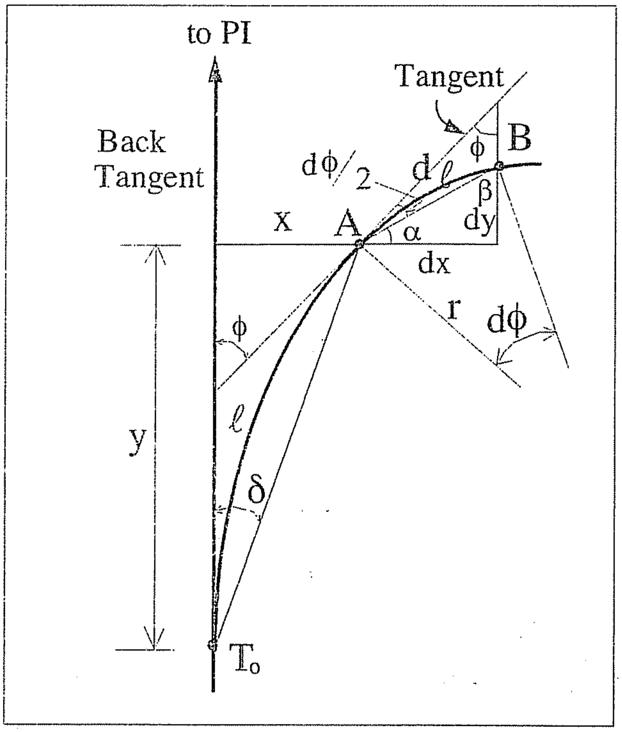

FIGURE 9.23: Coordinates of a point on a transition curve.

Using Mac Laurin's Series to expand $\sin\phi$ & $\cos\phi$:

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$$

$$\Rightarrow dx = d\ell\left[\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots\right]$$

And $$\Rightarrow dy = d\ell\left[1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots\right]$$

Substituting $\phi = \frac{\ell^2}{2K} \Rightarrow$

$$dx = d\ell\left[\frac{\ell^2}{2K} - \left(\frac{\ell^2}{2K}\right)^3\left(\frac{1}{6}\right) + \left(\frac{\ell^2}{2K}\right)^5\left(\frac{1}{120}\right) - \ldots\right]$$

$$dy = d\ell\left[1 - \left(\frac{\ell^2}{2K}\right)^2\left(\frac{1}{2}\right) + \left(\frac{\ell^2}{2K}\right)^4\left(\frac{1}{24}\right) - \left(\frac{\ell^2}{2K}\right)^6\left(\frac{1}{720}\right)\ldots\right]$$

By integration:

$$\Rightarrow \quad x = \frac{\ell^3}{6K} - \frac{\ell^7}{336K^3} + \frac{\ell^{11}}{42240K^5} + \ldots \quad \text{negligible terms} \qquad \ldots\ldots (9.26)$$

$$y = \ell - \frac{\ell^5}{40K^2} + \frac{\ell^9}{3456K^4} + \ldots\ldots \quad \text{negligible terms}$$

Since K = LR is large, the above equations can be approximated to:

$$x = \frac{\ell^3}{6K} \qquad \ldots\ldots (9.27)$$

$$y = \ell \qquad \ldots\ldots (9.28)$$

Substituting K = LR into Equation (9.27)

$$\Rightarrow \quad x = \frac{\ell^3}{6LR} \qquad \ldots\ldots (9.29)$$

Equation (9.29) is called the equation of Cubic Spiral, and substituting $y = \ell$ (i.e. Equation 9.28) into this equation:

$$\Rightarrow \quad x = \frac{y^3}{6LR} \qquad \ldots\ldots (9.30)$$

Equation (9.30) is called the equation of ***Cubic Parabola***. This equation can be used to calculate the data for setting out the transition curve by offsets from the tangent similar to the way circular curves were set out, with y being the assumed distance on the back tangent, and x being the computed offset perpendicular to the tangent.

RELATION BETWEEN δ AND ϕ (FIGURE 9.23):

Substitute $\phi = \dfrac{\ell^2}{2LR}$ into Equations (9.26):

$$\Rightarrow \quad x \cong \frac{\ell^3}{6K}\left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440}\right)$$

$$y \cong \ell\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216}\right)$$

And from Figure 9.23,

$$\tan\delta = \frac{x}{y} = \frac{\ell^2}{6K} \frac{\left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440}\right)}{\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216}\right)}$$

$$\cong \frac{\phi}{3}\left(1 + \frac{\phi^2}{35}\right)$$

For small angles, which is the case with transition curves, the term $\frac{\phi^2}{35}$ becomes negligible and $\tan\delta = \delta$ (ϕ and δ are in radian).

$$\Rightarrow \delta = \frac{\phi}{3} \qquad \text{(9.31)}$$

SHIFT:

Where transition curves are introduced between the tangents and a circular curve of radius R (see Figure 9.24), the circular curve is shifted inwards from its original position by an amount AG = S (the shift) so that the transition and circular curves can meet tangentially. This is equivalent to having a circular curve of radius (R + S) connecting the tangents replaced by transition curves and a circular curve of radius R, even though the tangent points are not the same.

Referring to Figure 9.24, consider the two triangles BFA and OFT_1.

$$\text{Angle } F\hat{A}B = \text{Angle } O\hat{T_1}F = 90°$$
$$\text{Angle } B\hat{F}A = \text{Angle } O\hat{F}T_1$$
$$\Rightarrow \text{Angle } C\hat{O}T_2 = \text{Angle } A\hat{O}T_1 = \text{Angle } F\hat{B}A = \phi$$

Also:

$$AH = JT_1 = \text{maximum offset on transition curve} = \frac{L^3}{6LR}$$

$$\text{Shift (S)} = AG = AH - GH = JT_1 - GH$$
$$\Rightarrow \quad S = JT_1 - (GO - HO) = JT_1 - (R - R\cos\phi)$$

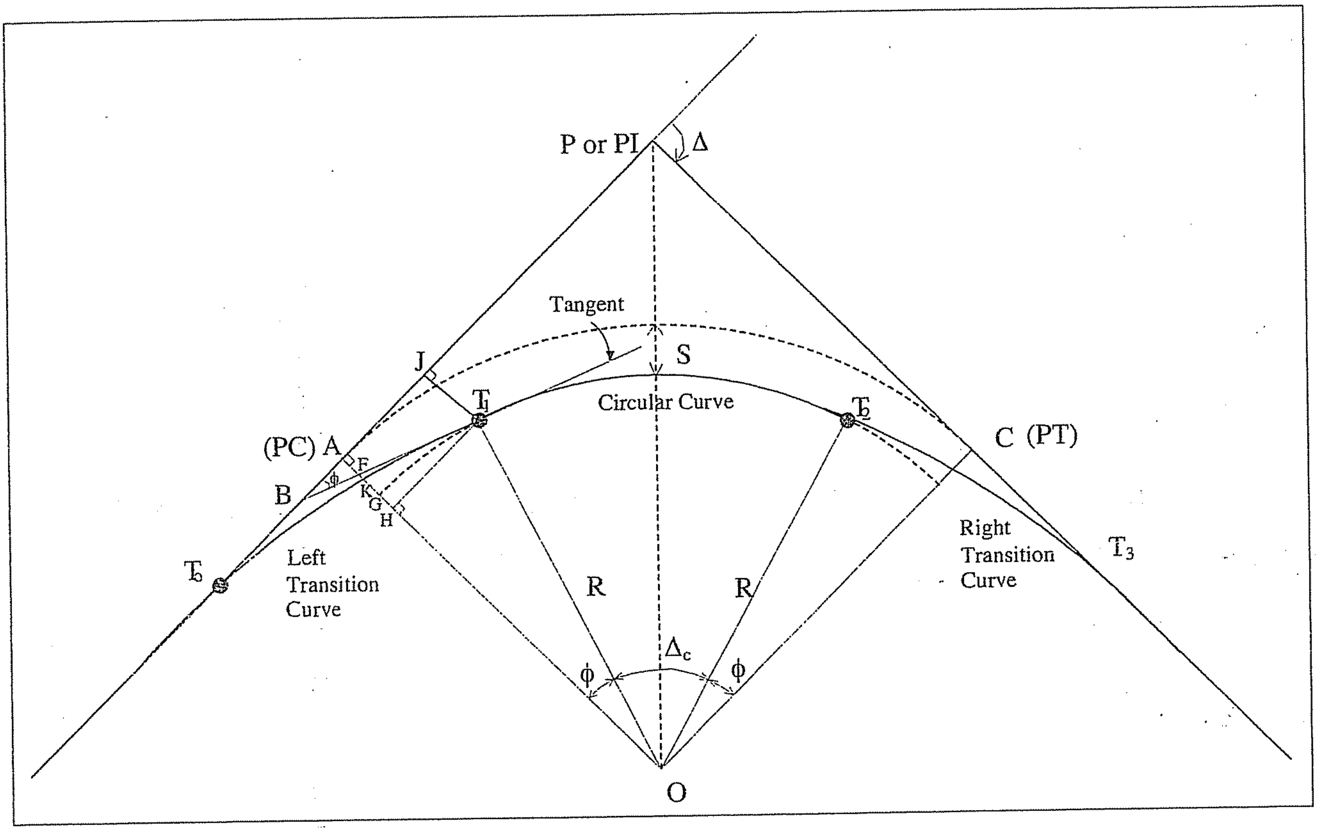

FIGURE 9.24: A circular curve connected to transition curves from both sides.

Substitute the values of JT_1 and cos ϕ:

$$JT_1 = \frac{L^3}{6LR}$$

$$\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots$$

$$\Rightarrow \quad S = \frac{L^3}{6LR} - \left[R - R\left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots\right)\right]$$

$$= \frac{L^3}{6LR} - R + R - \frac{R\phi^2}{2!} + \frac{R\phi^4}{4!} \dots$$

Ignoring the powers of ϕ which are higher than 2,

$$\Rightarrow \quad S = \frac{L^3}{6LR} - \frac{R\phi^2}{2} \qquad (\text{where } \phi = \frac{L^2}{2LR})$$

$$\Rightarrow \quad S = \frac{L^3}{6LR} - \frac{R}{2}\left(\frac{L^2}{2LR}\right)^2 = \frac{L^2}{24R}$$

$$\therefore \quad S = \frac{L^2}{24R} \quad \dots\dots\dots\dots \quad (9.32)$$

SHIFT IN THE POINT OF TANGENCY (AT_0):

From Figure 9.24, the curve T_1K from the transition curve is almost equal to T_1G from the circular curve:

$$T_1K \cong T_1G$$

But $T_1G = R\phi$ (where $\phi = \dfrac{L^2}{2LR}$)

$$\Rightarrow \quad T_1K \cong R\phi = R\left(\frac{L^2}{2LR}\right) = \frac{L}{2}$$

Therefore, K is the mid-point of the transition curve, and since the deviation of this point from the tangent is small, then:

$$T_1K = T_0K = T_0A = \frac{L}{2}$$

$$\Rightarrow \quad AT_0 \cong \frac{L}{2} \qquad \text{.......... (9.33)}$$

9.2.2.4 TRANSITION CURVE LAYOUT USING THE THEODOLITE

The following procedure is used for laying out a combination of circular and transition curves. Equipment required include: one theodolite, tapes, arrows and ranging rods.

A. Preliminary work and calculations:

1) Find D_a or R for the circular curve to be connected by the transition curves from both sides.
2) Measure the angle of intersection (Δ).
3) If the length of the transition curve (L) is not given, calculate it using Equation (9.24): $L = \dfrac{V^3}{aR}$
4) Calculate the shift (S) from Equation (9.32): $S = \dfrac{L^2}{24R}$
5) Calculate the total length of the tangent PT_0 (see Figure 9.24).

$$PT_0 = (R + S)\tan\left(\frac{\Delta}{2}\right) + \frac{L}{2} \qquad \text{.......... (9.34)}$$

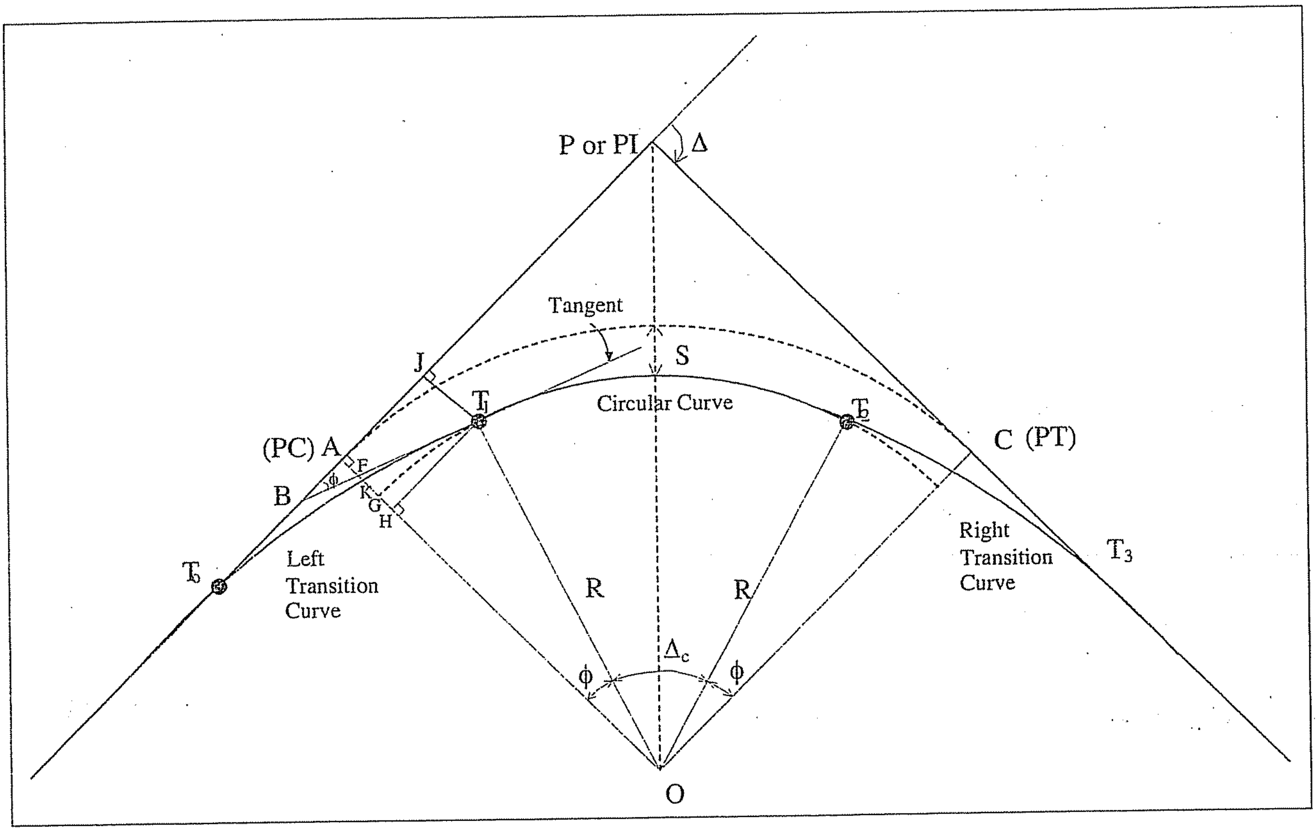

FIGURE 9.24: A circular curve connected to transition curves from both sides.

Substitute the values of JT_1 and $\cos\phi$:

$$JT_1 = \frac{L^3}{6LR}$$

$$\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \cdots$$

$$\Rightarrow \quad S = \frac{L^3}{6LR} - \left[R - R\left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \cdots\right)\right]$$

$$= \frac{L^3}{6LR} - R + R - \frac{R\phi^2}{2!} + \frac{R\phi^4}{4!} \cdots$$

Ignoring the powers of ϕ which are higher than 2,

$$\Rightarrow \quad S = \frac{L^3}{6LR} - \frac{R\phi^2}{2} \qquad \left(\text{where } \phi = \frac{L^2}{2LR}\right)$$

$$\Rightarrow \quad S = \frac{L^3}{6LR} - \frac{R}{2}\left(\frac{L^2}{2LR}\right)^2 = \frac{L^2}{24R}$$

$$\therefore \quad S = \frac{L^2}{24R} \qquad \cdots\cdots (9.32)$$

SHIFT IN THE POINT OF TANGENCY (AT_0):

From Figure 9.24, the curve T_1K from the transition curve is almost equal to T_1G from the circular curve:

$$T_1K \cong T_1G$$

But $\quad T_1G = R\phi \qquad (\text{where } \phi = \frac{L^2}{2LR})$

$$\Rightarrow \quad T_1K \cong R\phi = R\left(\frac{L^2}{2LR}\right) = \frac{L}{2}$$

Therefore, K is the mid-point of the transition curve, and since the deviation of this point from the tangent is small, then:

$$T_1K = T_0K = T_0A = \frac{L}{2}$$

$$\Rightarrow \quad AT_0 \cong \frac{L}{2} \qquad \ldots\ldots (9.33)$$

9.2.2.4 TRANSITION CURVE LAYOUT USING THE THEODOLITE

The following procedure is used for laying out a combination of circular and transition curves. Equipment required include: one theodolite, tapes, arrows and ranging rods.

A. Preliminary work and calculations:

1) Find D_a or R for the circular curve to be connected by the transition curves from both sides.
2) Measure the angle of intersection (Δ).
3) If the length of the transition curve (L) is not given, calculate it using Equation (9.24): $\quad L = \frac{V^3}{aR}$
4) Calculate the shift (S) from Equation (9.32): $S = \frac{L^2}{24R}$
5) Calculate the total length of the tangent PT_0 (see Figure 9.24).

$$PT_0 = (R + S)\tan\left(\frac{\Delta}{2}\right) + \frac{L}{2} \qquad \ldots\ldots (9.34)$$

6) Locate T_0 and T_3 by measuring a distance = PT_0 from P (or PI) on both tangents.

7) Choose the partial chords and calculate the deflection angles for both transition and circular curves as follows:

1. Transition curve:

Choose the partial chord length $C \le \frac{R}{40}$ so that chord length $C \cong$ arc length ℓ. Choose C_1 , C and C_2 (all $\le \frac{R}{40}$). C_1 is selected such that the station (chainage) of the first point on the transition curve becomes an even integer divisible by 5 or 10. This allows all intermediate points to have even stations. The partial chord C_2 is equal to $(L - (C_1 + nC))$, where n is the number of intermediate partial chords. The deflection angles are calculated in an accumulative way as follows:

From Equation (9.31) $\Rightarrow \delta_i = \frac{\phi_i}{3} = \frac{\ell_i^2}{3(2LR)} = \frac{\ell_i^2}{6LR}$

Where ℓ_i = length of the transition curve from T_0 up to point i.

$$\delta_1 = \frac{C_1^2}{6LR} \cdot \frac{180°}{\pi} \qquad \text{(in degrees)}$$

$$\delta_2 = \frac{(C_1 + C)^2}{6LR} \cdot \frac{180°}{\pi}$$

$$\delta_3 = \frac{(C_1 + 2C)^2}{6LR} \cdot \frac{180°}{\pi}$$

.

.

$$\delta_{T_1} = \frac{L^2}{6LR} \cdot \frac{180°}{\pi} = \frac{L}{6R} \cdot \frac{180°}{\pi} = \frac{\phi}{3} \quad \text{(last point on the left transition curve)}$$

2. Circular curve:

The deflection angle for the circular part $\Delta_c = (\Delta - 2\phi)$. Now given R and Δ_c, the preliminary work and calculations are done according to section (9.2.1.5).

B. Setting out the curves:

a. Left transition curve:

1. Set up the theodolite at T_0 and bring it to read zero while sighting the PI.
2. Rotate the theodolite in a clockwise direction an angle equal to δ_1. The rear chain-man holds the zero of the tape over T_0 and the leader holds the tape at a length equal to C_1 and begins moving left and right while extending the tape until seen by the theodolite man. At this point he drives an arrow into the ground marking the first point on the curve (point 1 in Figure 9.25).
3. Rotate the theodolite in a clockwise direction until the horizontal angle becomes δ_2. The rear man moves to point 1 holding the zero of the tape, while the leader moves left and right with the tape extended a distance equal to C, horizontally. When the leader is along the line of sight of the theodolite, he drives another arrow into the ground to mark point 2.
4. Continue on the above procedure until T_1 is reached.

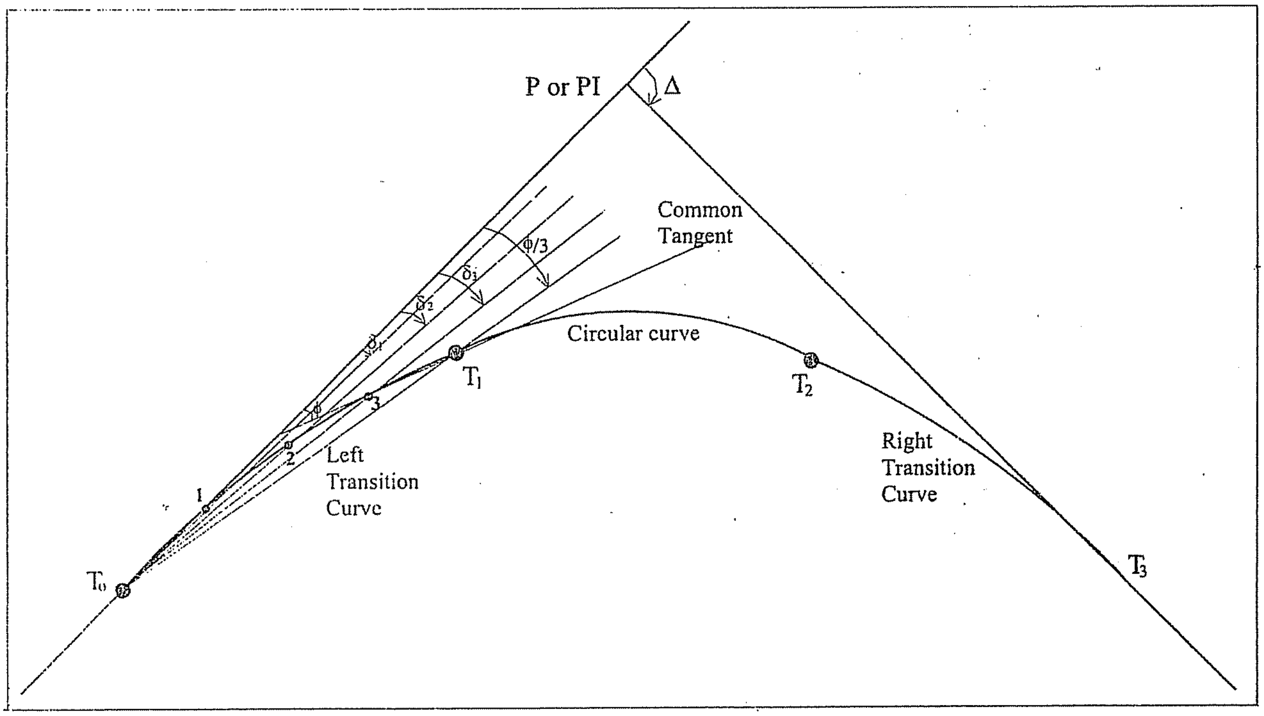

FIGURE 9.25: Layout of the left transition curve.

b. Common tangent between the transition and circular curves:

As shown in Figure 9.26, the angle between the line joining T_0 & T_1 and the common tangent is $\frac{2\phi}{3}$. To locate the direction of the common tangent, simply set up the theodolite at point T_1, direct the telescope to point T_0, and with the horizontal circle reading zero, invert the telescope to point in the direction T_1V. Rotate the theodolite in a clockwise direction an angle equal to $\frac{2\phi}{3}$. The resulting line of sight will be in the direction of the common tangent T_1 W.

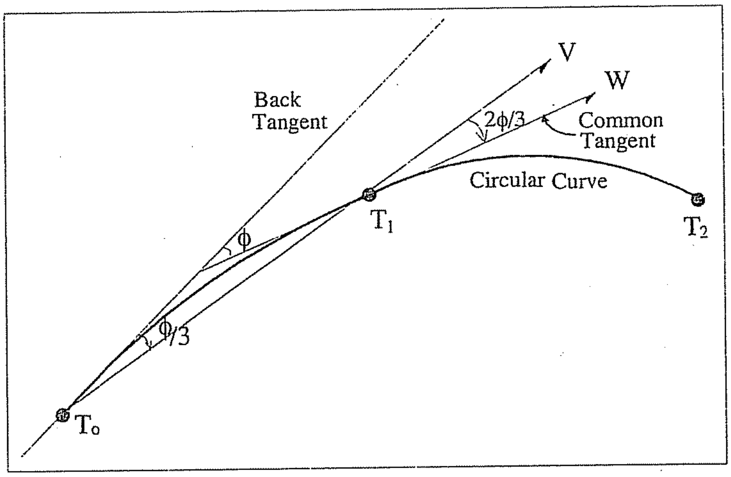

FIGURE 9.26: Common tangent between the transition and circular curves.

c. Circular curve:

- Deflection angle for the circular curve $(\Delta_c) = \Delta - 2\phi$
- Length of circular curve = $L_c = \frac{\pi R}{180°} \cdot \Delta_c = \frac{\pi R}{180°}(\Delta - 2\phi)$
- Partial chords C_1, C and $C_2 \leq \frac{R}{20}$
- Deflection angles (Equation 9.16 or 9.17)
- See section (9.2.1.5) for setting out circular curves. This curve is laid out with the theodolite stationed at point T_1 and first directed towards the direction of the common tangent as described above.

d. Right transition curve:

This curve is laid out the same way as the left transition curve. The only difference is that the theodolite is set up at T_3 and brought to read zero while sighting the PI. Deflection angles are measured in a counterclockwise direction. The next example will summarize the procedure.

EXAMPLE 9.4:

Two tangents of a highway deflect in direction by an angle (Δ) = 24° 10' 30" and intersect at P whose chainage is 5236.10 m. It is required to connect those lines by a circular curve of radius (R) = 400.00 m, which is in turn connected to two transition curves; one from each side. Given that the design speed (V) = 90 km/hr and the rate of change of radial acceleration (a) = 0.4 m/sec³, calculate and arrange all the data required to lay out the curves.

SOLUTION:

1) Length of transition curve L:

$$V = 90 \text{ km/hr} = \frac{90 \times 1000}{60 \times 60} = 25 \text{ m/sec}$$

$$L = \frac{V^3}{aR} = \frac{(25)^3}{0.4 \times 400} = 97.65 \text{ m}$$

2) Shift S:

$$S = \frac{L^2}{24R} = \frac{(97.65)^2}{24 \times 400} = 0.99 \text{ m}$$

3) Total tangent length PT_0 or PT_3 :

$$PT_0 = PT_3 = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}$$

$$= (400 + 0.99)\tan\frac{24° \ 10' \ 30''}{2} + \frac{97.65}{2} = 134.70 \text{ m}$$

4) Chainage of T_0 = Chainage of P - PT_0

= 5236.10 - 134.70 = 5101.40 m

Chainage of T_1 = Chainage of T_0 + L

= 5101.40 + 97.65 = 5199.05 m

5) Lengths of partial chords for the left transition curve:

$$C \le \frac{R}{40} = \frac{400.00}{40} = 10.00 \text{ m}, \Rightarrow \text{choose } C = 10.00 \text{ m}$$

Choose station of point 1 on the transition curve to be 5110.00 m,

$\Rightarrow C_1 = 5110.00 - 5101.40 = 8.60$ m

No. of intermediate partial arcs = 8

$\Rightarrow C_2 = L - (C_1 + 8C) = 97.65 - (8.60 + 8 \times 10) = 9.05$ m

6) Deflection angles:

$$\delta_i = \frac{\ell_i^2}{6LR} \cdot \frac{180°}{\pi}$$

All the data required to lay out the left transition curve are tabulated in Table 9.3.

TABLE 9.3: Layout data for the left transition curve.

Point No.	Chord (m)	Chainage (m)	Transition curve length (ℓ) (m)	Total deflection angle (δ) ° ' "
T_0	0.00	5101.40	0.00	0 00 00
1	8.60	5110.00	8.60	0 01 05
2	10.00	5120.00	18.60	0 05 05
3	10.00	5130.00	28.60	0 12 00
4	10.00	5140.00	38.60	0 21 51
5	10.00	5150.00	48.60	0 34 39
6	10.00	5160.00	58.60	0 50 22
7	10.00	5170.00	68.60	1 09 02
8	10.00	5180.00	78.60	1 30 37
9	10.00	5190.00	88.60	1 55 09
T_1	9.05	5199.05	97.65	2 19 53

Check:

$$\phi_{T_1} = \frac{L}{2R} \cdot \frac{180°}{\pi} = \frac{97.65}{2 \times 400} \cdot \frac{180°}{\pi} = 6° \ 59' \ 38.7''$$

$$\delta_{T_1} = \frac{\phi_{T_1}}{3} = 2° \ 19' \ 52.9''$$ which is the same value in Table 9.3.

7) Circular curve:

- Central angle of the circular curve (Δ_c):

$$\Delta_c = \Delta - 2\phi \quad = 24° \ 10' \ 30'' - 2(6° \ 59' \ 38.7'') = 10° \ 11' \ 12.6''$$

- Length of circular curve L_c:

$$L_c = \frac{\pi \cdot \Delta_c}{180°} \cdot R = \frac{\pi(10° \ 11' \ 12.6'')}{180°} \cdot 400.00 = 71.12 \text{ m}$$

- Partial chords:

$$C \leq \frac{R}{20} = \frac{400.00}{20} = 20.00 \text{ m}$$

Choose $C = 20.00$ m, and

Choose the station of point 1 on the circular curve to be 5210.00m

$\Rightarrow C_1 = 5210.00$ - chainage of $T_1 = 5210.00 - 5199.05 = 10.95$ m

No. of intermediate partial chords = 2

$\Rightarrow C_2 = L_c - (C_1 + 2C) = 71.12 - (10.95 + 2 \times 20.00) = 20.17$ m

- Partial deflection angles: $d = \dfrac{28.648 \times C}{R}$ (Equation 9.17)

$\Rightarrow d = 1° \ 25' \ 56.6''$, $d_1 = 0° \ 47' \ 2.2''$, $d_2 = 1° \ 26' \ 40.8''$

To lay out the circular curve, first locate the direction of the common tangent. In order to do so, set up the theodolite at T_1 and sight T_0. Invert the telescope of theodolite by an angle = 180°, and then rotate it in a clockwise direction an angle equal to $\frac{2}{3}\phi = \frac{2}{3}(6° \ 59' \ 38.7'') = 4° \ 39' \ 45.8''$. The rest of the data needed to lay out the circular curve are tabulated in Table 9.4.

TABLE 9.4: Layout data for the circular curve.

Point No.	Chord (m)	Chainage (m)	Deflection angle (δ) ° ' "	Total deflection angle (δ) ° ' "
T_1	0.00	5199.05	0 00 00.0	0 00 00
1	10.95	5210.00	0 47 02.2	0 47 02
2	20.00	5230.00	1 25 56.6	2 12 59
3	20.00	5250.00	1 25 56.6	3 38 55
T_2	20.17	5270.17	1 26 40.8	5 05 36

Check:

$\delta_{T_2} = \dfrac{\Delta_c}{2} = \dfrac{10°11'12.6''}{2} = 5°5'36''$, which is the same value in Table 9.4.

> **Note:**
> Chainage of T_2 can also be calculated as:
> Chainage of T_2 = Chainage of T_1 + L_c
> $= 5199.05 + 71.12 = 5270.17$ m

8) Right transition curve:

- Locate T_3 by measuring a distance $= PT_0 = PT_3$ from P on the second tangent.
- The length and shift are the same as for the left transition curve: i.e., $L = 97.65$ m and $S = 0.99$ m
- Chainage of T_3 = Chainage of $T_2 + L = 5270.17 + 97.65 = 5367.82$ m
- Partial chords: as calculated before for the left transition curve, take $C = 10.00$ m

  Since the layout of this curve starts from T_3 and continues in a counterclockwise direction, C_1 is the chord near T_3 and C_2 is the chord near T_2.

  Choose C_2 so that the point that comes directly after point T_2 has an even chainage.

  Chainage of $T_2 = 5270.17$ m

  Let chainage of the next point $= 5280.00$ m,

  $\Rightarrow C_2 = 5280 - 5270.17 = 9.83 \text{ m} < \dfrac{R}{40}$

  No. of intermediate chords = 8,

  $\Rightarrow C_1 = L - (C_2 + 8C) = 97.65 - (9.83 + 8 \times 10.00) = 7.82$ m
- Deflection angles:

$$\delta_i = \frac{\ell_i^2}{6RL} \cdot \frac{180°}{\pi}$$

The data required to lay out the right transition curve are tabulated in Table 9.5.

TABLE 9.5: Layout data for the right transition curve.

Point No.	Chord (m)	Chainage (m)	Trans. curve length (ℓ) (m)	Total Def. angle (δ) ° ' "	Theodolite Reading ° ' "
T_3	0.00	5367.82	0.00	0 00 00	360 00 00
1	7.83	5360.00	7.82	0 00 54	359 59 06
2	10.00	5350.00	17.82	0 04 40	359 55 20
3	10.00	5340.00	27.82	0 11 22	359 48 39
4	10.00	5330.00	37.82	0 20 59	359 39 01
5	10.00	5320.00	47.82	0 33 33	359 26 27
6	10.00	5310.00	57.82	0 49 03	359 10 57
7	10.00	5300.00	67.82	1 07 29	358 52 31
8	10.00	5290.00	77.82	1 28 51	358 31 09
9	10.00	5280.00	87.82	1 53 09	358 06 52
T_2	9.83	5270.17	97.65	2 19 53	357 40 07

Notes:

1) The values given in the fifth column of Table 9.5 are the angles between the second tangent T_3P and the line of sight.

2) The values in the last column of Table 9.5 are those ones, which are to be read on the horizontal circle of the theodolite because it is rotated in a counterclockwise direction. The theodolite is assumed to have a least count of 1".

3) As a check:

$$\delta_{T_2} = \frac{\phi_{T_2}}{3} = \frac{6° \ 59' \ 38.7''}{3} = 2° \ 19' \ 53''$$

9.2.2.5 TRANSITION CURVE LAYOUT USING ELECTRONIC TOTAL STATIONS

This method is more computational than the previous method, but requires less effort in the field since all points on the left and right transition curves as well as the circular curve in between are located from the T_0. In principle, this method is similar to that explained in section 9.2.1.6 and has the same advantages over other methods. Moreover, the location of T_3 does not need to be known in advance.

In order to apply this method, the length of the line joining the T_0 to any point on the transition-circular-transition combination, as well as the deflection angle between this line and the back tangent need to be known. The following sections will outline the procedure:

a. Left Transition Curve:

Assume that the coordinates of the T_0 are (0.00 , 0.00). The coordinates of any point *i* on the left transition curve (x_i , y_i) can be calculated from equations (9.26). The horizontal distance d_i between points T_0 and *i* is equal to $\sqrt{x_i^2 + y_i^2}$ (Figure 9.27). The deflection angle (δ_i) between the back tangent and the line T_0-*i* is calculated from equations (9.25) and (9.31).

b. Circular Curve:

The coordinates (x_{T_1} , y_{T_1}) of the tangent point T_1 are known from equations (9.26). The angle between the back tangent and the common tangent between the left transition curve and the circular curve is equal to ϕ (Figure 9.27). If the azimuth of the back tangent is assumed to be zero, then the azimuth of the common tangent will be ϕ. If for any point *j* on the circular curve, the deflection angle from the common tangent is equal to θ_j, then the azimuth of line T_1-j will be equal to $(\phi + \theta_j)$. The length of line $T_1 - j$ will be $(2R \sin \theta_j)$. The coordinates of point *j* are now calculated from the equations developed in Chapter 7. Mainly,

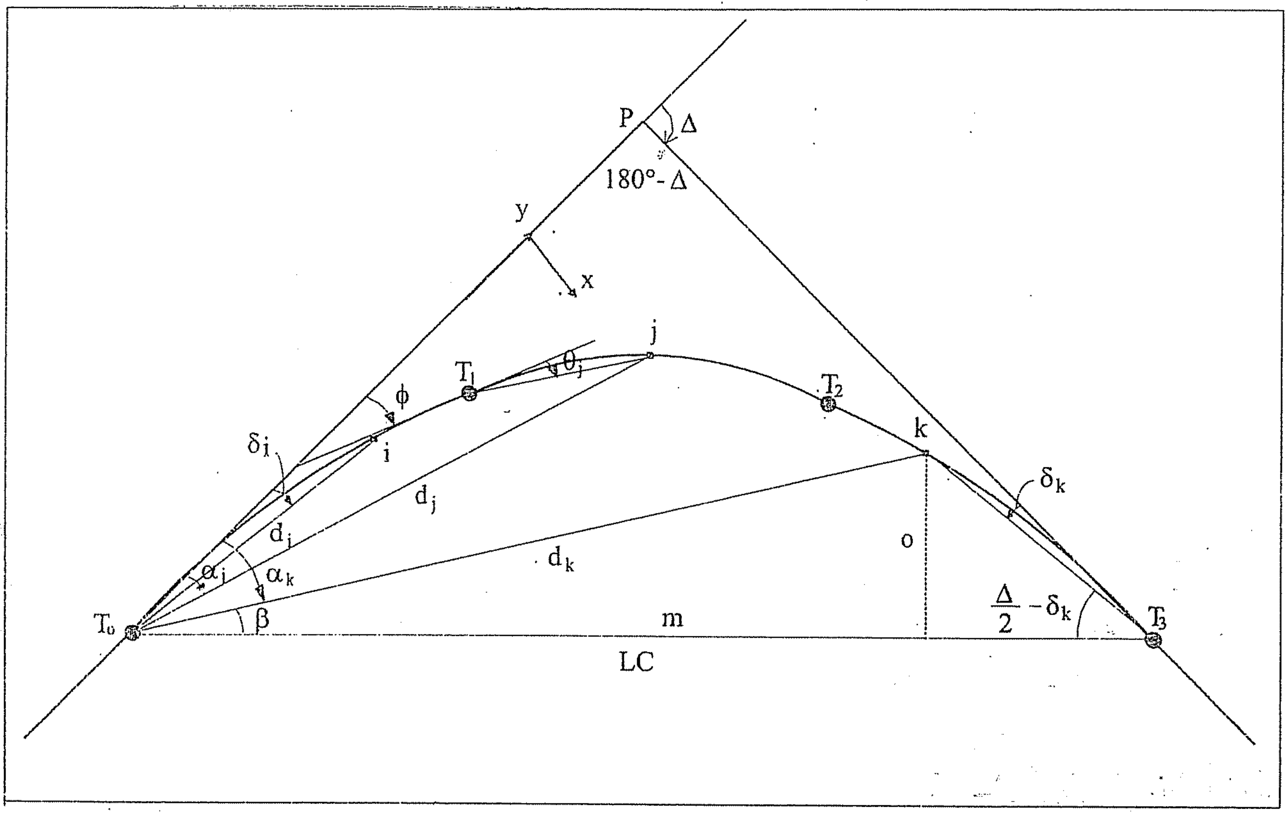

FIGURE 9.27: A circular curve connected to transition curves from both sides.

$$\begin{aligned} x_j &= x_{T_1} + d_{T_1\text{-}j}\sin(\phi+\theta_j) \\ y_j &= y_{T_1} + d_{T_1\text{-}j}\cos(\phi+\theta_j) \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.35)$$

The distance $T_0 - j$ is equal to $\sqrt{x_j^2 + y_j^2}$, and the deflection angle (α_j) between the back tangent and the line $T_0 - j$ is equal to $\tan^{-1}\left(\dfrac{x_j}{y_j}\right)$.

c. Right Transition Curve:

The deflection angle (α_k) and the length of the line (d_k) joining T_0 to point k are calculated as follows (see Figure 9.27):

1. Calculate the total tangent distance PT_0 using Equation (9.34).

2. Calculate the long chord (LC) using the Cosine law on triangle T_0PT_3 as follows:

$$LC = \sqrt{2\ (PT_0)^2\ -\ 2\ (PT_0)^2 \cos\ (180°\text{-}\Delta)}$$
$$= \sqrt{2(PT_0)^2.[1-\cos\ (180°-\Delta)]} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.36)$$

LC can also be calculated from the sine law on the same triangle.

3. Calculate the deflection angle δ_k of the line T_3k as explained in section 9.2.2.4.

4. Calculate the distance and offset of point k (x'_k , y'_k) from the forward tangent using Equations (9.26). The distance T_3k is equal to $\sqrt{x_k'^2 + y_k'^2}$.

5. Calculate the distance and offset of point *k* (m,o) (Figure 9.27) from the long chord starting at T_0 as follows:

$$m = LC - T_3k\ .\ \cos\left(\frac{\Delta}{2} - \delta_k\right)$$
$$o = T_3k\ .\ \sin\left(\frac{\Delta}{2} - \delta_k\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.37)$$

6. Calculate the coordinates of point k using Equations (7.6) developed in Chapter 7:

$$x_k = m.\sin\ (\Delta/2)\ -\ o.\cos\ (\Delta/2)$$
$$y_k = m.\cos\ (\Delta/2)\ +\ o.\sin\ (\Delta/2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.38)$$

7. Finally, calculate the distance and deflection angle (d_k and α_k) of line T_0k as follows:

$$d_k = \sqrt{x_k^2 + y_k^2}$$
$$\alpha_k = \tan^{-1}\left(\frac{x_k}{y_k}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.39)$$

Alternatively, d_k and α_k can also be calculated from the following equations:

$$d_k = \sqrt{m^2 + o^2}\ , \quad \text{and}\ \alpha_k = \frac{\Delta}{2} - \beta = \frac{\Delta}{2} - \tan^{-1}\frac{o}{m}$$

Tables 9.6, 9.7 and 9.8 show the data needed to locate the transition-circular-transition curve in Example 9.4 using an electronic total station.

TABLE 9.6: Layout data for the left transition curve.

Point No.	Partial Chord (m)	Chainage (m)	Trans. curve length (ℓ) (m)	Total Def. angle (δ) ° ' "	Chord dist. From T_0 to point
T_0	0.00	5101.40	0.00	0 00 00	0.00
1	8.60	5110.00	8.60	0 01 05	8.60
2	10.00	5120.00	18.60	0 05 05	18.60
3	10.00	5130.00	28.60	0 12 00	28.60
4	10.00	5140.00	38.60	0 21 51	38.60
5	10.00	5150.00	48.60	0 34 39	48.60
6	10.00	5160.00	58.60	0 50 22	58.60
7	10.00	5170.00	68.60	1 09 02	68.59
8	10.00	5180.00	78.60	1 30 37	78.58
9	10.00	5190.00	88.60	1 55 09	88.56
T_1	9.05	5199.05	97.65	2 19 53	97.59

TABLE 9.7: Layout data for the circular curve.

Point No.	Chord (m)	Chainage (m)	Partial Def. angle (δ) ° ' "	Def. angle from back tangent ° ' "	Dist. from T_0 to point (m)
T_1	0.00	5199.05	0 00 00.0	2 19 53	97.59
1	10.95	5210.00	0 47 02.2	2 52 47	108.49
2	20.00	5230.00	1 25 56.6	3 59 8	128.36
3	20.00	5250.00	1 25 56.6	5 10 43	148.15
T_2	20.17	5270.17	1 26 40.8	6 26 20	168.02

TABLE 9.8: Layout data for the right transition curve.

Point No.	Chord (m)	Chainage (m)	Trans. curve length (ℓ) (m)	Def. angle from back tangent ° ' "	Dist. from T_0 to point (m)
T_2	0.00	5270.17	97.65	6 26 20	168.02
1	9.83	5280.00	87.82	7 04 05	177.71
2	10.00	5290.00	77.82	7 42 25	187.50
3	10.00	5300.00	67.82	8 20 21	197.27
4	10.00	5310.00	57.82	8 57 31	207.03
5	10.00	5320.00	47.82	9 33 38	216.78
6	10.00	5330.00	37.82	10 08 27	226.53
7	10.00	5340.00	27.82	10 41 46	236.27
8	10.00	5350.00	17.82	11 13 25	246.02
9	10.00	5360.00	7.82	11 43 15	255.78
T_3	7.82	5367.82	0.00	12 05 15	263.43

9.3 VERTICAL CURVES

9.3.1 INTRODUCTION

Following the plotting of the ground profile along the selected route of a highway or railroad in a new location or along an existing line of transportation that is to be improved, a study is made of the various grade-lines. Those grade-lines are usually connected by what is called vertical curves, which are generally parabolic in shape and have tangents of equal length. Two broad cases of grade-lines occur: those meeting at summits (crests), and those meeting at sags, as shown in Figure 9.28.

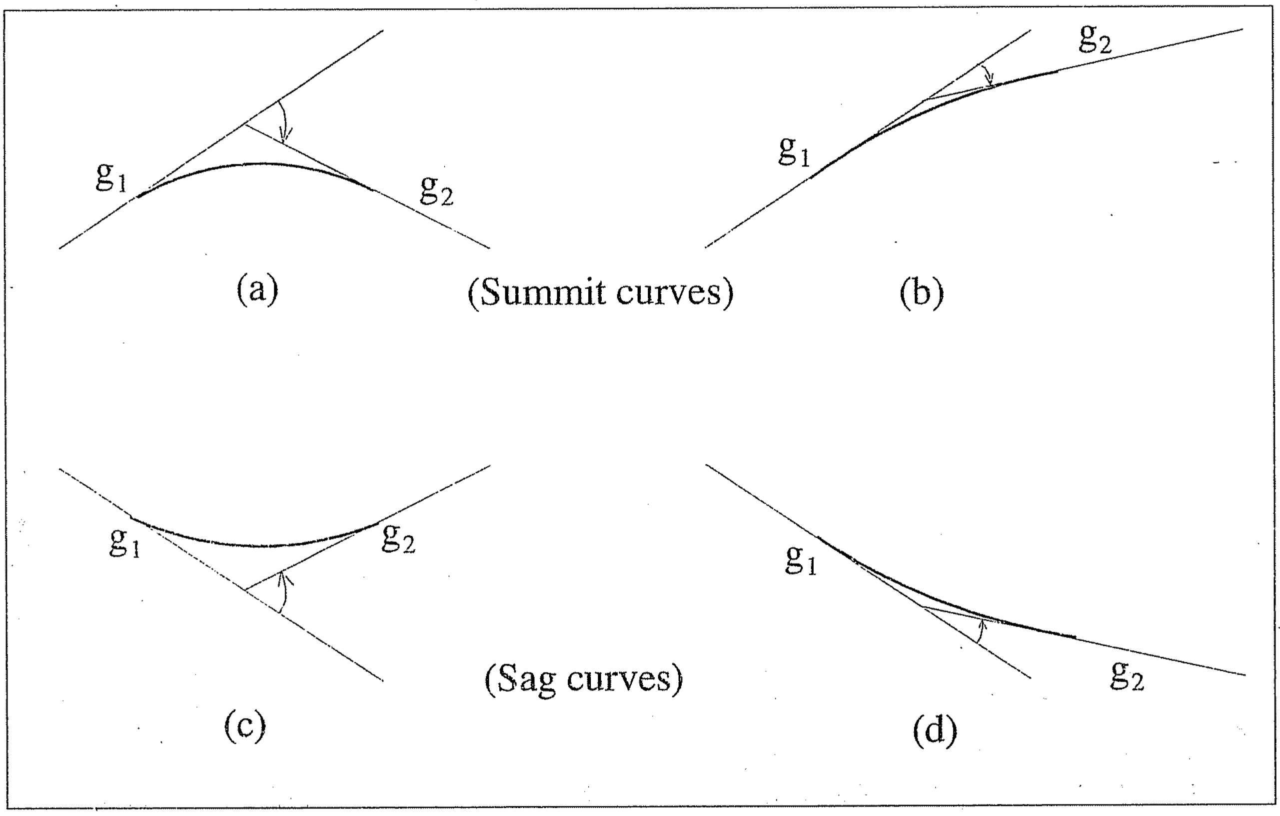

FIGURE 9.28: Types of vertical curves (summit and sag curves).

The gradients (g_1 & g_2) are conveniently expressed as percentages (e.g. 4%). A gradient is considered positive if it is rising upwards and negative if it is falling downwards.

In Figure 9.28 a: g_1 is positive & g_2 is negative
b: g_1 is positive & g_2 is positive
c: g_1 is negative & g_2 is positive
d: g_1 is negative & g_2 is negative

9.3.2 SHAPES OF VERTICAL CURVES

Where the ratio of the length of a curve to its radius is less than 0.10, there is no practical difference between the shapes of a circle, a parabola and an ellipse. Since this condition is usually encountered with vertical curves, the parabola will be used for its simplicity, easy riding and sight conditions.

The data that are usually needed for the calculation of the layout data of the vertical curves is:

1. The gradients, in percent, of the grade-lines
2. The elevation and chainage of the vertical point of intersection (VPI), and
3. A selected length of the vertical curve. This is the horizontal distance from VPC to VPT (vertical points of curvature and tangency).

9.3.3 DERIVATION OF THE VERTICAL CURVE EQUATIONS

The following assumptions are made in the derivation of vertical curve equations:

1. All distances x (Figure 9.29) are measured in a horizontal plane.
2. The vertical curve is symmetrical about the VPI, i.e. the VPI is in the center of the curve.
3. The vertical curve to be used is the parabola.

The general equation of a parabola is:

$$\frac{d^2y}{dx^2} = r, \qquad \text{where } r \text{ is a constant}$$

y: vertical distance from the x-axis to the curve

By integration:

$$\frac{dy}{dx} = rx + C_1$$

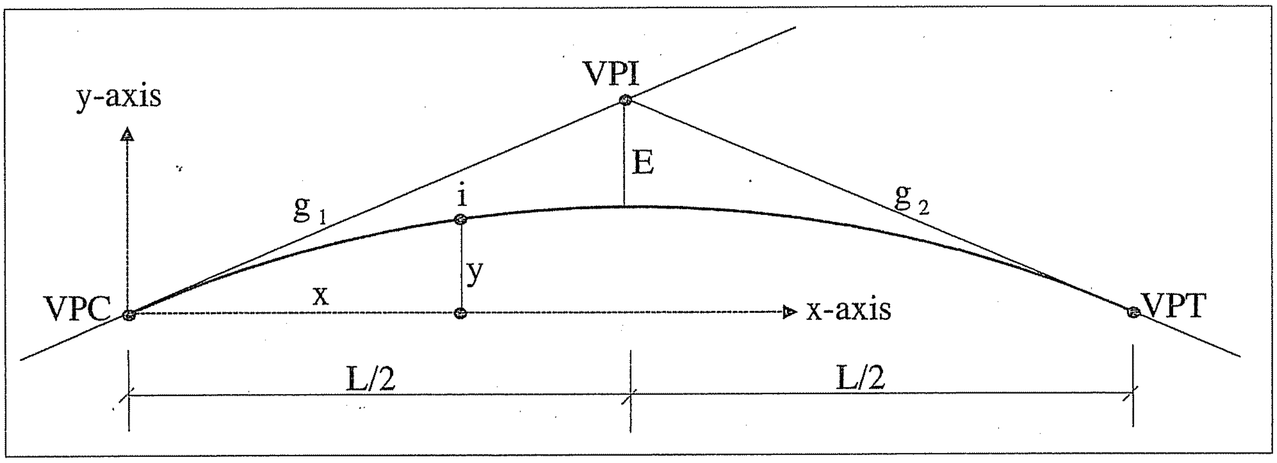

FIGURE 9.29: Vertical curve.

Substitute the following two conditions: $x = 0, \ \dfrac{dy}{dx} = g_1$, and

$x = L, \ \dfrac{dy}{dx} = g_2$

$$\Rightarrow \quad C_1 = g_1 \quad \text{and } r = \frac{g_2 - g_1}{L}$$

Let $A = g_2 - g_1$, $\Rightarrow r = A/L$, and

$$\frac{dy}{dx} = \frac{A}{L}x + g_1 \qquad \text{.......................(9.40)}$$

Integrating a second time:

$$\Rightarrow \quad y = \frac{A}{L}\cdot\frac{x^2}{2} + g_1 x + C_2$$

When $x = 0, y = 0 \Rightarrow C_2 = 0$

$$\Rightarrow \quad y = \frac{A}{2L}x^2 + g_1 x \qquad \text{.......................(9.41)}$$

To find the location of the maximum or minimum elevation on the vertical curve:

Solve Equation (9.40)

$$\frac{dy}{dx} = \frac{A}{L}x_{max/min} + g_1 = 0$$

$$\Rightarrow \quad x_{max/min} = \frac{-g_1 L}{A} \qquad \text{.......................(9.42)}$$

Where $x_{max/min}$ is the x-coordinate of the location where a maximum or minimum vertical curve height occurs.

9.3.4 CALCULATIONS AND DESIGN OF VERTICAL CURVES

The objective of the calculations is to determine the elevations of selected points on the vertical curve, which are usually 10 m or 20 m apart depending on the length of the curve. Referring to Figure 9.29, the elevation of a point i (H_i) on the curve is equal to the elevation of the vertical point of curvature (H_{VPC}) plus the vertical distance y (i.e.: $H_i = H_{VPC} + y$). Now, substituting the value of y from equation (9.41),

$$\Rightarrow \quad H_i = H_{VPC} + g_1 x + \frac{A}{2L} x^2 \quad \text{.............................(9.43)}$$

Where: H_{VPC} = elevation of VPC
$A = g_2 - g_1$
H_i = elevation of any point i on the vertical curve.

Referring to Figure 9.29, the elevation and chainage of the vertical point of curvature are calculated as follows:

$$\left.\begin{aligned} H_{VPC} &= H_{VPI} - g_1 \frac{L}{2} \\ Ch_{VPC} &= Ch_{VPI} - \frac{L}{2} \end{aligned}\right. \quad \text{.............................(9.44)}$$

Likewise, the elevation and chainage of the vertical point of tangency are calculated as follows:

$$\left.\begin{aligned} H_{VPT} &= H_{VPI} + g_2 \frac{L}{2} \\ Ch_{VPT} &= Ch_{VPI} + \frac{L}{2} \end{aligned}\right. \quad \text{.............................(9.45)}$$

EXAMPLE 9.5:

Design a vertical curve 200.00 m long connecting a rising gradient of 1 in 50 with a falling gradient of 1 in 75, which meet at a summit of RL = 30.35 m and chainage = 2752.00 m. Give offsets at 20.00 m intervals. Also calculate the elevation of the highest point on the curve.

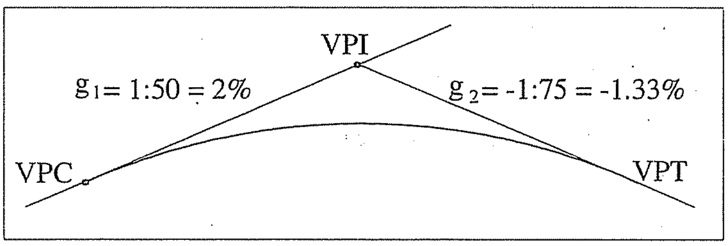

FIGURE 9.30: Vertical curve of Example 9.5.

SOLUTION:

$$g_1 = +\frac{1}{50} = +2.00\%$$
$$= +0.02$$
$$g_2 = -\frac{1}{75} = -1.33\%$$
$$= -0.0133$$
$$A = g_2 - g_1 = -0.0133 - 0.02 \quad = -0.0333$$

Chainage of VPC = Chainage of VPI - L/2
= 2752.00 - 200.00/2 = 2652.00 m

Elevation of VPC $= h_{VPC} = h_{VPI} - g_1 \cdot \frac{L}{2}$
$= 30.35 - 0.02 \times 100.00 = 28.35$ m

Equation (9.43) becomes:

$$H = 28.35 + 0.02\,x + \frac{(-0.0333)}{2 \times 200.00} \cdot x^2$$

Elevations at selected points 20 m apart are shown in Table 9.9.

TABLE 9.9: Vertical curve data of Example 9.5.

Point No.	Chainage (m)	Horizontal distance (x) (m)	Elevation (H)
VPC	2652.00	0.00	28.35
1	2660.00	8.00	28.50
2	2680.00	28.00	28.84
3	2700.00	48.00	29.12
4	2720.00	68.00	29.32
5	2740.00	88.00	29.46
6	2760.00	108.00	29.54
7	2780.00	128.00	29.54
8	2800.00	148.00	29.49
9	2820.00	168.00	29.36
10	2840.00	188.00	29.17
VPT	2852.00	200.00	29.02

Maximum elevation occurs at: $x = \frac{-g_1 L}{A}$

$$\Rightarrow \quad x_{max} = \frac{-\,0.02 \times 200.00}{-\,0.0333} = 120.00 \text{ m}$$

$$H_{max} = 28.35 + 0.02\,(120.00) + \frac{(-\,0.0333)}{2 \times 200.00} \cdot (120.00)^2 = 29.55 \text{ m}$$

EXAMPLE 9.6:

Design a 200.00 m long vertical sag curve, which connects a falling gradient of 1 in 50 with a rising gradient of 1 in 75. The R.L. of the intersection point of the gradients is 30.35 m and the chainage is 2750.00 m. Give offsets at 20.00 m intervals. Calculate also the elevation of the lowest point on the curve.

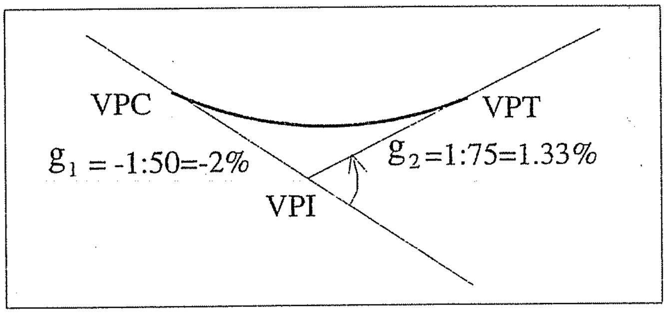

FIGURE 9.31: Vertical curve of Example 9.6.

SOLUTION:

$$g_1 = -\frac{1}{50} = -2.00\%$$
$$= -0.02$$
$$g_2 = +\frac{1}{75} = +1.33\%$$
$$= +0.0133$$
$$A = g_2 - g_1 = 0.0133 - (-0.02) = +0.0333$$

Chainage of VPC = Chainage of VPI - L/2
= 2750.00 - 200.00/2 = 2650.00 m

Elevation of VPC = $h_{VPC} = h_{VPI} - g_1 \cdot \frac{L}{2}$
$= 30.35 - (-0.02) \times 100 = 32.35$ m

Equation (9.43) becomes:

$$H = 32.35 - 0.02\,x + \frac{(0.0333)}{2 \times 200.00} x^2$$

Elevations at selected points 20 m apart are shown in Table 9.10.

Minimum elevation occurs at $x = \frac{-g_1 L}{A}$

$$\Rightarrow \quad x_{min} = \frac{-(-0.02) \times 200.00}{0.0333} = 120.00 \text{ m}$$

$$H_{min} = 32.35 - 0.02\,(120.00) + \frac{(0.0333)}{2 \times 200.00} \cdot (120.00)^2 = 31.15 \text{ m}$$

Elevations at selected points 20 m apart are shown in Table 9.10.

TABLE 9.10: Vertical curve data of Example 9.6.

Point No.	Chainage (m)	Horizontal distance (x) (m)	Elevation (H)
VPC	2650.00	0.00	32.35
1	2660.00	10.00	32.16
2	2680.00	30.00	31.82
3	2700.00	50.00	31.54
4	2720.00	70.00	31.36
5	2740.00	90.00	31.22
6	2760.00	110.00	31.16
7	2780.00	130.00	31.16
8	2800.00	150.00	31.22
9	2820.00	170.00	31.35
10	2840.00	190.00	31.56
VPT	2850.00	200.00	31.68

9.3.5 LAYOUT OF VERTICAL CURVES

The purpose here is to modify the ground elevations by the cut and fill operations until the design elevation of the vertical curve points is reached. In order to illustrate this problem, let us refer to Figure 9.32. Assume that the ground elevations of points 1 & 2 are 29.14m and 28.44m, and the design elevations are 28.50m and 29.32m respectively. Assume also that a level was setup at a nearby location, a staff reading was taken at a close BM, and the instrument height was calculated and found to be 31.70m. Now, if a staff reading is taken at point 1 (before cutting), the reading will be 31.70 – 29.14 = 2.56m. The depth of cut needed here is 29.14 – 28.50 = 0.64m. As a result, the cutting process at this location starts and will be ceased only when the staff reading becomes 2.56 + 0.64 = 3.20m. However, at point 2, fill is needed instead. If a staff reading is taken on the ground before filling, the reading will be 3.26m (31.70 – 28.44). The depth of needed fill here is 29.32 – 28.44 = 0.88 m. The fill process starts and will be ceased when the staff reading becomes 3.26 – 0.88 = 2.38 m. This process is repeated for the whole project points given that both the ground and design levels are known.

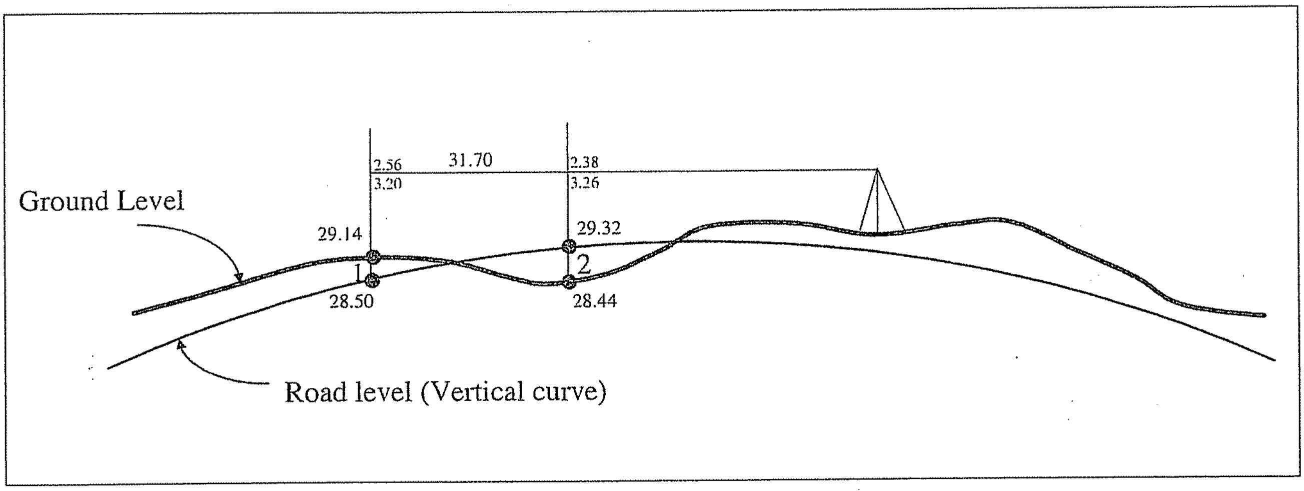

FIGURE 9.32: Layout of vertical curves.

9.3.6 LENGTH OF VERTICAL CURVES

The main factor affecting the length of a vertical curve, especially on crests, is the visibility condition, i.e. sight distance. For sag curves, centrifugal effect is the ruling factor. The other factor affecting the vertical curve length is the gradelines represented by $A = g_2 - g_1$. These two factors are discussed below.

1) Sight Distance:

Let two points on the vertical curve, one at height h_e and the other at height h_o from the surface of the road, be inter-visible, and let the horizontal distance between them be S (Figure 9.33). The sight line from the driver's eye to the object is taken to pass tangentially through point D on the curve. Thus, the sight distance represents the length of road over which an observer, whose eye level is h_e above the road surface, first sees an object whose height is h_o on the other side of the crest.

A value of 1.05 m is usually taken as the eye level height (i.e. h_e) above the road surface for an observer sitting in a motorcar. The sight distance (S) is calculated from the following expression:

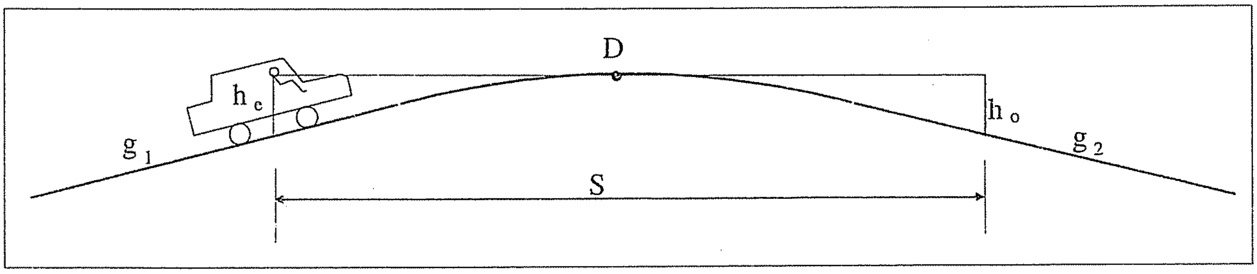

FIGURE 9.33: Sight distance on vertical curves.

$$S = V.t + \frac{V^2}{2g(f + i)} \qquad (9.46)$$

Where S = sight distance in meters (m)
V = velocity (m/s)
t = perception and reaction time ≅ 2.5 seconds
g = gravity = 9.81 m/sec²
f = coefficient of friction of the road surface
i = gradient= positive upwards
= negative downwards

Concerning the object to be seen by the observer on the other side of the road, the sight distance can be divided into two types:

1) Stopping sight distance. This is the braking distance that the driver needs to stop the car in order to avoid a crash with the viewed object. The height of the driver's eye (h_e) is assumed to be 1.05 m and the object height (h_o) is 0.15 m above the road surface. Stopping sight distance is calculated using Equation (9.46).

2) Passing sight distance. It is the clear distance that the driver must view in order to be able to safely pass the car in front of him. In this case, the object on the other side of the road is considered to be a possible oncoming car and the driver must complete the pass safely to avoid a crash with the oncoming car from the other direction. For this reason, the passing sight distance is taken to be *twice* the stopping sight distance with $h_e = h_o = 1.05$ m.

2) Gradelines:

The effect of the gradelines on the vertical curve length can best be understood be inspecting Figure 9.34. When $g_1 = g_2$ (meaning $A = g_2 - g_1 = 0$), no vertical curve is needed (Figure 9.34a). When the second gradeline starts deviating from the first one yielding a small A, a short vertical curve is needed (Figure 9.34b). When |A| becomes large, a longer vertical curve is required to satisfy sight conditions for summit curves and reduce the centrifugal effect for sag curves (Figure 9.34c).

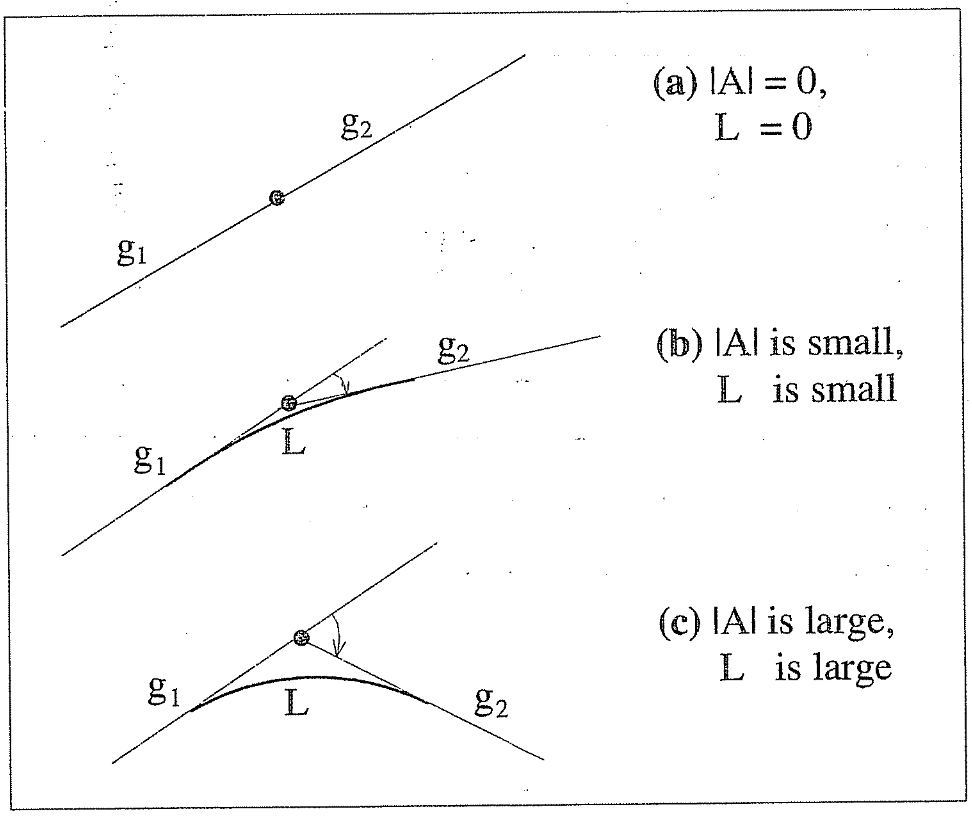

FIGURE 9.34: Relationship between the gradients and vertical curve length.

Taking the previous two factors into consideration, different countries have developed their own specifications for calculating the vertical curve length. As an example, the American Association of State Highway and Transportation Officials (AASHTO) uses the following equations for vertical curve length calculations:

1) For summit (crest) curves:

a. $S < L \Rightarrow L = \dfrac{|A| \cdot S^2}{2\left[\sqrt{h_e} + \sqrt{h_o}\right]^2}$ .... (9.47)

b. $S > L \Rightarrow L = 2S - \dfrac{2}{|A|}\left[\sqrt{h_e} + \sqrt{h_o}\right]^2$ .... (9.48)

Where: $A = g_2 - g_1$

$h_e = 1.05$ m

$h_o = 0.15$ m for stopping sight distance

$= 1.05$ m for passing sight distance

2) For sag (valley) curves:

a. $S < L \Rightarrow L = \dfrac{|A| \cdot S^2}{(1.22 + 0.035S)}$ .... (9.49)

b. $S > L \Rightarrow L = 2S - \dfrac{(1.22 + 0.035S)}{|A|}$ .... (9.50)

EXAMPLE 9.7:

For the grades given in Example 9.5, what length of vertical curve is required if two points 1.05 m above the road level and 200.00 m apart are to be inter-visible?

SOLUTION:

$$g_1 = +\frac{1}{50} = +2\% = +0.02$$

$$\Rightarrow A = -0.0333$$

$$g_2 = -\frac{1}{75} = -1.33\% = -0.0133$$

$S = 200.00$ m

The curve is summit (crest).

a. Try $S < L \Rightarrow$ use Equation (9.47)

$$L = \frac{|A| \cdot S^2}{2\left[\sqrt{h_e} + \sqrt{h_o}\right]^2} = \frac{0.0333\,(200.00)^2}{2\left(\sqrt{1.05} + \sqrt{1.05}\right)^2}$$

$$= 158.73 \text{ m}$$

$S = 200.00$ m is not less than $L = 158.73$ m, $\Rightarrow$ this assumption is wrong.

b. Try $S > L \Rightarrow$ use Equation (9.48)

$$L = 2S - \frac{2}{|A|}\left[\sqrt{h_e} + \sqrt{h_o}\right]^2$$

$$= 2 \text{ x } 200.00 - \frac{2}{0.0333}\left[\sqrt{1.05} + \sqrt{1.05}\right]^2$$

$$= 148.00 \text{ m} \Rightarrow \text{O.K.}$$

$$\Rightarrow L = 148.00 \text{ m}$$

Usually, tables are prepared which give the sight distances (both stopping and passing) for different values of speed, road width, coefficient of friction and longitudinal gradient.

PROBLEMS

9.1 For a simple circular curve, the following is known:
Angle of intersection = 60° 20' 43" .
External distance (E) = 40 m.
Calculate: the radius (R), the degree of curve (D_a), the long chord (LC), curve length (L), tangent distance (T) and the mid-ordinate (M).

9.2 a. The chainage of the point of curvature (PC) and the point of tangency (PT) are 320.34 m and 339.75 m respectively. Find the angle of intersection if the degree of the curve is 2 degrees.

b. If the angle of intersection in part (a) above is increased by 5 degrees by rotating the forward tangent clockwise around the point of intersection (PI), find the new chainage for PC and PT if the same radius as in part (a) will be used.

9.3 AC and BC are two tangents for a railroad curve. The chainage of point C (PI) is equal 174 chords (each chord = 30 m). The radius of the curve is equal to 600 m and the angle of intersection is equal to 44° 16'. Calculate:

a. The length of the long chord (LC).
b. The middle ordinate (M).
c. The chainage of the midpoint of the curve.
d. The total deflection angle required to locate the third point on the curve.
e. The chainage needed for locating the fourth point on the curve.

9.4 In Figure 9.35, given R=50.00 m and distance AE=25.00 m, Calculate the hatched area.

a. Try $S < L \Rightarrow$ use Equation (9.47)

$$L = \frac{|A| \cdot S^2}{2\left[\sqrt{h_e} + \sqrt{h_o}\right]^2} = \frac{0.0333\,(200.00)^2}{2\left(\sqrt{1.05} + \sqrt{1.05}\right)^2}$$

$= 158.73$ m

$S = 200.00$ m is not less than $L = 158.73$ m, $\Rightarrow$ this assumption is wrong.

b. Try $S > L \Rightarrow$ use Equation (9.48)

$$L = 2S - \frac{2}{|A|}\left[\sqrt{h_e} + \sqrt{h_o}\right]^2$$

$$= 2 \times 200.00 - \frac{2}{0.0333}\left[\sqrt{1.05} + \sqrt{1.05}\right]^2$$

$= 148.00$ m $\Rightarrow$ O.K.

$\Rightarrow L = 148.00$ m

Usually, tables are prepared which give the sight distances (both stopping and passing) for different values of speed, road width, coefficient of friction and longitudinal gradient.

PROBLEMS

9.1 For a simple circular curve, the following is known:
Angle of intersection = 60° 20' 43".
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Calculate: the radius (R), the degree of curve (D_a), the long chord (LC), curve length (L), tangent distance (T) and the mid-ordinate (M).

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9.4 In Figure 9.35, given R=50.00 m and distance AE=25.00 m, Calculate the hatched area.

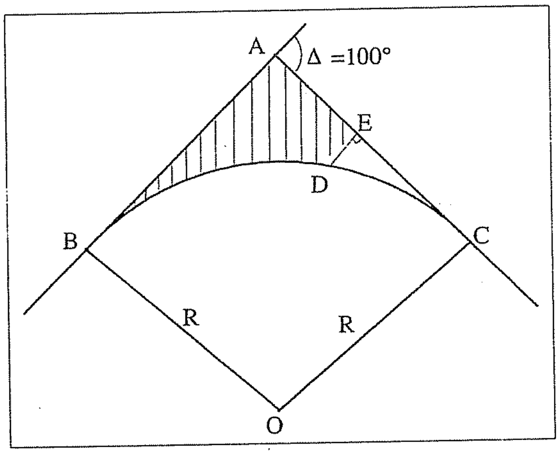


FIGURE 9.35

9.5 It is required to set out a simple circular curve which will be tangent to three lines, two of which, XY and YZ intersect at Y, and the third runs from A on XY to B on YZ, such that:
Angle BYA = 104° 36’
Angle BAX = 148° 54’
Angle ZBA = 135° 42’
The chainage at A is 12776 m and at Y is 14296 m. Calculate:
a. The chainage of PC on the line XY, and
b. The chainage of the point P where the curve touches the line AB.

9.6 In making a survey for a new road, the intersection point (I) of the two tangents, ABI and ICD, was found to be inaccessible. Four points A, B, C and D were selected, such that the distance BC was 118.926 m, the angle CBA was 169° 47’ and the angle DCB was 149° 24'. Prepare a table of deflection angles and chainages for setting out a 200 m radius curve using a theodolite. The chainage of point B is 840.950 m.

9.7 A right-hand circular curve is to connect two tangents AI and IB, the azimuths of which are 70° 42' and 130° 54' respectively. The curve is to pass through a point X such that IX is 441 m and the angle AIX is 34° 36'. If the chainage of the intersection point is 5251 m, determine the deflection angles and chord lengths required to set out the first three pegs on the curve at thorough chainages of 10 meters:
a. If a theodolite is to be used, and
b. If an electronic total station is to be used.

9.8 Tabulate all the data needed to lay out the simple circular curve shown in Figure 9.36 using a theodolite. Use a partial arc length = 20m (i.e., the distance between two consecutive points on the curve is 20 m). Calculate the data needed if this curve is to be laid out using an electronic total station.

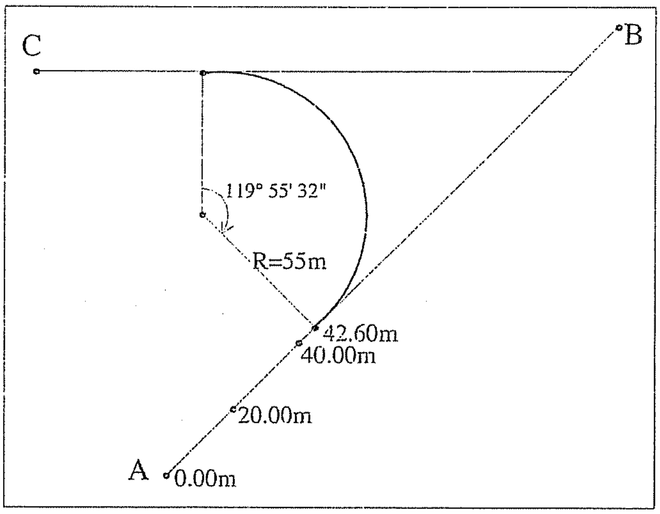


FIGURE 9.36

9.9 To locate the exact position of the point of tangency (PT) of an existing 250 m radius circular curve in a built-up area, points *a* and *d* were selected on the tangents close to the estimated positions of the two tangent points PC and PT respectively and a traverse *abcd* was run between them (see next table).

Station	*Length (m)*	*Deflection Angle*
a		9° 54' R
	89.00	
b		19° 36' R
	115.50	
c		30° 12' R
	101.50	
d		5° 18' R

The angles at *a* and *d* were relative to the tangents. Find the distance between PT and point *d*.

9.10 It is required to join two tangents having a deflection angle of 18° 36' 00" to the right by a circular curve of 457.2 m radius with a cubic spiral transition curve at each end. The design speed is 72 km/h and the rate of change of radial acceleration along the transition curve should not exceed 30.48 cm/s^3. Chainage of the intersection point is 781 m. Determine:

a. Length of transition curve.
b. Tangent length.
c. Chainages of tangent points and junction points, and
d. Tangent deflection angles to locate points on the transition and circular curves at 701.04 m and 760.00 m chainages.

9.11 Calculate all the data needed to lay out the transition-circular-transition curve combination in problem 9.10 if a total station is to be used.

9.12 It is required to replace a circular curve of degree equal 5.50° by three curves (i.e. Transition-Circular-Transition). The degree of the new circular curve is 5.55°.

a. Find the length of the transition curve.
b. The allowable vehicle speed, if the rate of change of radial acceleration = 0.4 m/s^3.
c. The deflection angle (Δ), if the length of the tangent for the first curve is equal to 893.0 m.
d. The central angle of the circular curve in the second case.
e. The tangent length for the second case.
f. Find the necessary super - elevation (e).

9.13 The limiting speed around a circular curve of 600m radius calls for a super-elevation of 1/25 across the 10-m wide road. Adopting the recommendation of a rate of 1 in 200 for the application of super-elevation along the transition curve leading from the tangent to the circular curve, calculate the deflection angles required to lay out the left transition curve using both a theodolite and an electronic total station.

9.14 Three straight lines AB, BC and CD have whole circle bearings (azimuths) of 30°, 90° and 45° respectively. AB is to be connected to CD by a continuous reverse curve formed of two circular curves of equal radius together with four transition curves. BC, which has a length of 800m, is to be the common tangent to the two inner transition curves. Determine the radius of the circular curves if the maximum speed is to be restricted to 80 km/h and the rate of change of radial acceleration is 0.3 m/s^3.

9.15 A curve connecting two straight lines which deflect through an angle of 12° is transitional throughout (circular curve length = 0.0 m). If the junction of the two transition curves is 5.00 m from the intersection point of the straight lines, determine the radius of curvature of the curve and the length of each tangent.

9.16 An uphill gradient of 1 in 100 meets a downhill gradient of 0.44 in 100 at a point where the chainage is 6100.00m and the reduced level is 126.00 m. If the rate of change of gradient is to be 0.18% per 30 m, prepare a table for setting out a connecting vertical curve at intervals of 30 m.

9.17 Design a vertical curve connecting two gradients of -2%, 7%. Its length = 400 m and R.L. of VPI = 16 m below mean sea level. Calculate the R.L. at each 100 m interval. Also find the R.L. and location of the lowest point.

If the gradients are not changed, and the level of VPI is as previous, knowing that the chainage of VPI is 1600.0 m and the curve is passing through point k where R.L. = 13.065 m below mean sea level and its chainage = 1580.0 m, find the new curve length.

9.18 Given:

- $g_1 = 2\%$, $g_2 = -3\%$
- Length of Vertical Curve (L) = 180.0 m
- The chainage and elevation of point A which lies on the first gradient are: 2140 m and 860.63 m respectively.
- The chainage and elevation of point B which lies on the second gradient are: 2260 m and 860.05 m respectively.

Calculate:

a. The chainage and elevation of VPI, VPC and VPT

b. The chainage and elevation of points required to set out the curve at 20.0m intervals, and

c. The chainage and elevation of the highest point on the vertical curve.

9.19 Two lines AB and BC intersect at point B whose chainage and elevation are 1200.0 m and 640.0 m respectively. The gradient of AB = -2%, and the gradient of BC = 0%. Line BC, in turn, intersects a third line CD whose gradient = 1%. The chainage of point C is 1500.0 m. It is required to make a table of all the information needed to set out a 600.0 m length vertical curve which is tangential to the three lines.

9.20 A sag vertical curve PQ is continuous with a summit vertical curve QR, the relevant tangents being PA, AQB, and BR respectively, with PA and BR falling to the right. The gradients of PA, AQB and BR are 1 in 40, 1 in 50, and 1 in 30 respectively, and curve PQ has a length of 200 m. If the difference in level between the lowest point on curve PQ and the highest point on curve QR is 2.09 m, determine the length of curve QR, and thence the reduced level of the curve point midway between P and R. The reduced level of tangent point P is 80.00 m above datum.

NAT

10 HORIZONTAL CONTROL FRAMEWORKS

10.1 INTRODUCTION

A horizontal control framework is a network of evenly distributed and well monumented survey points on the surface of the earth for which the horizontal positions (y & x) are known. The surveying process required to establish the horizontal positions of these points is known as a horizontal control survey. Such points, after their positions have been determined, are referred to as horizontal control points.

The horizontal control network forms the basis for all the surveying and mapping activities in a country. It facilitates the integration of land information from different sources. Assume, for example, that the city of Nablus has prepared a zoning map for the different areas of the city and wants to know in which zone every land parcel is located. If both the zoning map and the land parcels map (called cadastral map) are referenced to the same horizontal control network, then it will be easy to get the answer by superimposing the two maps on top of each other using common control points (Figure 10.1). This can be easily done in a computer environment using a geographic information system (GIS), or simply a computer-aided design/drafting program such as AutoCAD.

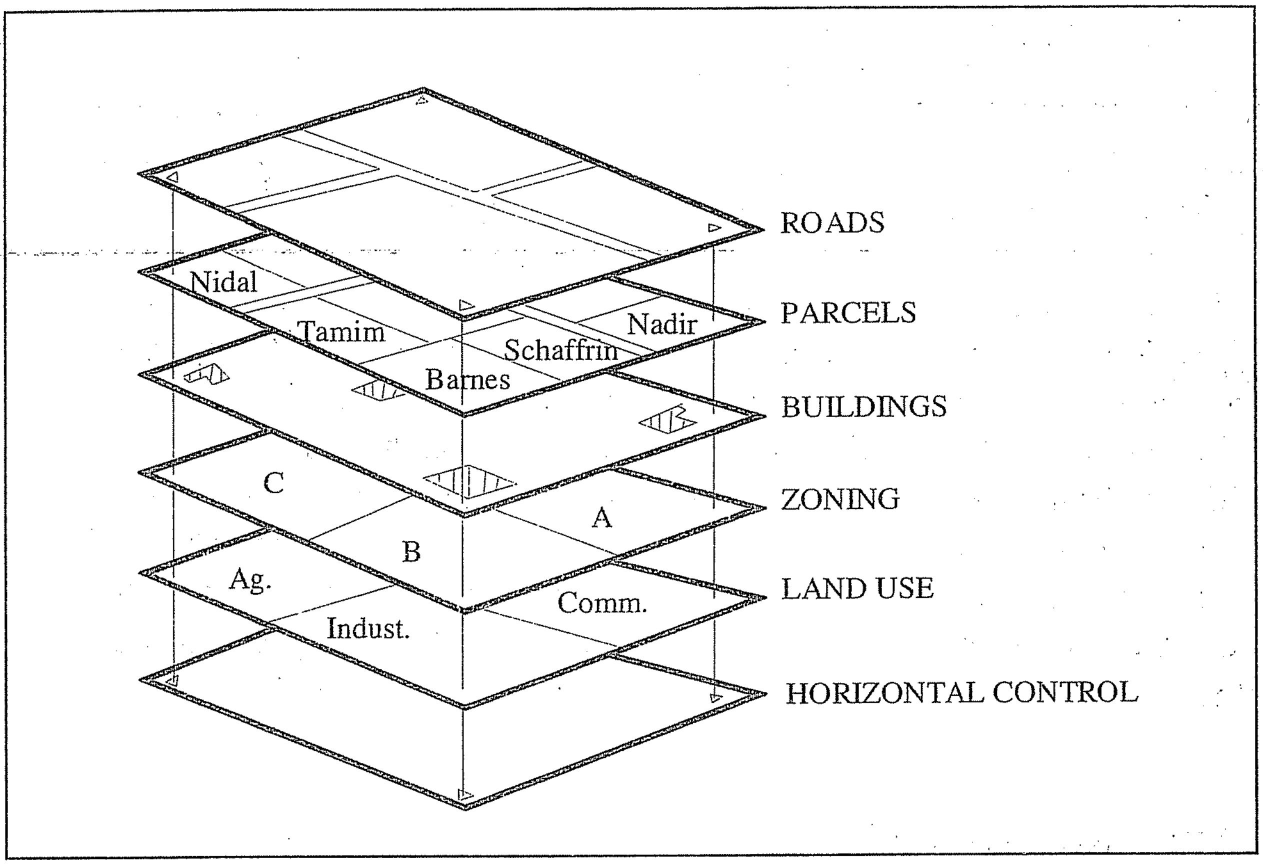


FIGURE 10.1: Integration of land information using the horizontal control network as a basis.

One of the first steps involved in an engineering project is usually the establishment of a horizontal control survey. For example, in a dam construction project, a control survey is conducted to establish a number of control points within the project area. These points are then used together with aerial photographs to prepare topographic maps of the area, from which the watershed or catchment area can be calculated. After choosing the location of the dam, and preparing the final design drawings, the same control points are then used to locate the dam and other engineering facilities on the ground. Thus, these control points provide the important link between the physical surface of the earth and the engineering designs. Even after the completion of the project, these control points are often used for long-term monitoring of the performance of the engineering facilities, such as the settlement of the dam in this case.

A horizontal control survey usually involves the measurement of angles and distances. For a small project area, the control survey may simply involve one or more interconnecting traverses (Chapter 7). For larger areas, a horizontal control network consisting of triangles and quadrilaterals is usually required to achieve the required positioning accuracy.

Before the development of EDM instruments, it was extremely difficult to measure long distances with a high degree of accuracy. Therefore, most horizontal control networks established in the past relied on the measurement of angles and a method called triangulation. With the advent and wide availability of EDM instruments, most horizontal control surveys now involve both angle and distance measurements. The current evolution in the global positioning systems technology using signals received from orbiting satellites has made it even easier, faster and more accurate to build horizontal control networks.

10.2 TRIANGULATION

Triangulation is a method of surveying which is used to determine the horizontal positions of points on the surface of the earth. This method is based on the trigonometric proposition that if one side and the three angles of a triangle are known, the remaining sides can be computed by the law of sines. Moreover, if the direction of one side is known, then the directions of the remaining sides can be determined.

Figure 10.2, shows a triangulation network consisting of six survey points: A, B, C, D, E & F. The length of line AB is measured with high precision. Since this line forms the base and provides the scale for the network, it is called the baseline. All the horizontal angles subtended at each survey point in the network are accurately measured. These angles, together with the baseline measurement, are then used to compute the horizontal positions of the survey points by trigonometry or least square adjustment as well be indicated later in this chapter.

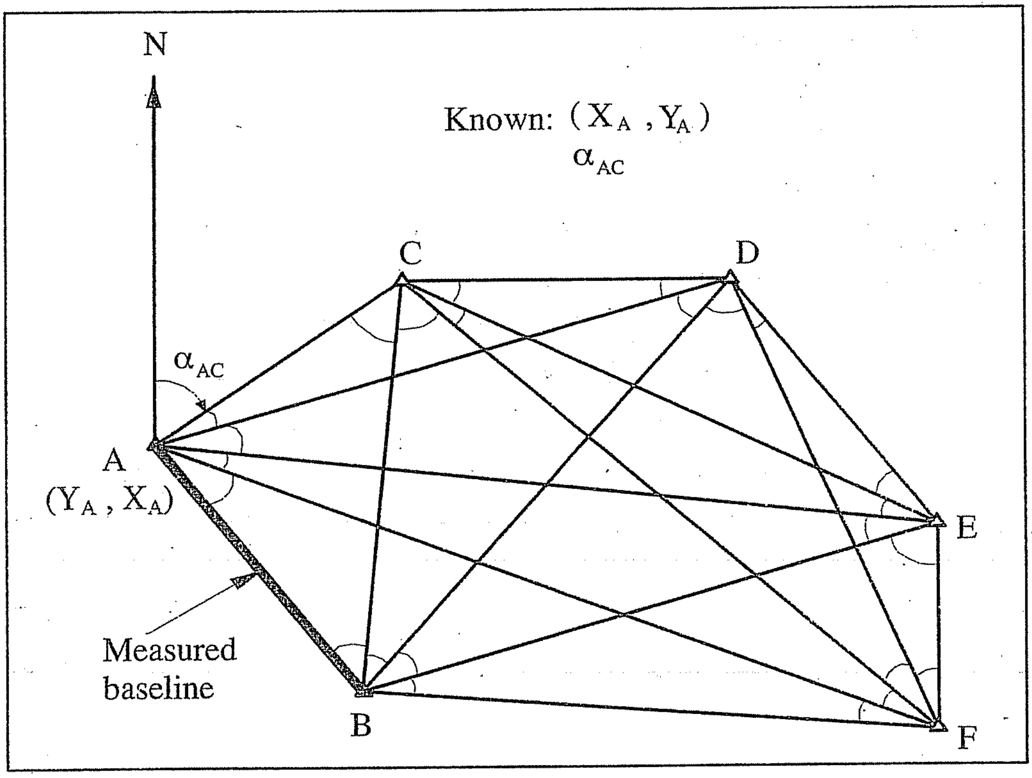


FIGURE 10.2: Triangulation network

Before the advent of long-range EDM instruments, baselines were usually measured using invar tapes. Such baselines were usually kept very short and had to be located in relatively flat terrain to achieve high accuracy. In large triangulation networks, usually more than one baseline was required.

To define the survey datum for the triangulation network, the position of at least one point and the direction of at least one line must be either known or arbitrarily defined as shown in Figure 10.2. The knowledge of the coordinates of two inter-connected survey points should also be sufficient to define the survey datum instead. This will make it possible to calculate the coordinates of all other points and the azimuth of all other lines in the network. The triangulation method was used in the late 1920's and early 1930's (at the time of the British Mandate on Palestine) to establish the Palestinian control network.

10.3 TRILATERATION

The advent and availability of electronic distance measuring equipment (EDM) made it more convenient to measure distances instead of angles. It is possible to determine the positions of new control points by measuring distances only. For example, instead of measuring the horizontal angles in the network in Figure 10.2, the lengths of all the lines can be measured. However, these distances should be corrected for earth curvature and atmospheric refraction as explained in chapter 6. Angles can be calculated later by the cosine law. A control network in which only distances are measured is called a trilateration network, and the survey process is called trilateration.

10.4 BASIC FIGURES USED IN TRIANGULATION AND/OR TRILATERATION

Depending on the area for which control is to be established, one of the following shapes may be used:

(1) Chain of triangles (Figure 10.3): This is a simple, fast and relatively less costly method of surveying to cover a narrow strip of terrain, like a river valley or a highway path. It is not as accurate as other methods, and baselines must be introduced frequently if the accumulation of errors is not to become excessive. It is important in this system that no small angles (less than 30°) be permitted as will be explained later.

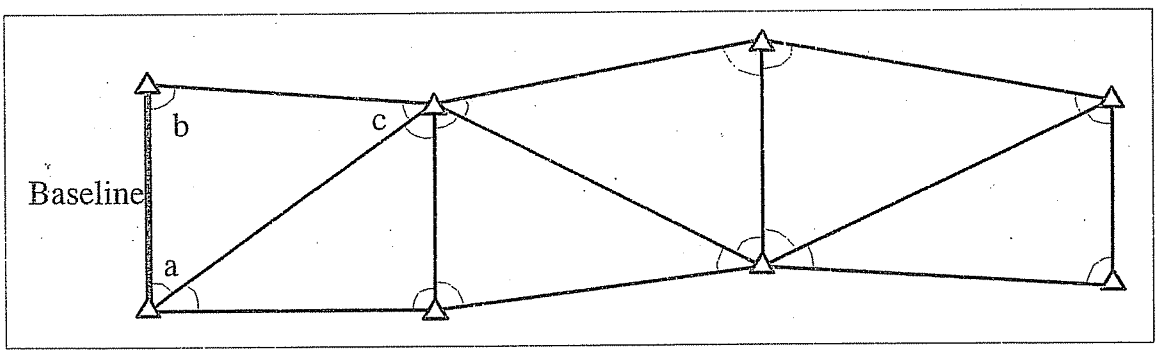


FIGURE 10.3: Chain of triangles.

(2) Chain of quadrilaterals (Figure 10.4): This system is preferred to the chain of triangles since the computed lengths of the sides can be carried out by different combinations of sides and angles (routes), and thus providing checks on the calculations. For example, side BC can be calculated using triangles DAB and ABC or triangles ADC and DCB. The individual triangles here overlap each other, but the intersection point of the two diagonals in the 4-sided quadrilateral is not known.

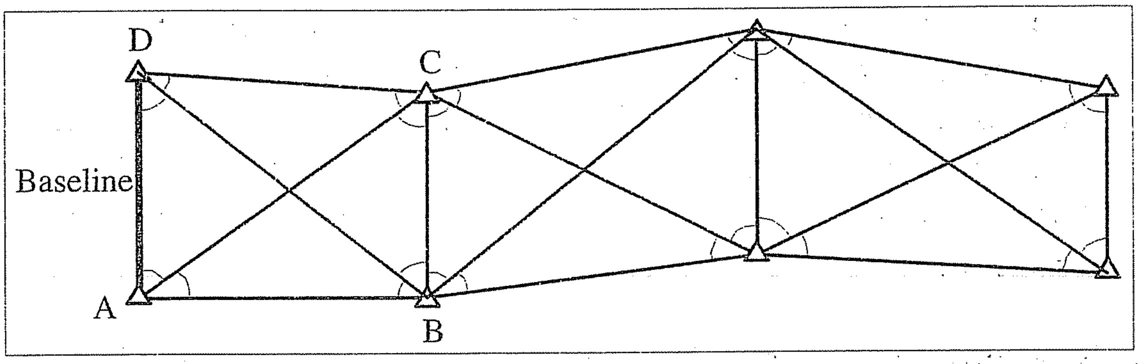


FIGURE 10.4: Chain of quadrilaterals.

(3) Central Point Figures (Figure 10.5): This system is usually used in the case where a wide area, such as a city, a county or even a country, is to be covered with a relatively dense distribution of control points.

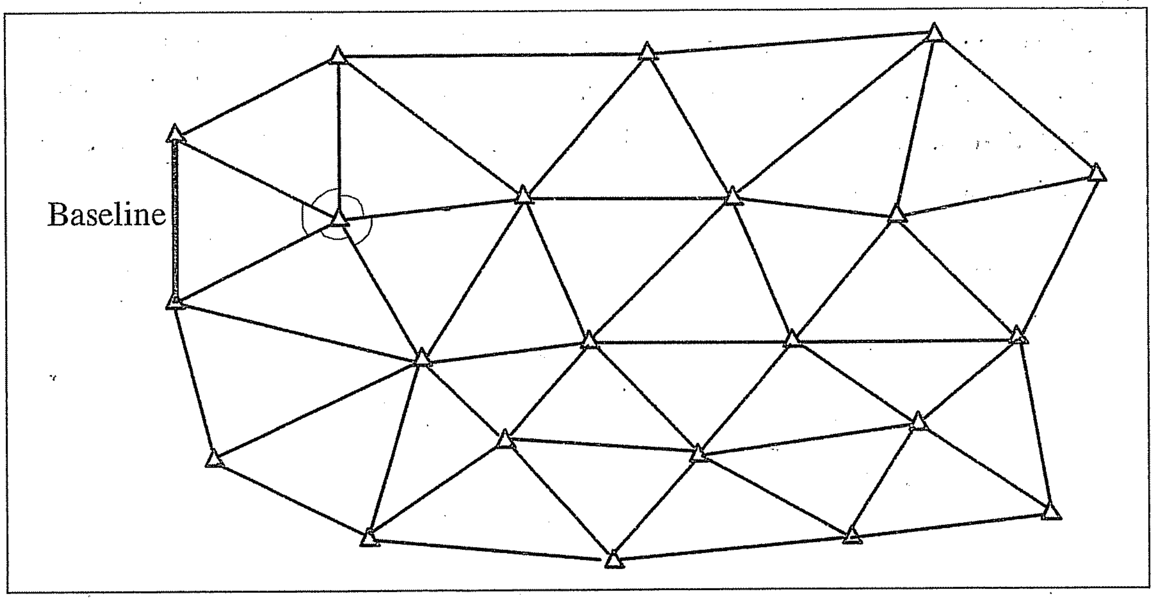


FIGURE 10.5: Central point figures.

10.5 NETWORK DESIGN AND PLANNING

During the design and planning of a horizontal control survey, the following points must be considered:

1. The required positioning accuracy of the new survey stations.
2. The accuracy, distribution and number of existing control points, if any.
3. The coordinate system to be used.
4. The location where the new survey points are to be placed.
5. The angles and distances to be measured.
6. The instruments to be used for the field measurements, as well as their accuracy requirements.
7. The number of repetitions to be made for each angle and distance in the control network.

The proper design of control survey networks requires a good understanding of the practical limitations of field conditions and surveying personnel.

In selecting the triangulation survey stations, certain considerations should be kept in mind that may be summarized as follows:

(a) Every station should be visible from the adjacent stations. To overcome the problem of weak visibility, uneven terrain and the effect of earth curvature, special towers of reasonable height that are built over the survey stations are usually used (Figure 10.6).

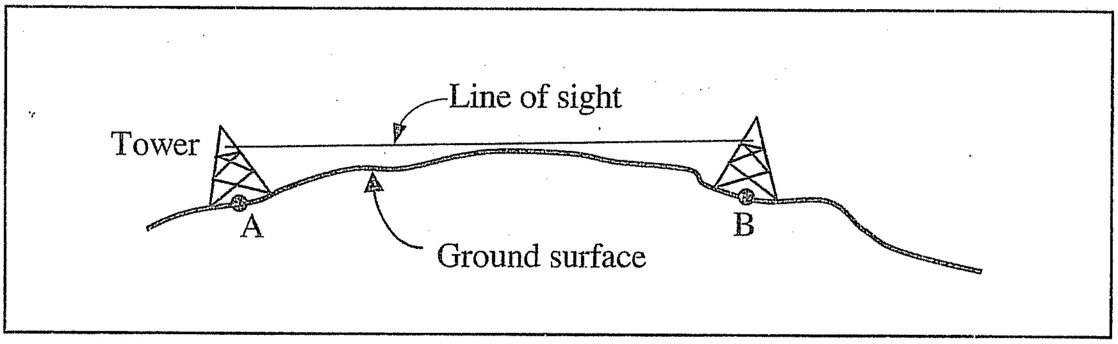


FIGURE 10.6: Towers to overcome visibility problems.

(b) The triangles formed by the survey stations should be well-conditioned, that is, as nearly equilateral as possible. No angles should be less than 30°, if at all possible. The effect of the shape of a triangle upon the accuracy with which the length of a side can be calculated is termed the ***strength of figure***. In any system of triangulation, the lengths of the triangle sides are computed by the law of sines. Since for a given uncertainty in the angle, the sine of a small angle changes more rapidly than that of a large one, it is evident that the percentage of error in the computed side of a triangle will be larger if the side is opposite to a small angle than if it is opposite to a larger angle. It is to be assumed that the accuracy with which an angle is measured is independent of its size.

Mathematically, let us consider the triangle in Figure 10.7a. Considering side b to be a base line, then from the Sine law, side $c = b.\dfrac{\sin\theta}{\sin\beta}$.

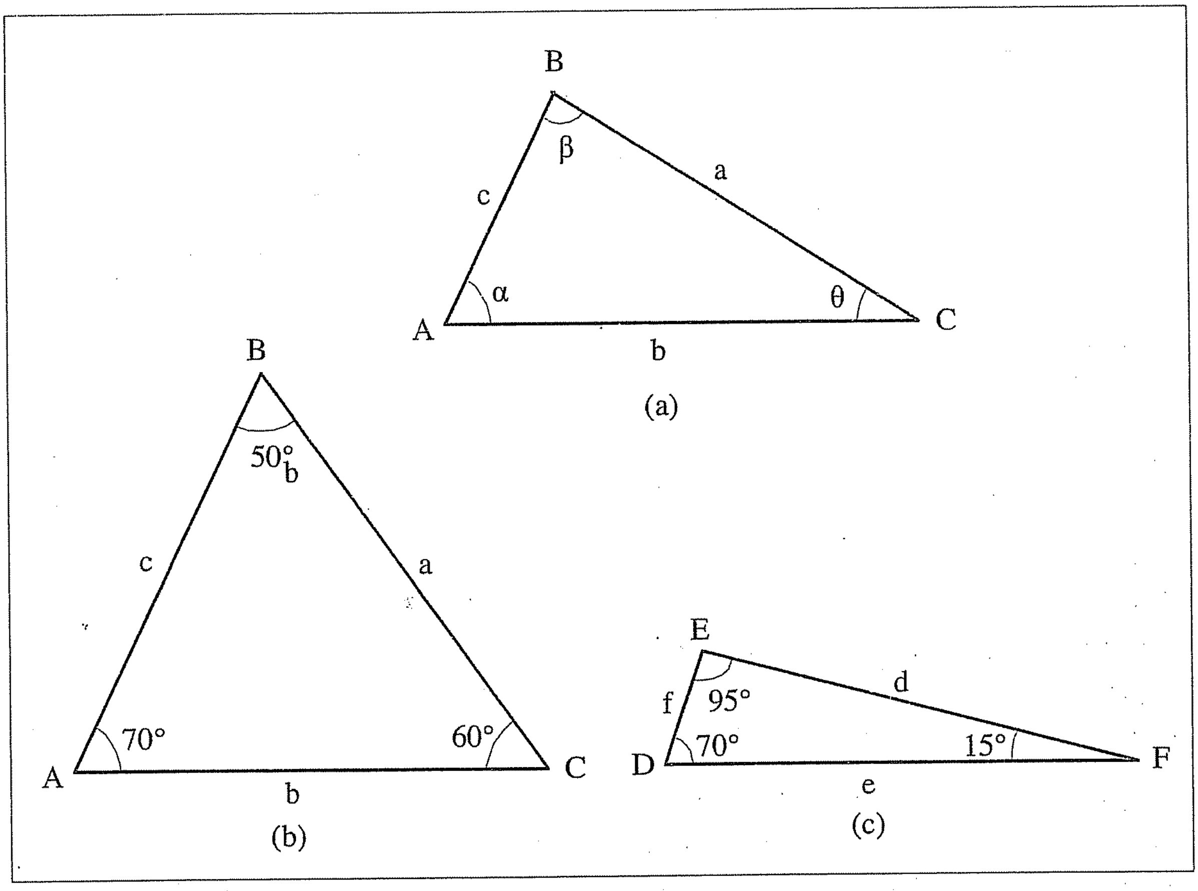


FIGURE 10.7: Strength of figure.

Now, to study how the accuracy of side c is affected by the size of angle θ opposite to it, we will consider β and b to be free of error (their $\sigma \approx 0$). Then by the law of propagation of random errors (Chapter 2):

$$\sigma_c^2 = \left(\frac{\partial c}{\partial \theta}\right)^2 \sigma_\theta^2 = b^2.\left(\frac{\cos\theta}{\sin\beta}\right)^2 \sigma_\theta^2, \Rightarrow \sigma_c = b.\left(\frac{\cos\theta}{\sin\beta}\right)\sigma_\theta$$

$$\text{The relative accuracy of side c} = \frac{1}{c/\sigma_c} = \frac{1}{\dfrac{b.\sin\theta/\sin\beta}{b.\cos\theta\,.\,\sigma_\theta/\sin\beta}} = \frac{1}{\tan\theta/\sigma_\theta}$$

As an example, let us assume that:

$$\sigma_\theta = 20'' = \frac{20}{3600}.\frac{\pi}{180} = 9.696\text{x}10^{-5}\,\text{radians}, \text{ then:}$$

- For θ = 60°, the relative accuracy of the opposite side a (Figure 10.7b) is ≈ 1/18,000.

- For θ = 15°, the relative accuracy of the opposite side f (Figure 10.7c) is ≈ 1/3,000.

This shows that the smaller the angle opposite to a line, the larger is the percentage of error in the computed length of this line. In general, keeping this angle larger than 30° or less than 120° will reduce the effect of the angle on the accuracy of the computed length opposite to this angle.

10.6 MONUMENTATION

A horizontal control point must be located on a geologically stable ground and constructed in a way so that its anticipated movement will stay within the accuracy with which it is surveyed. It must also be protected from damage throughout the expected duration of the engineering project.

Figure 10.8 shows a basic design for a concrete horizontal control monument. It consists mainly of 60 cm x 60 cm x 15 cm thick base buried below the frost line and a 20 cm diameter column. A brass survey marker is embedded in the concrete at the top of the column. An indentation mark on the brass marker indicates the exact location of the survey point. To minimize the possibility of damage by surface vehicles, the top surface of the column is positioned about 10 cm below the ground surface and protected by a 9-inch diameter outer pipe and an access cover. A witness post is sometimes installed within about 75 cm from the concrete column to make it easy to be located by the surveyors. Regardless of whether a witness post is installed or not, a good sketch should always be drawn to illustrate the exact location of the survey monuments.

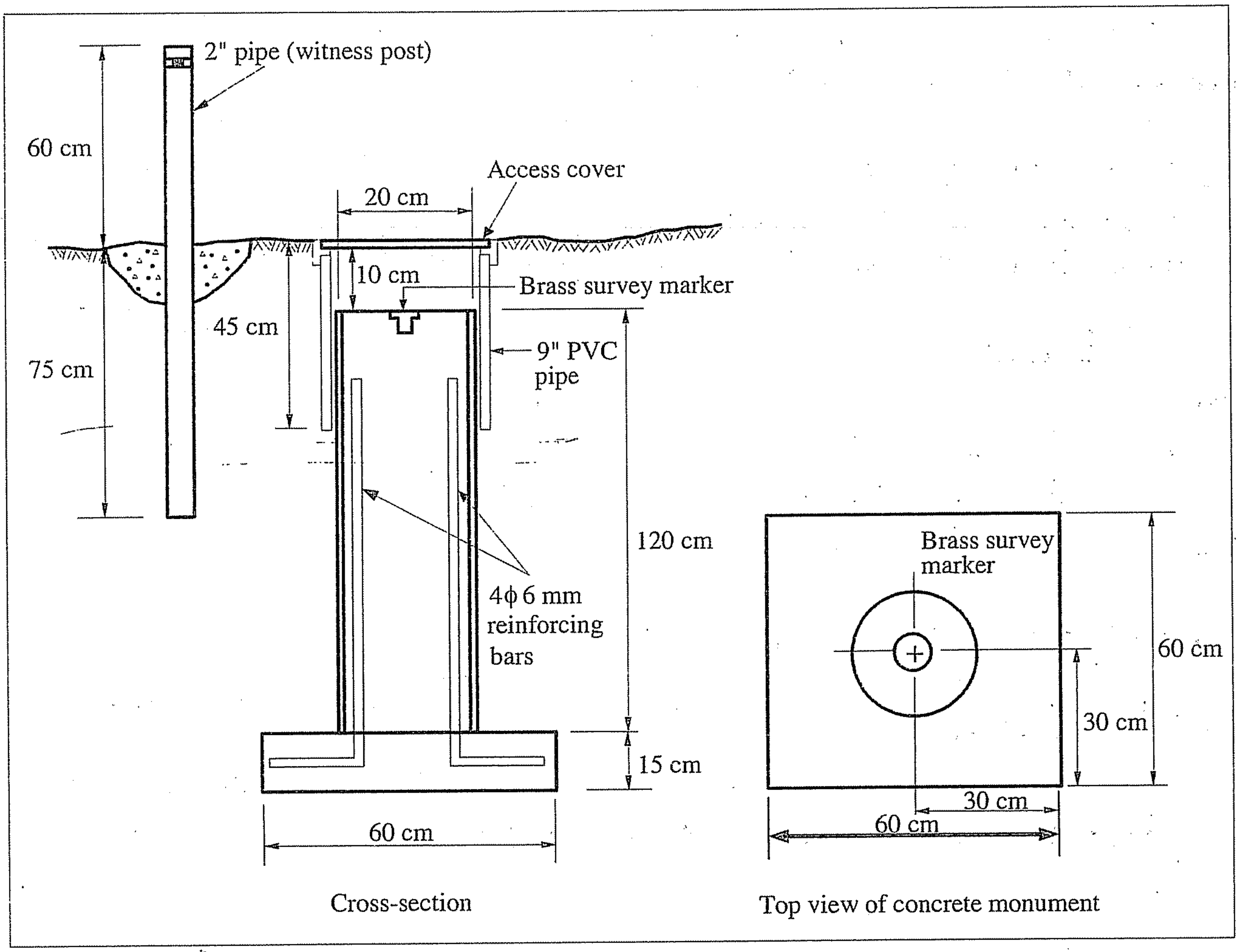


FIGURE 10.8: Concrete horizontal control monument.

10.7 PRELIMINARY CHECKS OF FIELD MEASUREMENTS

In control surveys, several repetitions are usually made of each angle and distance that is to be measured. The number of repetitions needed depends on the precision of the surveying instruments and the accuracy requirements of the project. To minimize the effects of atmospheric conditions on the survey operations, measurements are conducted on two or more different days and different times of the day. The mean and standard deviation are then computed in the field for each group of repeated measurements to assure that the expected precision is achieved.

In order to avoid making expensive return trips back to the project area to repeat some measurements if blunders or large errors are found, it is preferable to perform preliminary checks on the field measurements while the measuring program is in progress. The accuracy of the field measurements can be checked using the following two geometric conditions:

(1) The sum of the three angles in a spherical triangle must be equal to 180° + ε, where ε is called the spherical excess and is due to the curvature of the earth. It can be calculated from the following expression:

$$\varepsilon = \frac{A}{R^2 \sin 1''} \qquad (10.1)$$

Where A = area of triangle
R = mean radius of the earth ≅ 6372200 m
ε = spherical excess in seconds of arc

For a given triangle having an area of 200 km²,

$\varepsilon = \dfrac{200}{(6372.2)^2 . \sin 1''} \cong 1''$. Thus for most control surveys associated with engineering projects where the distance between control points does not exceed 20 km, the spherical excess is small and can be safely ignored.

(2) Given any two angles in a triangle and the length of one side, the lengths of the two remaining sides can be computed by the sine law.

For example, consider the quadrilateral shown in Figure 10.9. The averaged values of the measured angles and distances are:

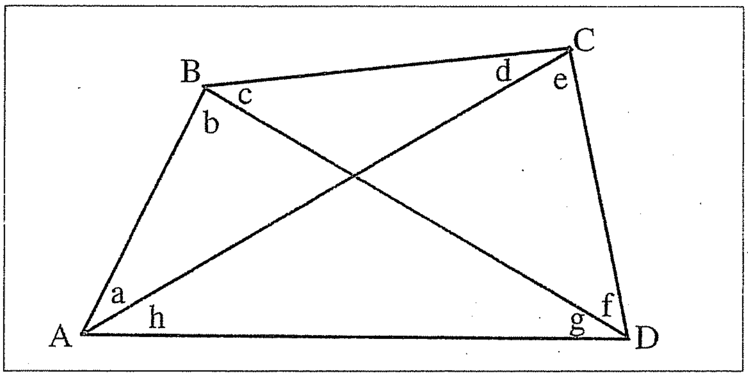


FIGURE 10.9: A quadrilateral.

Angles	Distances (m)
a = 29° 25' 34"	AD = 984.684
b = 58 41 20	AB = 890.540
c = 69 36 05	DB = 1087.883
d = 22 17 02	AC = 1843.258
e = 30 29 25	BC = 1153.857
f = 57 37 33	DC = 1280.632
g = 50 35 44	
h = 41 17 21	

The following closure checks can be performed on the angles:
(a + b + c + d) - 180° = +0° 00' 01"
(e + f + g + h) - 180° = +0° 00' 03"
(a + b + g + h) - 180° = -0° 00' 01"
(c + d + e + f) - 180° = +0° 00' 05"

If all of the angles are measured with the same degree of accuracy and each has a standard error of σ, then the sum of the four angles in each triangle will have a standard error of $\sqrt{4} \cdot \sigma = 2\sigma$ (law of propagation of random errors). The maximum error in the sum is 3(2σ) = 6σ. Thus the maximum closure error in any triangle in a quadrilateral should not exceed 6σ. In this example if σ = ± 1", then the maximum allowed closure error in the triangle is ± 6".

To check the measured distances in the quadrilateral, one of the distances is assumed to be correct and the lengths of the remaining sides are computed by the sine law. Assuming that the measured value of distance AD is correct, the length of side AB would be:

$$AB = \frac{AD \cdot \sin g}{\sin b} = 890.552 \text{ m}$$

The difference (ΔAB) between the measured and computed values of AB = 890.540 - 890.552 = -0.012 m. For the remaining sides in the quadrilateral:

$$\Delta DB = 1087.883 - \frac{AD \cdot \sin(a+h)}{\sin b} = +0.011\text{m}$$

$$\Delta AC = 1843.258 - \frac{AD \cdot \sin(g+f)}{\sin e} = -0.104\text{m}$$

$$\Delta BC = 1153.857 - \frac{890.552 \cdot \sin a}{\sin d} = +0.024\text{m}$$

$$\Delta DC = 1280.632 - \frac{AD \cdot \sin h}{\sin e} = +0.058\text{m}$$

The differences between the measured and computed distances are caused by errors in both the measured angles and distances. Generally, these differences are considered acceptable if they do not exceed two times the expected maximum errors in the distance measurements (i.e. $2x3\sigma_d = 6\sigma_d$). For example, if the distances were all measured with a relative precision of 1/100,000 at 1σ, then the maximum expected error (3σ) in the distances AB, DB, AC, BC and DC are ± 0.03, 0.03, 0.06, 0.03 and 0.04 m respectively. The acceptable differences between the measured and computed values of these distances should then be less than ± 0.06, 0.06, 0.12, 0.06 and 0.08 m respectively.

10.8 ORDERS OF ACCURACY

Accuracies required for horizontal control depend on the type of survey and the ultimate use of the control points. In the United States, for example, the following orders of accuracy were established by the American Federal Geodetic Control Committee:

1) First-order
2) Second Order, Class I
3) Second Order, Class II
4) Third Order, Class I
5) Third Order, Class II

The following Table gives some details on these orders of accuracy and their uses.

TABLE 10.1: Accuracy standards for horizontal control surveys (According to the Federal Geodetic Control Committee)

	First-Order	Second-Order		Third-Order	
		Class I	Class II	Class I	Class II
Relative positioning accuracy between directly connected adjacent points at the 2σ level	1/100,000	1/50,000	1/20,000	1/10,000	1/5,000
Angle Measurements					
1. Least count of instrument	0.2"	0.2"	1"	1"	1"
2. No. of sets (one direct and one reverse per set)	16	16	12	4	2
3. Maximum triangle closure seldom to exceed	3"	3"	5"	5"	10"
Recommended Uses	Primary national control network, metropolitan control networks; crustal movement studies; large engineering projects	Secondary national control network; metropolitan control networks; large engineering projects	Controls along coastlines and inland waterways; highway construction; land subdivision	Local engineering projects; topographic and hydrographic surveys	
Recommended Density of Points	Stations at 12-20 km. Urban control 3-8 km	Stations at 10-13 km. Urban control 1-3 km	About 5 km or as required	As required	As required

10.9 ANGLE ADJUSTMENT OF TRIANGULATION NETWORKS

Two cases for angle adjustment of triangulation surveys will be considered. These are:

Case I: The triangulation network consists of a chain of triangles (see Figure 10.3).

In this case, the sum of the three angles of each triangle is calculated and compared to 180° to compute the closure error (ε). The correction (c = -ε) is then distributed equally between the three angles.

EXAMPLE 10.1:

If the sum of the three angles (a + b + c) in Figure 10.3 was found to be 179° 59' 45". Correct these angles.

SOLUTION:

The error (ε) = 179° 59' 45" - 180° 00' 00" = -15"
The total correction (c) = -(-15") = +15"
Correction/angle = +15"/3 = +5",
⇒ Add 5" for each of the three angles.

Case II: The triangulation consists of quadrilaterals (Figure 10.4) or central point figures (Figure 10.5).

In this case, and in addition to the satisfaction of the angle conditions, i.e. the sum of the angles in each triangle = 180°, other trigonometric conditions, called side conditions should be satisfied. Accordingly, the following geometric conditions need to be considered:

A. Angle Conditions:

Three general types of angle conditions can be identified here. These are:

(1) The sum of the measured interior angles in a polygon should be equal to some multiple (k) of 180° (i.e. Σ interior angles = k . 180°).

(2) If one or more directly observed angles α_i at a station can be expressed as a function of other angles β_j also observed at the same station, then a station equation should be satisfied.

(3) If all the angles about a point are observed, then a center-point equation should be satisfied. This condition states that the sum of these angles should be equal to 360°.

Let us first consider the number of angle conditions in a polygon. Start with one line AB as shown in Figure 10.10a, where there are no angles. Now add one other line BC to yield one angle α_1 (Figure 10.10b), so that there are two lines and one angle. Next, add a third line AD (Figure 10.10c) to get α_2, so that there are three lines and two angles. If this process is continued, then the resulting figure will have (L) lines and (L-1) angles to be determinant. Any angles measured in excess of those (L-1) angles will be redundant and need condition equations. For instance, in a plane triangle, L = 3, therefore two angles are needed to be measured to produce a determinant triangle. If all the three angles are measured, there will be one redundancy, for which the angle condition is: $\alpha_1 + \alpha_2 + \alpha_3 = 180°$.

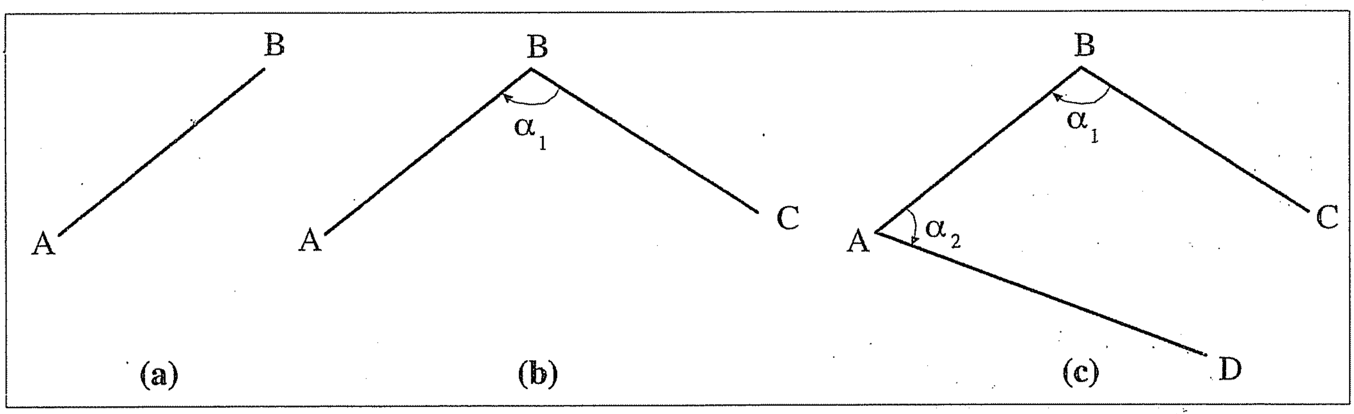


FIGURE 10.10: Angle conditions in a polygon.

In general, let C_A = total number of angle conditions in a polygon
L = number of lines in the polygon
A = number of measured angles in the polygon

Then,

$$C_A = A - (L-1) = A - L + 1 \qquad (10.2)$$

EXAMPLE 10.2:

Consider the triangle with a center point illustrated in Figure 10.11. Calculate the number of angle conditions for this figure and list these conditions.

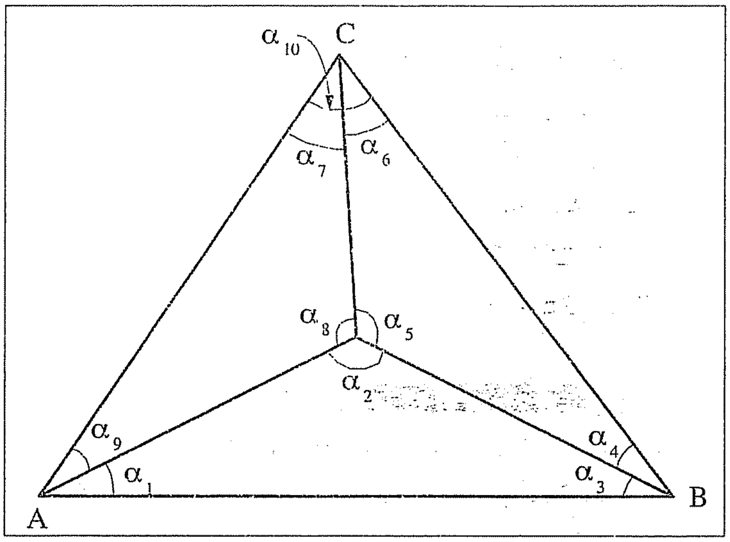


FIGURE 10.11: Center point triangle.

SOLUTION:

$L = 6$
$A = 10$

$\Rightarrow$ Number of angle conditions (C_A) = 10 - 6 + 1 = 5
This includes a center point equation and a station equation.

The basic angle conditions that can be written for this figure are:

$\alpha_1 + \alpha_2 + \alpha_3 = 180°$ ....(a)
$\alpha_4 + \alpha_5 + \alpha_6 = 180°$ ....(b)
$\alpha_7 + \alpha_8 + \alpha_9 = 180°$ ....(c)
$\alpha_1 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_7 + \alpha_9 = 180°$ ....(d)
$\alpha_2 + \alpha_5 + \alpha_8 = 360°$ ....(e)
$\alpha_6 + \alpha_7 - \alpha_{10} = 0°$ ....(f)

in which equations (a), (b), (c), (e) and (f); (b), (c), (d), (e) and (f); or (a), (b), (d), (e) and (f) are possible sets of independent angle-condition equations that are required to be satisfied to adjust this figure. Other equations can be written, but these equations will result from adding or subtracting to or from each other. This means that the equations will be linearly dependent on each other.

B. Side Conditions:

The angle conditions in a figure can be satisfied without having consistent lengths in the sides. Equation (10.2) gives only the number of angle conditions needed to be satisfied in a triangulation figure. However, the total number of conditions required to be satisfied is usually more than C_A. This total number can be calculated in one of two ways:

(1) The difference between the number of measured angles and the number of unknown coordinates in the figure.

(2) The difference between the number of measured angles and the minimum number of angles required constructing the figure.

For example, consider the braced quadrilateral shown in Figure 10.12, with the coordinates of points A and B known (i.e. length AB is known). For this figure:

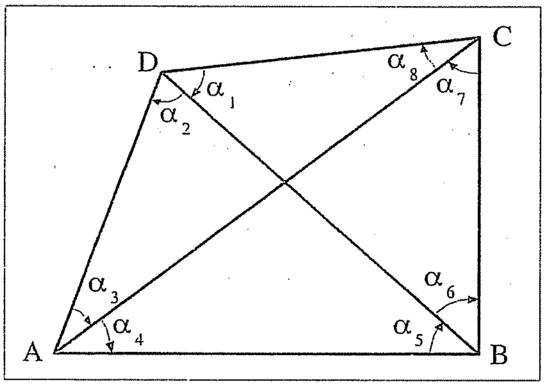


FIGURE 10.12: Braced quadrilateral.

- Number of angles (A) = 8
- Number of lines (L) = 6
- Minimum number of angles required to construct the figure = 4 (i.e., α3, α4, α5 and α6)
- Number of unknown coordinates = 4 (i.e., X_c, Y_c, X_d and Y_d)

$\Rightarrow$ $C_A = 8 - 6 + 1 = 3$

The total number of conditions needed for the complete adjustment = 8 - 4 = 4. If only the three angle conditions are satisfied, then the lengths of line CD calculated from different routes will be different (i.e. length of CD calculated using the sine law from triangles ABC and BCD will be different from the length obtained from triangles BAD and ADC). To avoid this inconsistency, a side condition needs to be satisfied.

In general, the number of side conditions (C_s) can be calculated from the following equation:

$$C_s = L - 3 - 2(S-3) = L - 2S + 3 \qquad \text{(10.3)}$$

Where C_s = number of side conditions in a figure
L = number of lines in the figure
S = total number of stations in the figure

For the quadrilateral in Figure 10.12, $C_s = 6 - 2 \times 4 + 3 = 1$

The logic behind Equation (10.3) is that three points are connected by three lines to form a triangle with no side conditions. Any two extra lines will add one station to keep the figure determinant. Any line that does not contribute to adding new points to the figure but connects already existing points will be an extra or redundant line and needs a side condition. An example of this kind of line is side DC in Figure 10.12. The following is a derivation of the side condition for line DC. Under the assumption that the length of line AB is known, and using the sine law:

- From triangle ABC $\Rightarrow BC = AB \cdot \dfrac{\sin\alpha_4}{\sin\alpha_7}$
- From triangle BCD $\Rightarrow DC = BC \cdot \dfrac{\sin\alpha_6}{\sin\alpha_1} = AB \cdot \dfrac{\sin\alpha_4 \cdot \sin\alpha_6}{\sin\alpha_1 \cdot \sin\alpha_7}$ ...(10.4)

- From triangle ABD $\Rightarrow AD = AB \cdot \dfrac{\sin\alpha_5}{\sin\alpha_2}$
- From triangle ACD $\Rightarrow DC = AD \cdot \dfrac{\sin\alpha_3}{\sin\alpha_8} = AB \cdot \dfrac{\sin\alpha_3 \cdot \sin\alpha_5}{\sin\alpha_2 \cdot \sin\alpha_8}$ ...(10.5)

Equating (10.4) and (10.5):

$$\Rightarrow \frac{\sin\alpha_1 \cdot \sin\alpha_3 \cdot \sin\alpha_5 \cdot \sin\alpha_7}{\sin\alpha_2 \cdot \sin\alpha_4 \cdot \sin\alpha_6 \cdot \sin\alpha_8} = 1 \qquad \text{(10.6)}$$

This is the required side condition equation. The side and angle conditions are simultaneously solved using the least squares adjustment technique (next section) to get the most probable adjusted values for the measured angles.

10.10 LEAST SQUARES ADJUSTMENT

In order to understand the principle of least squares adjustment, let us consider the following simple example. Assume that you have two control points i and j and that you want to calculate the coordinates of point k. For this purpose you measured the angles β and γ, and the two distances d_{ik} and d_{jk} (Figure 10.13). Now the coordinates of point k can be calculated using one of four different options (see Chapter 7):

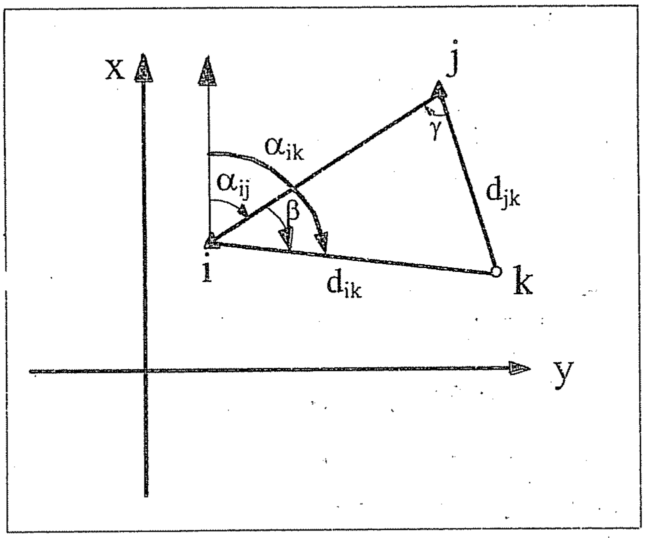


FIGURE 10.13: Measuring the coordinates of point k.

1) Location by angle β and distance d_{ik}.
2) Location by angle γ and distance d_{jk}.
3) Intersection by angles β and γ, and
4) Intersection by distances d_{ik} and d_{jk}.

The previous four different options will yield four different conflicting or unequal pairs of coordinates. This is due to two main reasons: the existence of random errors in the measured angles and distances, and the fact that more measurements are made than needed (two measurements are needed while four measurements are made). This is usually the problem encountered in horizontal control networks where more angles and distances are measured than the absolute minimum number required for calculating the unknown parameters (coordinates). So, which pair of these four pairs of coordinates is to be adopted? Actually, none of them, but the one resulting from the least squares adjustment solution.

The method of least squares is used almost universally to determine the most probable solution of the station coordinates from the angle and distance measurements. Basically, in a least squares adjustment, an equation is written for each measured angle or distance in the network. All of the resulting equations are then used to compute the unknown point coordinates by minimizing the sum of squares of the residual errors (see Chapter 2) in the measurements - that is, $\sum v_i^2$ for measurements made with the same degree of care (equally weighted), or $\sum w_i v_i^2$ for weighted measurements, where v_i is the residual error in the i-th measurement and w_i is the weight for that measurement. The following examples will give some understanding of the method.

EXAMPLE 10.3:

A distance was measured *n* times: $d_1, d_2, d_3 \dots d_n$. If you know that these measurements were made with the same degree of care, derive the formula for the most probable value (μ) of the distance.

SOLUTION:

The errors in these *n* measurements are: $v_1, v_2, v_3 \dots v_n$, where:

$$v_1 = d_1 - \mu$$
$$v_2 = d_2 - \mu$$
$$v_3 = d_3 - \mu$$
$$\cdot$$
$$\cdot$$
$$v_n = d_n - \mu$$

Now we minimize $\sum v_i^2$:

$$\sum v_i^2 = (d_1 - \mu)^2 + (d_2 - \mu)^2 + (d_3 - \mu)^2 + \cdots + (d_n - \mu)^2$$

Derive with respect to μ and set the derivative to be equal to zero.

$$\Rightarrow -2(d_1 - \mu) - 2(d_2 - \mu) - 2(d_3 - \mu) - \cdots - 2(d_n - \mu) = 0$$

Re-arranging the terms of this equation $\Rightarrow \mu = \frac{\sum_{i=1}^{n} d_i}{n}$ = the simple mean ($\bar{d}$) as explained in Chapter 2. This means that the simple mean is resulting from a least squares adjustment.

EXAMPLE 10.4:

Assume that the measurements in the previous example were made with different degrees of care and that they have the weights: $w_1, w_2, w_3 \ldots w_n$, derive the formula for the most probable value (m) of the distance.

SOLUTION:

In this case, we minimize $\sum w_i v_i^2$,

$$\sum w_i v_i^2 = w_1(d_1 - m)^2 + w_2(d_2 - m)^2 + w_3(d_3 - m)^2 + \cdots + w_n(d_n - m)^2$$

Derive with respect to m and set the derivative to be equal to zero.

$\Rightarrow$

$$-2w_1(d_1 - m) - 2w_2(d_2 - m) - 2w_3(d_3 - m) - \cdots - 2w_n(d_n - m) = 0$$

Re-arranging the terms of this equation $\Rightarrow m = \frac{\sum_{i=1}^{n} w_i d_i}{\sum w_i}$ = the weighted mean ($\hat{d}$) as explained in Chapter 2. This means that the weighted mean is also resulting from a least squares adjustment.

EXAMPLE 10.5:

Let us have the following system of equations consisting of 3 observations (ℓ_1, ℓ_2, ℓ_3) and 2 unknowns (x_1, x_2) such that:

$3x_1 + x_2 = \ell_1 = 4$ ....(1)

$2x_1 + x_2 = \ell_2 = 2$ ....(2)

$3x_1 + 2x_2 = \ell_3 = 1$ ....(3)

Calculate the most probable values for the unknowns x_1 and x_2.

SOLUTION:

Solving equations (1) & (2) $\Rightarrow$ $x_1 = 2$, $x_2 = -2$

Solving equations (1) & (3) $\Rightarrow$ $x_1 = 2\,^1\!/_3$, $x_2 = -3$

Solving equations (2) & (3) $\Rightarrow$ $x_1 = 3$, $x_2 = -4$

To get the most probable values for the unknowns, we write the following observation equations:

$v_1 = 4 - (3x_1 + x_2)$

$v_2 = 2 - (2x_1 + x_2)$

$v_3 = 1 - (3x_1 + 2x_2)$

Let us now minimize $\sum v_i^2 = 22x_1^2 + 6x_2^2 + 22x_1x_2 - 38x_1 - 16x_2 + 21$

Derive partially with respect to x_1 and x_2 and set the partial derivatives to be equal to zero.

$$\frac{\partial \sum v_i^2}{\partial x_1} = 44x_1 + 22x_2 - 38 = 0 \qquad \text{.........(a)}$$

$$\frac{\partial \sum v_i^2}{\partial x_2} = 22x_1 + 12x_2 - 16 = 0 \qquad \text{......... (b)}$$

Solving the above two equations (a) and (b) $\Rightarrow$ $x_1 = {}^{26}\!/_{11}$, $x_2 = -3$

Notice that these values of x_1 and x_2 lie between the three pairs of solutions given above.

Let us now generalize the procedure given in example 10.5. The following system of equations can be written for n observations and u unknown parameters ($n>u$):

$$V = L - AX, \quad \text{where } L = \underset{n\times 1}{\begin{bmatrix} \ell_1 \\ \ell_2 \\ \cdot \\ \cdot \\ \ell_n \end{bmatrix}}, \quad V = \underset{n\times 1}{\begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_n \end{bmatrix}}, \quad X = \underset{u\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_u \end{bmatrix}}$$

$$A = \begin{bmatrix} \frac{\partial \ell_1}{\partial x_1} & \frac{\partial \ell_1}{\partial x_2} & \cdots & \frac{\partial \ell_1}{\partial x_u} \\ \frac{\partial \ell_2}{\partial x_1} & \frac{\partial \ell_2}{\partial x_2} & \cdots & \frac{\partial \ell_2}{\partial x_u} \\ \cdot & \cdot & \cdots & \cdot \\ & \cdot & \cdots & \cdot \\ \frac{\partial \ell_n}{\partial x_1} & \frac{\partial \ell_n}{\partial x_2} & \cdots & \frac{\partial \ell_n}{\partial x_u} \end{bmatrix}$$

Minimizing $\sum v_i^2 = V^T V = (L - AX)^T (L - AX)$,

$$\Rightarrow X = (A^T A)^{-1} A^T L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10.7)$$

Where A^T is the transpose of matrix A.

EXAMPLE 10.6:

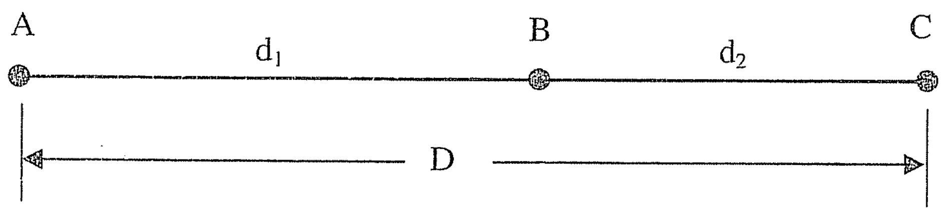
Solve the system of equations in example 10.5 using equation (10.7).

SOLUTION:

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad L = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}, \quad \Rightarrow \quad X = (A^T A)^{-1} A^T L = \begin{bmatrix} 26/11 \\ -3 \end{bmatrix}$$

EXAMPLE 10.7:

For the following figure, the following measurements were made:
$d_1 = 40.05$ m, $d_2 = 30.02$ m, $D = 70.00$ m



Using the method of least squares adjustment, calculate the most probable values of d_1 & d_2. Assume that all the three distances were measured with the same degree of care.

SOLUTION:

The following observation equations can be written:

$$\begin{aligned} d_1 + 0.d_2 &= 40.05 \\ 0.d_1 + d_2 &= 30.02 \\ d_1 + d_2 &= 70.00 \end{aligned}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 40.05 \\ 30.02 \\ 70.00 \end{bmatrix}, \quad X = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$X = (A^T A)^{-1} A^T L$$

$$A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$

$$(A^T A)^{-1} = \frac{1}{4-1} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$(A^T A)^{-1} A^T = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & 1/3 \end{bmatrix}$$

$$X = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = (A^T A)^{-1} A^T L = \begin{bmatrix} 2/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 40.05 \\ 30.02 \\ 70.00 \end{bmatrix} = \begin{bmatrix} 40.03 \\ 30.00 \end{bmatrix}$$

$$\Rightarrow d_1 = 40.03\,\text{m}, \qquad d_2 = 30.00\,\text{m}$$

The method explained above is called the method of observation equations and it is given here only for observations with equal weights. A modification is made to Equation (10.7) if the weights of observations are to be considered. Another method called the conditions equations method is used alternately to solve for the unknown parameters. The subject of adjustment computations is a big one, and it is beyond the scope of this book to deal with it with greater detail. However, for more information, the reader can refer to the references listed at the end of the book.

Computer programs for performing least-squares adjustment of horizontal control networks are now available. Such programs usually provide a general solution to the problems of horizontal control networks; that is, the solution is independent of the geometric shape of the survey network, which can be a traverse, a quadrilateral, or a network of triangular figures. The only requirement is that sufficient angle and distance measurements exist to determine the coordinates of the new survey stations. The input to such a computer solution usually includes the angle and distance measurements, estimated standard errors of these measurements, measured azimuths, and known coordinates of the existing stations. The output usually includes the most probable coordinates of the new survey stations as well as the estimated standard errors of these computed coordinates. An example of such programs is the HCONTROL program written by the author using the FORTRAN language.

10.11 GLOBAL POSITIONING SYSTEMS

Traditional methods of horizontal control surveys are both time consuming and labor intensive. Even for moderate size engineering projects, horizontal control surveys may requires 3 to 6 months to complete. The surveying process can be particularly difficult under severe weather and/or terrain conditions. Thus the development of global positioning systems (GPS), which are capable of determining the three-dimensional coordinates of survey points without measuring either angles or distances, represents a major breakthrough in surveying and mapping. To the surveyor, such a positioning system can be considered a "black box". It usually consists of a hardware component (satellite receiver), which is used to collect field data, and a software component, which consists of computer programs used to compute the positions of the survey points. By setting up two miniature portable antennas over two points to continuously track a group of satellites for a short period of time, it is possible to determine the relative positions of the two points with an accuracy equal to, if not exceeding, that of the conventional ground survey methods. This new technology is just starting to be known and used by the Palestinian Survey Department as well as local surveyors. By positioning the GPS over a survey point for a short period of time, it will likely be possible to obtain the geographic position of the point with an error of less than 1 cm in all three coordinates. More on the theory and concept of global positioning systems will be explained in Chapter 12.

PROBLEMS

10.1 What is meant by a horizontal control survey? Give two practical examples which demonstrate the importance of horizontal control networks.

10.2. What is the difference between triangulation and trilateration?

10.3 What are the basic figures used in the establishment of horizontal control surveys?

10.4 What are the main factors that should be considered when choosing the location of horizontal control points?

10.5 Given that angles can be measured with a standard deviation equal to ± 5", what will be the relative accuracy of a line opposite to a 25° angle?

10.6 In a triangulation network, the measured angles were estimated to have a standard error of ± 1". What should be the maximum allowable closure error for the triangles in a quadrilateral?

10.7 The following are the measured angles and distances for a quadrilateral as shown in Figure 10.9:

Angles	Distances (m)
a = 52° 11' 06"	AD = 1159.22
b = 32 17 04	AB = 1963.62
c = 23 47 32	DB = 2153.77
d = 71 44 13	AC = 1715.80
e = 39 28 25	BC = 1633.53
f = 44 59 47	DC = 932.05
g = 64 47 29	
h = 30 44 21	

a. Given that $\sigma_\theta = \pm 1''$, perform closure checks on the angles.

b. Given that $\sigma_d = \pm(0.003 \text{ m} + 5 \text{ ppm})$, perform checks on the distances by assuming the distance AD to be free of error.

10.8 Figure 10.14 shows three configurations for triangulation. For each of them, assume that the heavy line is the measured baseline and that all angles are observed. Determine:

a) The total number of angle conditions, and
b) The total number of side conditions.
c) Write down the angle equations, for the three figures, that need to be satisfied.
d) Derive the side conditions for Figures 10.14 a and b.

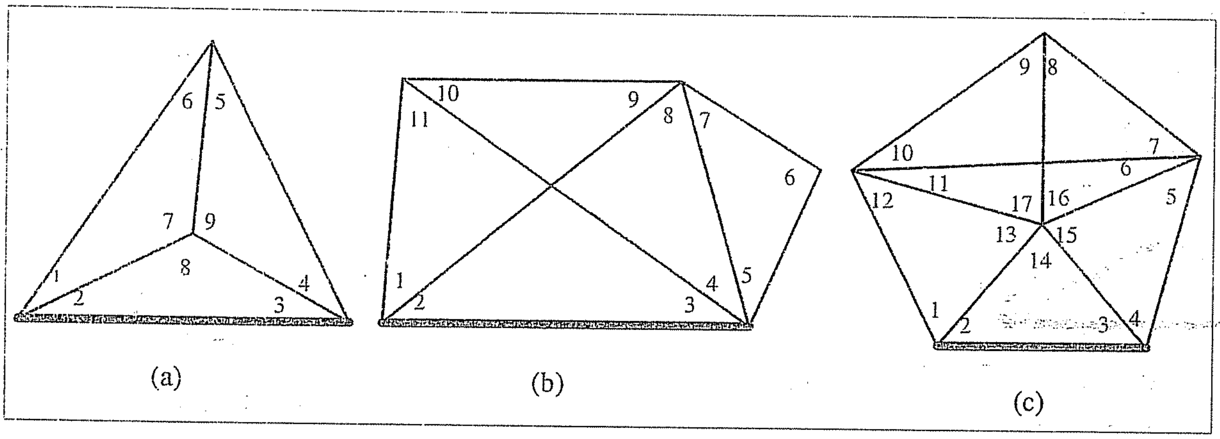


FIGURE 10.14

10.9 The following measurements were taken for a circular tract of land:

Perimeter (P) = 942.502 m
Diameter (D) = 300.015 m

If you know that both the perimeter and the diameter were measured with the same degree of care, calculate the most probable value of the radius (R) of the tract of land using the method of observation equations.

10.10 Given 3 points A, B and C that are located on the earth surface. Point A is a BM whose elevation is 500.00 m AMSL. The following elevation differences were measured: $\Delta H_{AB} = 5.62$ m, $\Delta H_{BC} = 4.38$ m and $\Delta H_{AC} = 9.97$ m. Using the method of least-squares adjustment (with matrices), calculate the most probable values of H_B & H_C. Assume that all the three elevation differences were measured with the same degree of care.

11 PHOTOGRAMMETRY

11.1 INTRODUCTION

Photogrammetry is the art, science and technology of obtaining reliable information about the physical objects and the environment by means of photographic and electromagnetic images. It includes: (a) photographing an object; (b) processing and studying the geometry of the photograph; (c) measuring the image of the object, and (d) reducing the measurements to some useful form such as a topographic map.

Photogrammetry embraces two broad categories of practice:

(1) Metrical photogrammetry: where quantitative measurements are made, and from which ground positions and elevations, distances, areas or volumes can be computed, or from which planimetric and topographic maps can be drawn. In order to reflect the method of measurement and reduction of data obtained from the photograph, metrical photogrammetry can be further broken into three main categories:

- *Analog or Instrumental photogrammetry*: where the land is photographed, and then the photographs are used to reconstruct a scaled three-dimensional optical model of the land's surface using an instrument called a stereoplotter. The output of this process is usually a planimetric or topographic map.

- *Analytical photogrammetry*: where rigorous mathematical methods are used to make precise three-dimensional measurements from photographs. The three-dimensional positions of survey stations located on the ground can be determined from aerial photographs by a process called photo-triangulation. Second order accuracy can be achieved.

- *Digital photogrammetry*. This is a modernized approach of photogrammetry that incorporates the tasks of both analog and analytical photogrammetry into one computerized system using digital raster images instead of hard copy photographs. The source of these digital images can range from digitized photographs, to digital cameras, to electro-optical scanners.

(2) Photo-interpretation or Remote Sensing: where the photographs are evaluated in a qualitative manner in order, for example, to detect water pollution, classify soils, interpret geological formations, identify crops or obtain military intelligence.

The term *aerial photogrammetry* is that branch of photogrammetry wherein photographs of the terrain in an area are taken in an orderly sequence by a camera mounted in an aircraft flying over the area. *Terrestrial photogrammetry*, on the other hand, is that branch wherein photographs are taken from a fixed, and usually known, position on or near the ground surface with the camera axis being horizontal or nearly so.

11.2 ORTHOGRAPHIC VERSUS PERSPECTIVE PROJECTION

A map can be defined as a reproduction, at a reduced scale, of an orthographic projection of the terrain onto a reference datum plane (Figure 11.1a). Terrain points are projected by parallel lines that are perpendicular to the reference plane. A distance measured between any two points on the map when multiplied by the scale of the map will be equal to the corresponding horizontal distance measured directly in the field. To simplify the illustration, the scale of the projection shown in Figure 11.1a is drawn to be 1:1 with respect to the terrain. Thus, ab = A'B'; bc = B'C'. Also ab/bc = A'B'/B'C'. Furthermore, an angle measured at a point on the map between any two lines is the same as the horizontal angle measured with a theodolite at that point on the ground between the two corresponding lines.

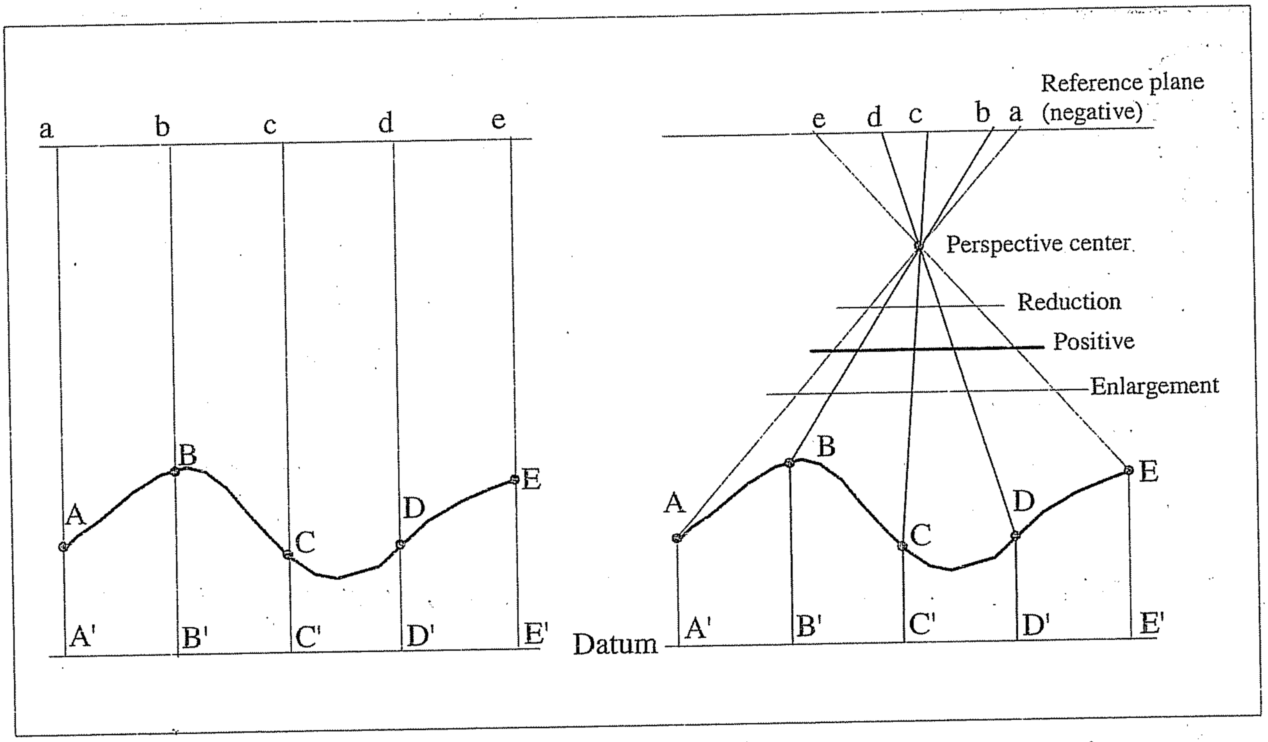


FIGURE 11.1: Orthographic versus perspective projection.

On the other hand, a perspective or central projection, is one in which all of the points are projected onto the reference plane through one point called the perspective center, as shown in Figure 11.1b. In photography, the sensitized film occupies the reference plane where the negative is formed behind the perspective center. Other reference planes are located between the object and the perspective center as shown in Figure 11.1b. The three projections shown in the figure represent a 1 to 1 reproduction of the negative, which is called a positive, a reduction in size from the negative, and an enlargement. Since the points lie at different elevations, the projected points will not bear the same relation to one another as the original points. In other words, in Figure 11.1b, even though A′B′ = B′C′ on the ground, ab ≠ bc on the photograph. Moreover, the angles on the photograph will not necessarily be equal to their counterparts on the terrain. Figure 11.2 indicates a third difference between the two projections by showing how three different objects look like both on a map and on a photograph. As can be seen, the map location and size of the three objects (Figure 11.2a) correspond to the bottom surfaces of the objects shown in perspective (Figure 11.2b), which all lie at the same elevation. A fourth difference between a map and a photograph is that the photograph does not contain map symbols, which are essential to the map.

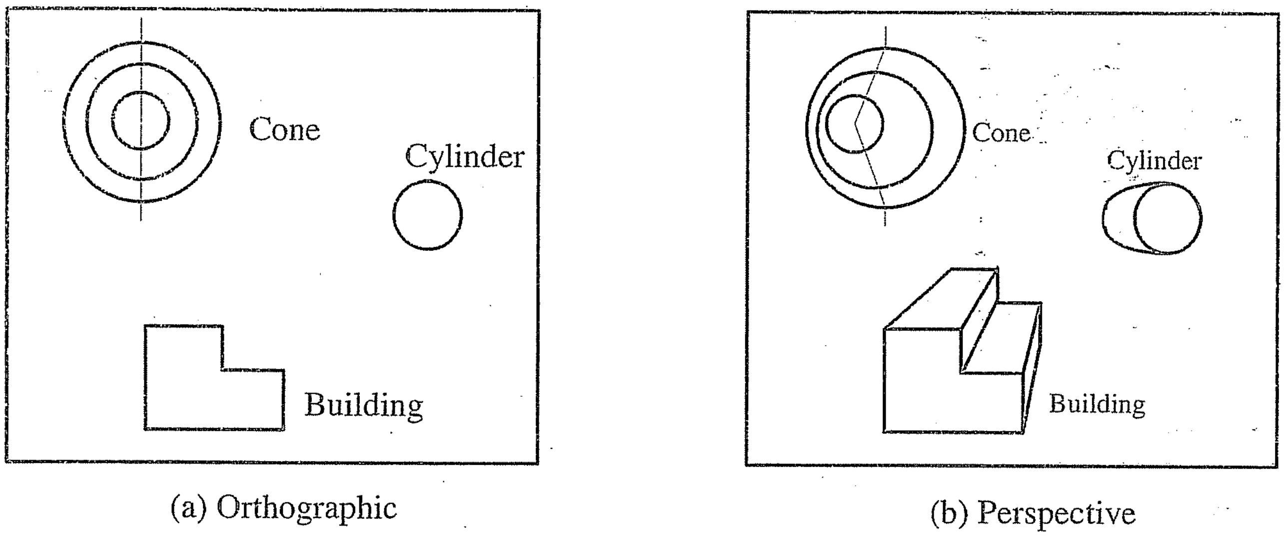


FIGURE 11.2: Idealized objects in orthographic and perspective projection.

11.3 AERIAL MAPPING CAMERAS

Figure 11.3 shows the basic elements of an aerial camera. Typically, the lens assembly has a focal length of 3, 6, 8 or 12 inches and an aperture ranging from f/4.0 to f/6.3, where f is the focal length.

Usually, one index mark (*called fiducial mark*) is mounted on the center of each edge of the image plane or in the plane corners. The images of these marks appear in every photograph as shown in Figure 11.4. By joining the opposite pairs of fiducial marks with thin lines on the photograph, the exact location of the principal point (being the point of intersection) can be defined. This is the point at which the optical axis intersects the plane of the image. Typical mapping projects require that the two intersecting lines be perpendicular to within ± 1 minute of arc and that their intersection defines the true location of the principal point to within ± 0.030 mm.

Also shown in Figure 11.4 is the relationship between the negative image formed on the film plane and a positive print of the same image. For illustration purposes it is sometimes more convenient to use the positive image in drawings.

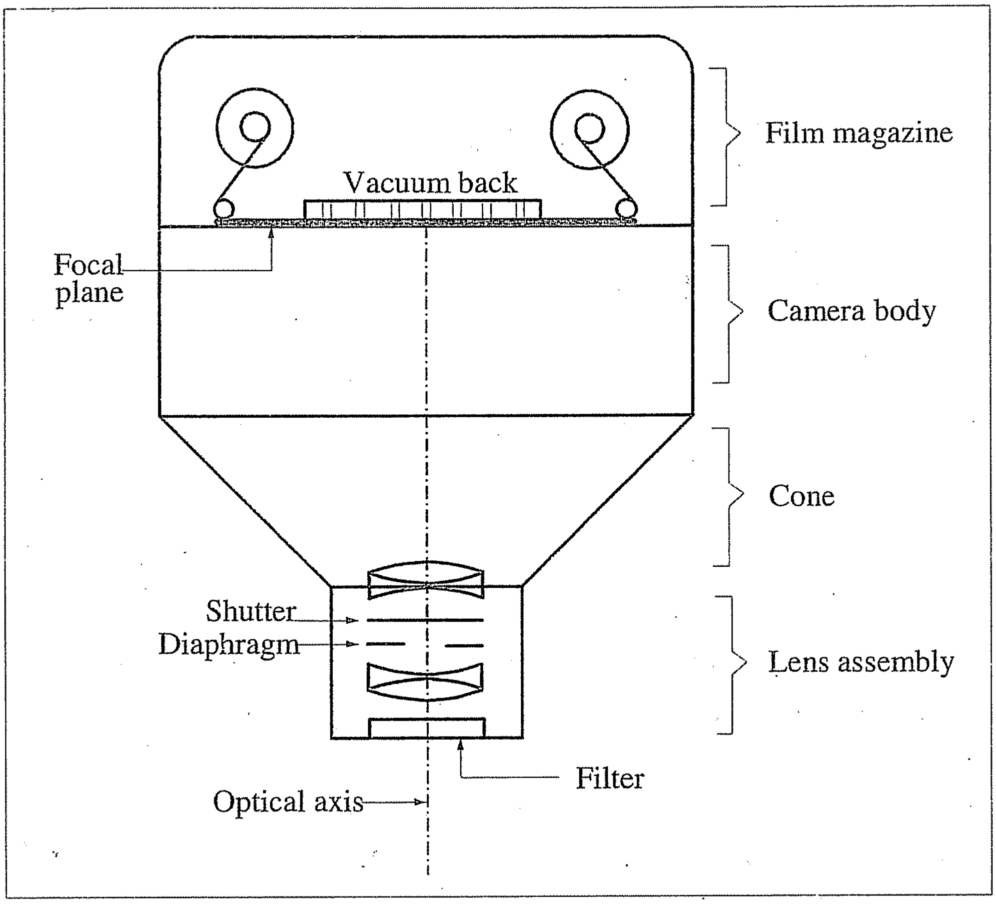


FIGURE 11.3: Elements of an aerial mapping camera.

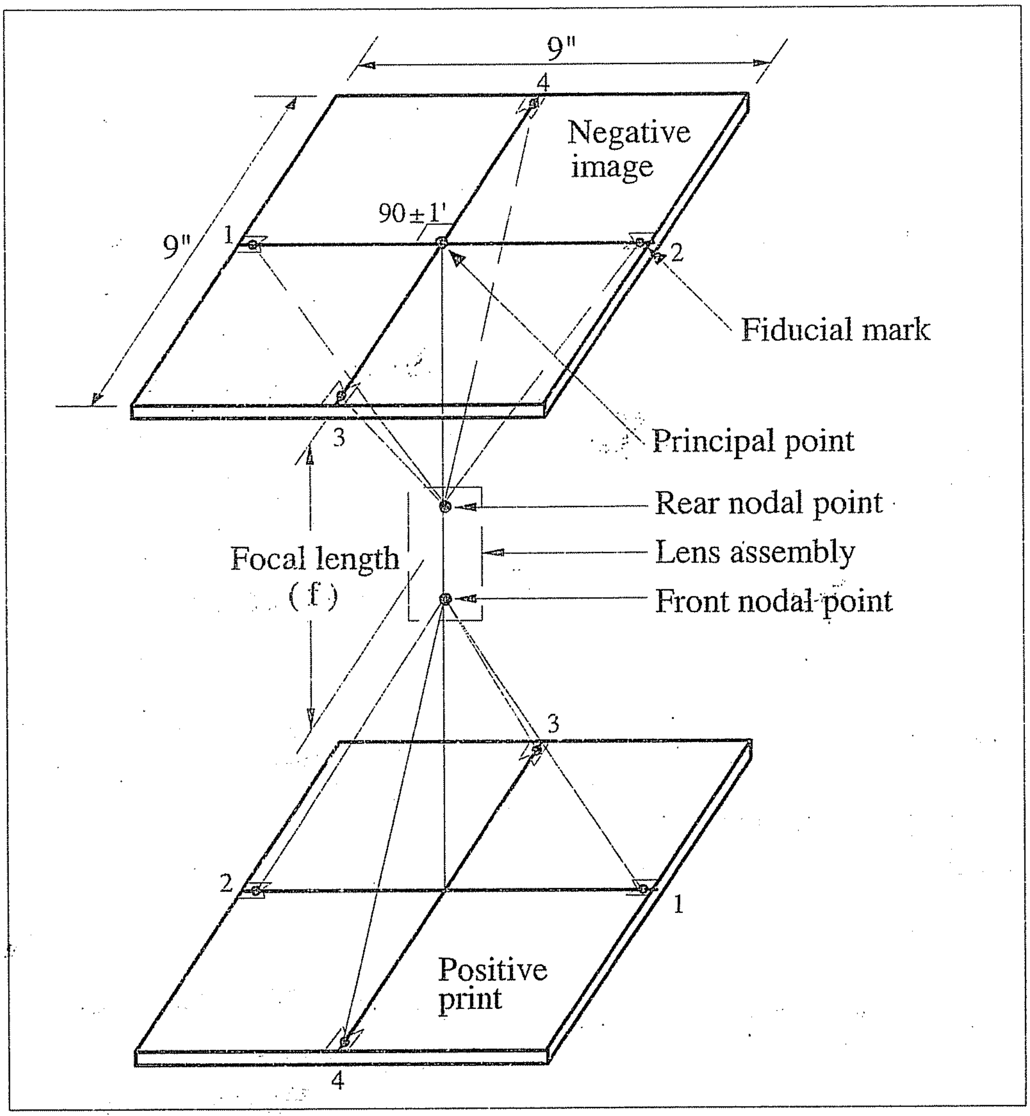


FIGURE 11.4: Fiducial marks and principal point

In topographic mapping, it is important that the aerial photographs be taken with the camera optical axis pointing as nearly vertical as possible. The maximum tolerance for tilting the optical axis is usually less than 5 degrees. The camera can also be mounted on a gyro-controlled platform that maintains the optical axis in a near vertical direction. Figures 11.5 and 11.6 show two examples of vertical photographs. The first one is for a part of the city of Chicago in the USA, and the second for a part of the city of Nablus in Palestine. These photographs were originally printed with a size 9″x9″ (almost 22.5 cm x 22.5 cm), but are printed here to fit the page of the book.

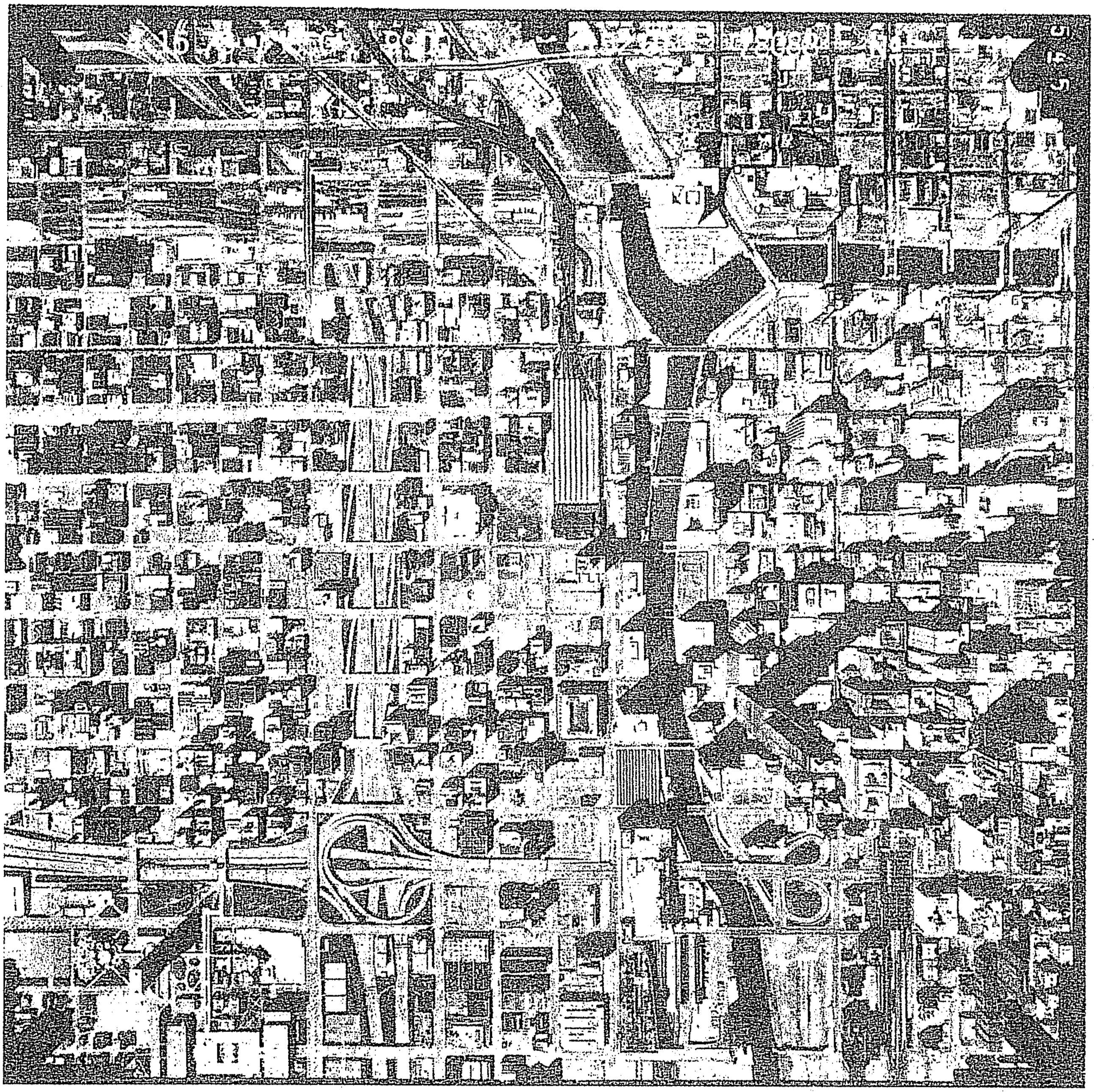


FIGURE 11.5: A vertical photograph that shows a part of the city of Chicago in the USA.

FIGURE 11.6: An old vertical photograph that shows a part of the city of Nablus in Palestine.

11.4 SCALE OF A VERTICAL AERIAL PHOTOGRAPH

The photographic scale can be defined as the ratio between a distance measured on the photograph and its corresponding ground distance. As mentioned earlier, the photograph is a perspective projection of the ground onto the focal plane of the camera. Unlike a topographic map, which is an orthogonal projection and has a uniform scale everywhere on the map, an aerial photograph does not have a uniform scale. The scale at any image point on the photograph depends on the distance of the corresponding ground point to the focal plane of the camera at the moment the photograph was taken.

The relationship between photographic scale and the flying height of the aircraft is illustrated in Figure 11.7. Let H represent the flying height of the aircraft above the elevation datum, h_A and h_B represent the ground elevations of points A and B respectively, and f be the focal length of the camera lens. By geometric proportion, the following relationships can be derived:

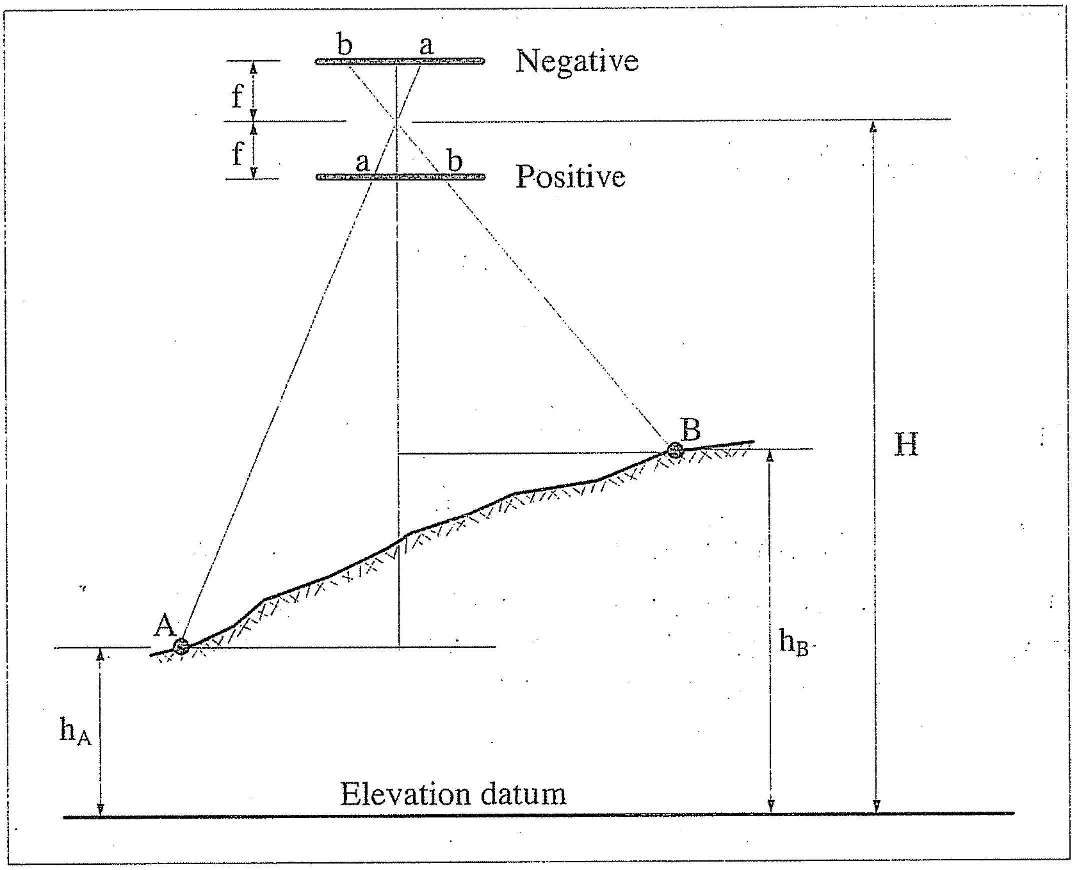


FIGURE 11.7: Scale of an aerial photograph.

$$\text{Scale of photo at image point a} = \frac{f}{H - h_A} = \frac{1}{\left(\dfrac{H - h_A}{f}\right)} \quad \ldots\ldots (11.1)$$

$$\text{Scale of photo at image point b} = \frac{f}{H - h_B} = \frac{1}{\left(\dfrac{H - h_B}{f}\right)} \quad \ldots\ldots (11.2)$$

The units for H, h_A, h_B and f must be identical. For example, suppose that H = 2400m, h_A = 600m, and f = 15 cm. Then the scale of the photograph at image point a is:

$$\frac{1}{\left(\dfrac{2400 - 600}{0.15}\right)} = \frac{1}{12{,}000}$$

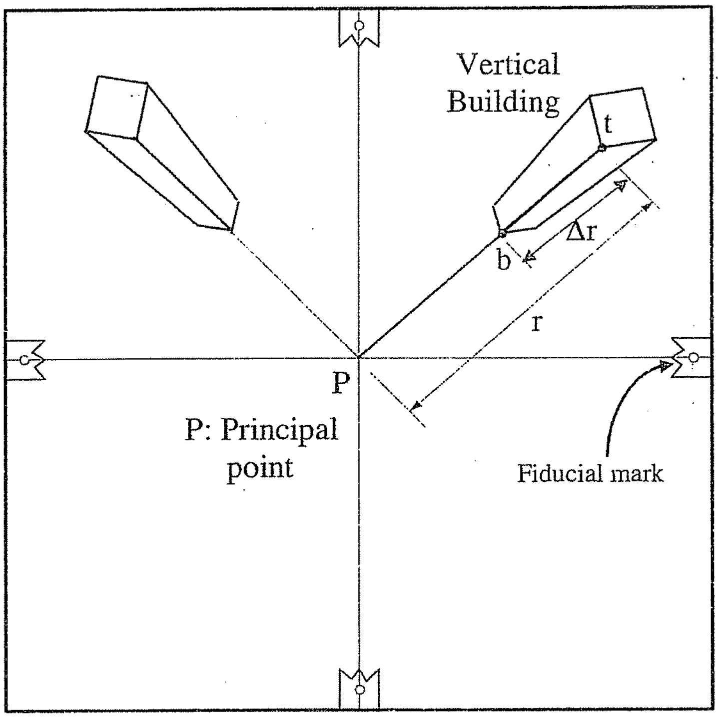
For convenience, the scale of an aerial photograph is often referred to by its average scale, which is computed by using the average ground elevation (h_{avg}); that is:

$$\text{Average scale of a photograph} = \frac{1}{\left(\dfrac{H - h_{avg}}{f}\right)} \quad \ldots\ldots\ldots\ldots (11.3)$$

11.5 HEIGHT DETERMINATION FROM A SINGLE PHOTOGRAPH

For a perfectly vertical aerial photograph, the images of vertical objects such as buildings and utility poles radiate outward from the principal point (Figure 11.8). The reason is that all rays pass through the perspective center at the time of photography, and for a perfectly vertical image, the perspective center coincides with the principal point. This geometric condition can be used to measure the height of vertical objects from a single aerial photograph.

Figure 11.9 separates one building of those shown in Figure 11.8. Let T and B represent respectively the top and bottom of a corner edge of the building, and t & b represent the corresponding images. Let r represent the



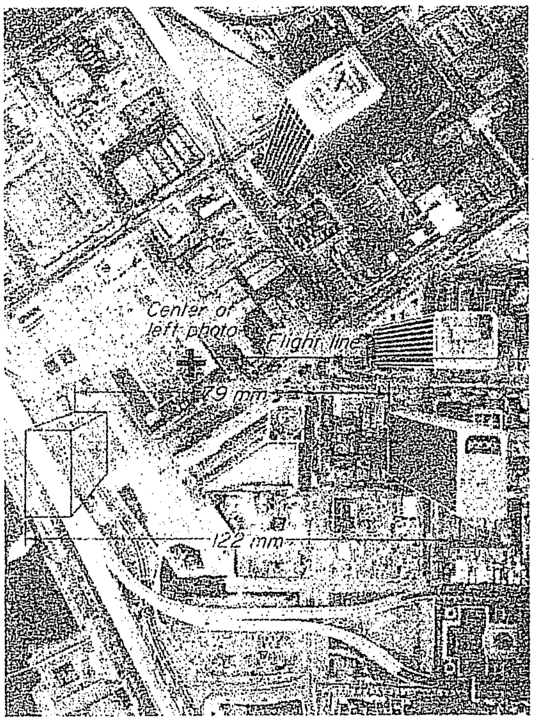


FIGURE 11.8: Height displacement radiates outwards from the principal point.

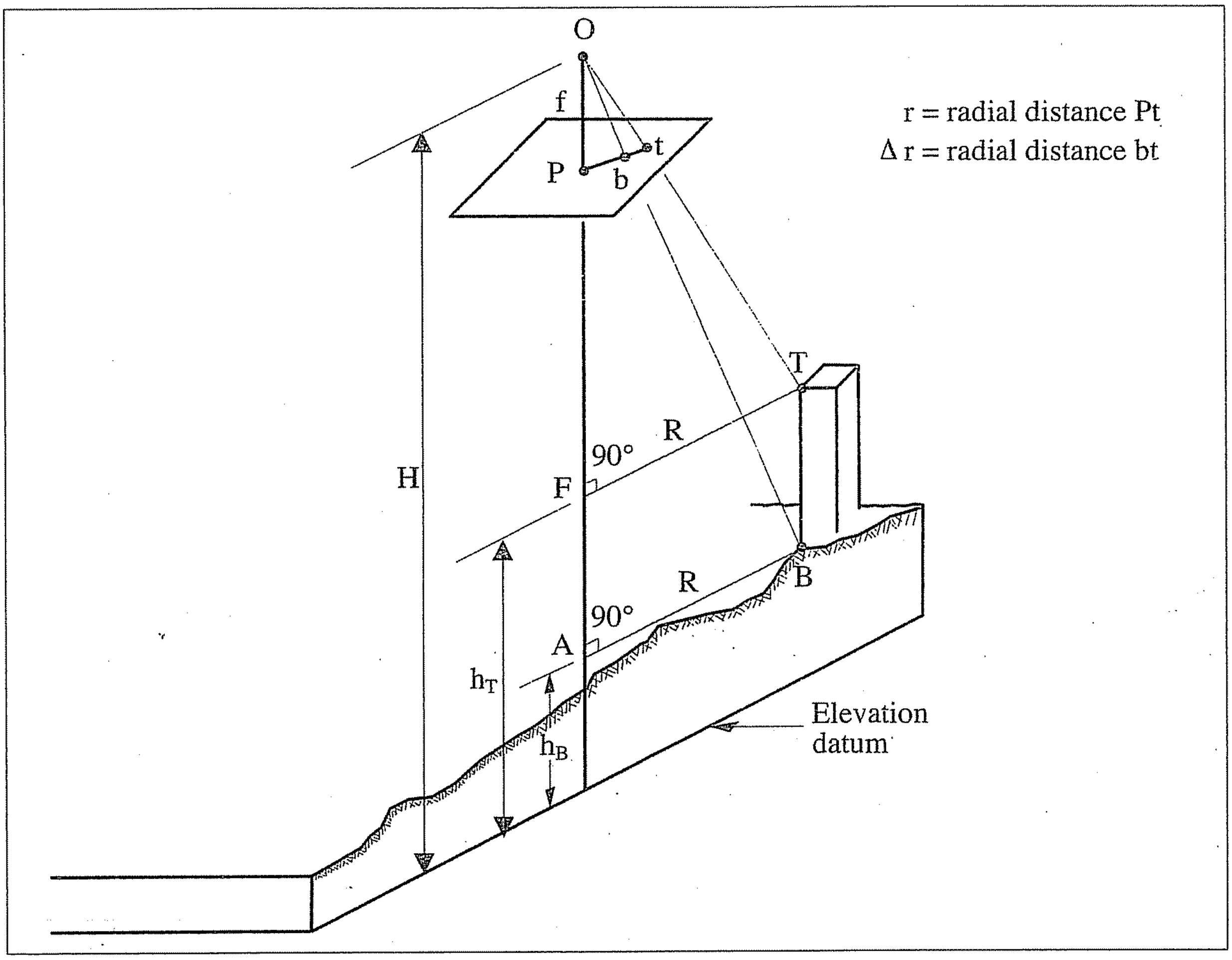


FIGURE 11.9: Geometry of height displacement

radial distance (Pt) from the principal point P to image point t; Δr represent the radial distance from the image point b to image point t. Furthermore, let H denote the flying height of the aircraft above an elevation datum, and h_B and h_T be the elevation of the bottom and top corners of the building above the same datum. Then from Figure 11.9:

ΔOPb and ΔOAB are similar triangles. Therefore:

$$\frac{R}{r - \Delta r} = \frac{H - h_B}{f}$$

Or $$H - h_B = \frac{f}{r - \Delta r} \cdot R \qquad \text{.............................(11.4)}$$

Similarly, from ΔOPt and ΔOFT:

$$\frac{R}{r} = \frac{H - h_T}{f}$$

Or $$H - h_T = \frac{f}{r} \cdot R \qquad \text{.............................(11.5)}$$

From Equations (11.4) and (11.5),

$$(H - h_B) - (H - h_T) = \frac{f}{r - \Delta r} \cdot R - \frac{f}{r} \cdot R$$

$$\Rightarrow \quad h_T - h_B = \frac{\Delta r \cdot f \cdot R}{r(r - \Delta r)} \qquad \text{.............................(11.6)}$$

From Equation (11.4),

$$R = \frac{r - \Delta r}{f}(H - h_B) \qquad \text{.............................(11.7)}$$

Substituting Equation (11.7) into (11.6) yields the following:

$$h_T - h_B = \frac{\Delta r}{r}(H - h_B)$$

Setting $\Delta h = h_T - h_B$ = height of the building

$$\Rightarrow \quad \Delta h = h_T - h_B = \frac{\Delta r}{r}(H - h_B) \qquad \text{........................(11.8)}$$

EXAMPLE 11.1:

In Figure 11.9, let $\Delta r = 1.25$ cm, $r = 11.20$ cm, $h_B = 210.52$ m and $H = 718.75$ m. Calculate the height of the building (Δh).

SOLUTION:

$$\Delta h = \left(\frac{1.25}{11.20}\right)(718.75 - 210.52) = 56.72 \text{ m}$$

The accuracy of the computed value of Δh depends largely on the accuracy of the measured values of Δr and r and on the accuracy to which $(H - h_B)$ is known. If h_B is not known, it is sufficiently accurate to use (h_{avg}) instead of h_B especially if H is large. Equation (11.8) assumes a perfectly vertical photograph. Therefore the actual tilt of the photograph contributes to the error in the computed height value.

11.6 STEREOSCOPIC VISION

Stereoscopy is the term given to the following natural phenomenon: when a person looks simultaneously at two photographs that have been taken of the same scene from two viewpoints, looking at one photograph with each eye, he can see an image of the scene in three dimensions.

Depth perception in humans is due to the fact that a given object is viewed simultaneously with both eyes, which are separated in space; hence the two rays of vision converge at an angle upon the viewed object. The angle of convergence of the two rays of vision is called the parallactic angle and its magnitude has an important effect upon the accuracy with which the observer can judge the distance of a given object.

Figure 11.10 illustrates the effect of parallactic angles on depth perception. Points I_1 and I_2 denote the positions of the two eyes, separated by an eye base, b. Points A and B are located in the field of vision. The two eyes subtend the parallactic angles ϕ_1 and ϕ_2 at points A and B respectively. Since the angle ϕ_2 is larger than ϕ_1, point B appears closer to the eyes than point A.

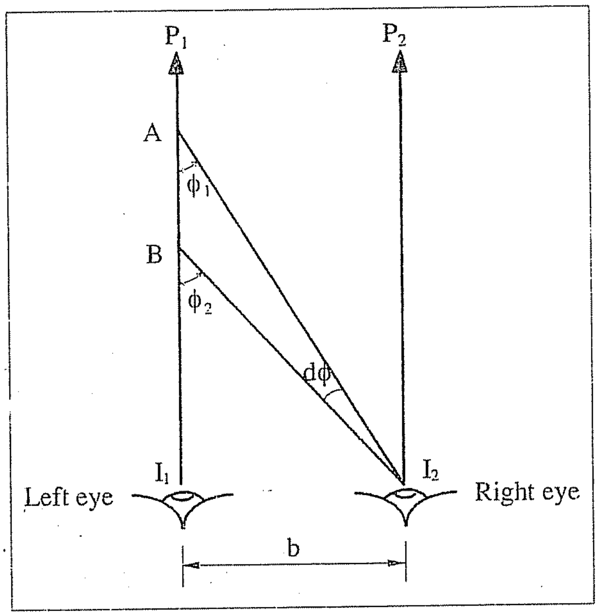


FIGURE 11.10: Parallactic angle.

The angular difference $d\phi = \phi_2 - \phi_1$ is called the differential parallax. It provides a direct measure of the difference in distance of the two objects from the two eyes, that is, the distance AB. The human brain recognizes this angular difference immediately and translates it to a difference in distance of the two objects.

It is evident that as $d\phi$ becomes small, there is a limiting value for ϕ_1 or ϕ_2 below which the sense of stereoscopic vision becomes nil and the observer is unable to judge which is the nearer of the two objects. This limiting value for most observers is about 20". If the distance between the observer's eyes is 6.3 cm, the rays, $I_1 P_1$ and $I_2 P_2$ meet at a distance of about 650 m. At that distance or beyond, the sense of stereoscopic vision becomes inoperative and the relative distances to objects must be judged by their apparent sizes or by other factors. However, the range and intensity of stereoscopic perception can be increased in two ways - either by apparently increasing the base between the viewpoints, or by magnifying the field of view by the use of lenses.

In aerial photogrammetry, an area to be mapped is photographed from two different viewpoints, as shown in Figure 11.11. With the help of an optical system, the left photograph is presented to the left eye while the right photograph is presented separately to the right eye. The human brain then merges the two images into a three-dimensional vision of the ground. This is one of the fundamental principles of photogrammetry.

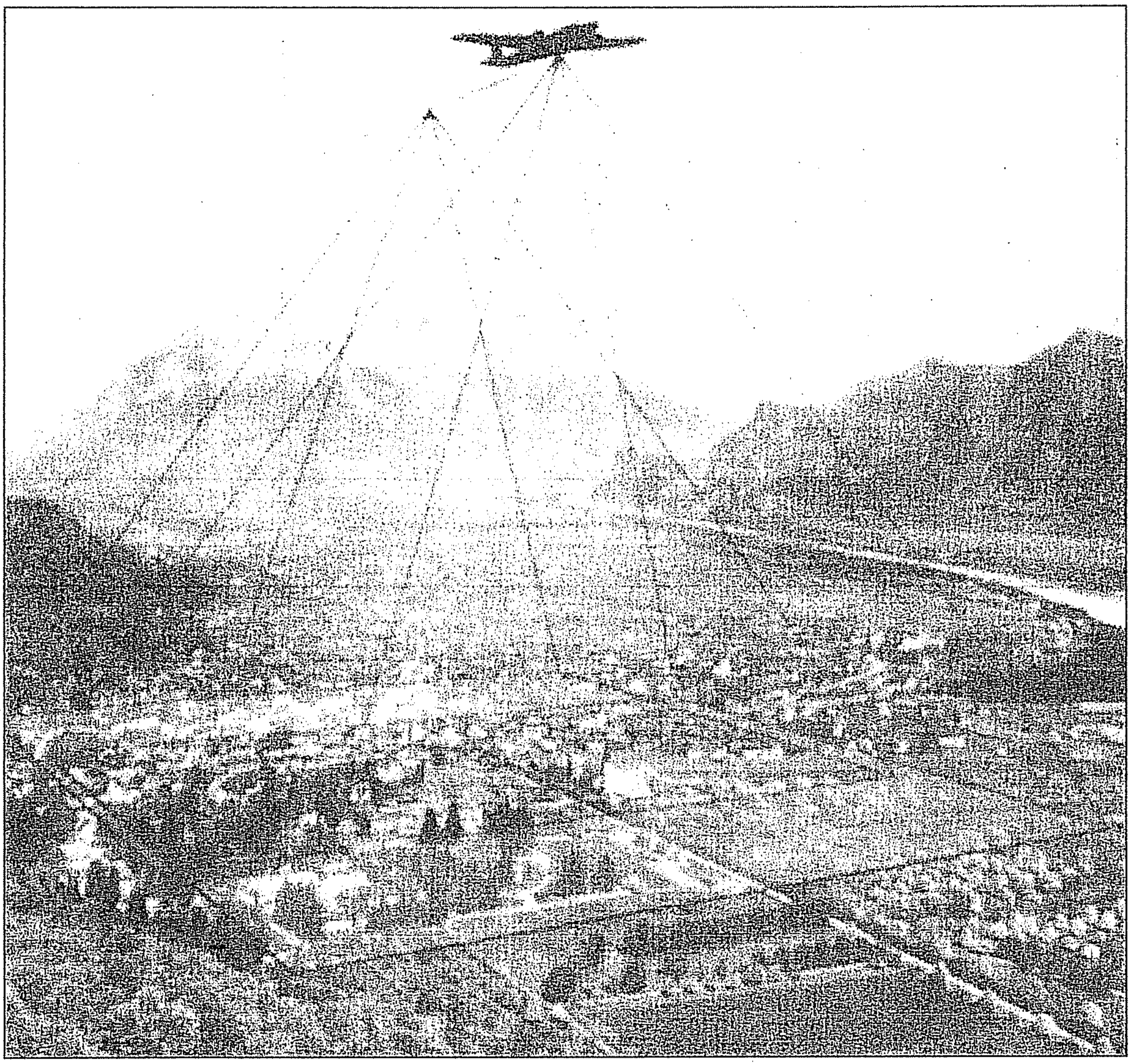


FIGURE 11.11: Stereoscopic coverage of aerial photographs.

To illustrate the stereoscopic viewing process, a simple experiment can be conducted with Figure 11.12. By looking at the two images of a truncated pyramid (image I) in the normal manner with both eyes, no stereoscopic vision is possible since both eyes see both images simultaneously. Now, hold a piece of cardboard perpendicular to the page between the two images over the dotted line, so that the right image is seen only by the right eye and the left image is seen only by the left eye. Focus both eyes on the respective images. After a short time, you will be able to see a 3-D stereoscopic view of the truncated pyramid. Repeat the same procedure for image II.

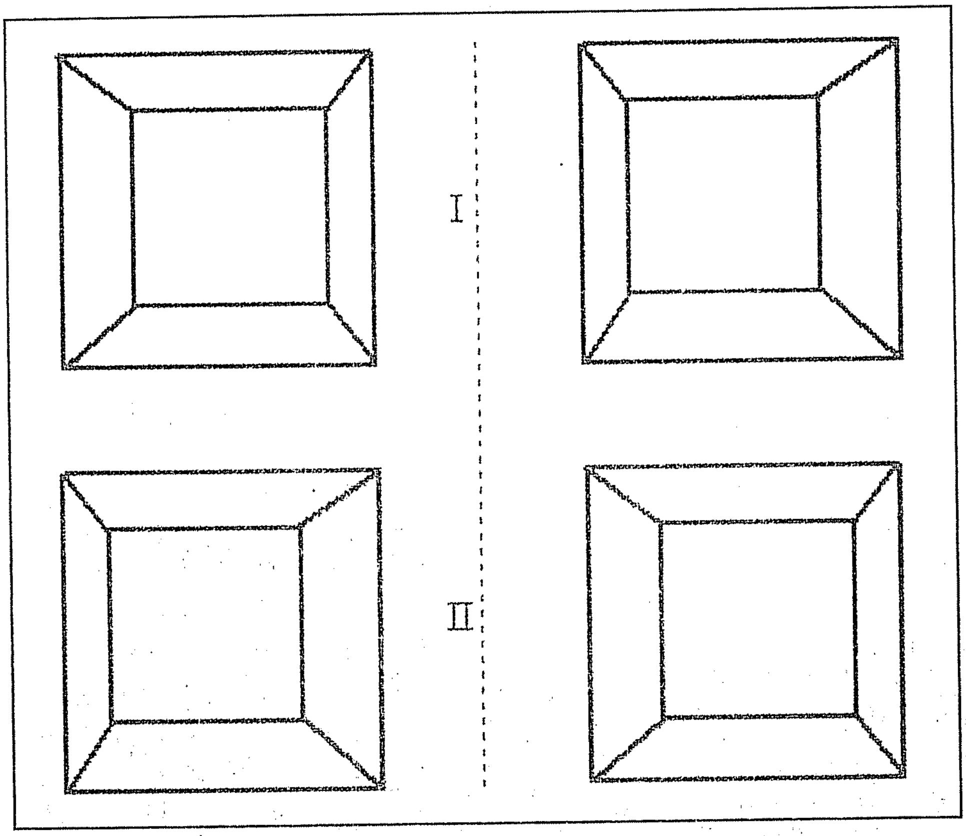


FIGURE 11.12: Pairs of images suitable for stereoscopic viewing.

11.7 PARALLAX IN AERIAL STEREOSCOPIC VIEWS

The ideal conditions for obtaining aerial stereoscopic views of the ground surface are:

(a) Two pictures are taken that overlap each other;
(b) The elevations of the two camera positions O_1 and O_2 are the same; and
(c) The camera axis is vertical, and therefore the picture planes lie in the same horizontal plane.

This geometry is illustrated in Figure 11.13. The two images of an object P, which appear in both photographs, are P_1 for the left photograph and P_2 for the right photograph. The x-coordinate of this object is x_1 for view O_1, and x_2 for view O_2, and the difference between the coordinates $p = x_1 - x_2$ is called the x-parallax of that image. Parallax can be defined as the change in position of the image of a point on two subsequent photographs, due to the change in position of the cameras. It may be noted that for any stereoscopic pair of photographs for which the ideal conditions stated in the preceding paragraphs exist, the parallaxes of the images for all of the points having the same elevation will be equal.

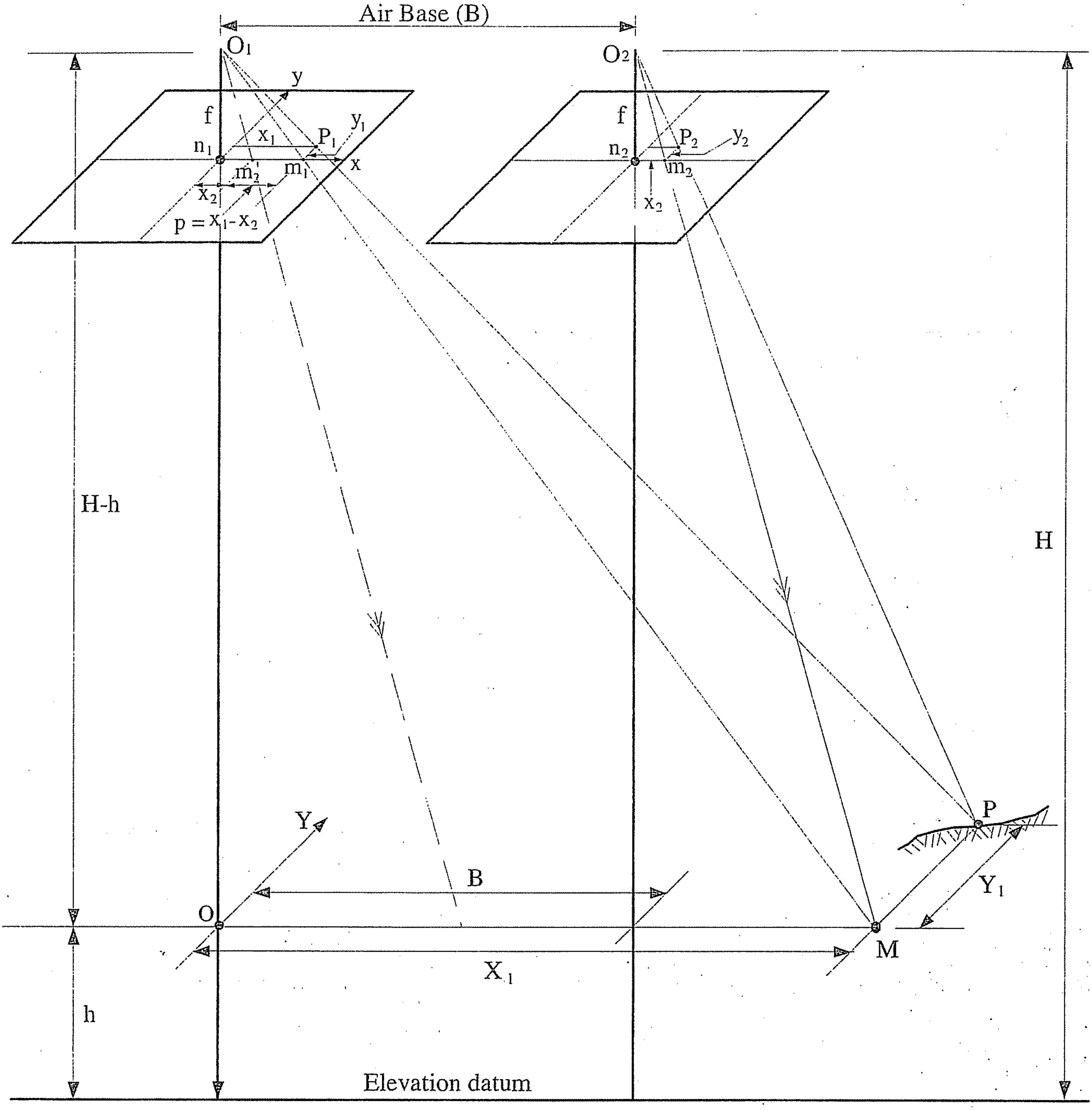


FIGURE 11.13: Parallax in aerial stereoscopic views.

11.8 GROUND COORDINATES FROM MEASUREMENTS ON A VERTICAL PHOTOGRAPH

In Figure 11.13, and for the left vertical photograph, the X and Y ground coordinate axes coincide in direction with the photograph x and y-axes. From the similar triangles $O_1n_1m_1$ & O_1OM and $O_1m_2'm_1$ & O_1MO_2, the following equations can be derived:

$$\frac{X_1}{x_1} = \frac{H - h}{f} = \frac{B}{p}, \quad \text{Where } p = x_1 - x_2$$

$$\Rightarrow \quad X_1 = \frac{B}{p} \cdot x_1 \qquad \text{.......(11.9)}$$

$$h = H - \frac{B}{p} \cdot f \qquad \text{.......(11.10)}$$

Similarly,

$$\frac{Y_1}{y_1} = \frac{H - h}{f} = \frac{B}{p}$$

$$\Rightarrow \quad Y_1 = \frac{B}{p} \cdot y_1 \qquad \text{.......(11.11)}$$

11.9 MEASUREMENT OF PARALLAX

Parallax measurements on paper photographs are usually performed using a small device called parallax bar. The most common type of parallax bar is diagrammed in Figure 11.14(a). It contains the bar proper, which holds a fixed plate of transparent material (such as plastic or glass) near the left end and a movable plate towards the right end. Identical reference marks are located at the underneath surface of each plate. The movable plate is moved by means of a micrometer screw, the total movement being about 25 mm to 40 mm, depending on the manufacture of the instrument. The micrometer is usually read to the nearest 0.01 mm.

In Figure 11.14(b), the principal point of the right photograph O_2 is located on the left photograph and is called the conjugate principal point of O_1. Similarly O_1 is located on the right photograph and is called the conjugate of

O_2. A fine line is then drawn on a sheet of heavy drafting paper and the left-hand photograph is oriented on the paper so that the flight line, defined by the principal point and the conjugate principal point, is in exact coincidence with the line on the paper. This orientation may be obtained by laying a straight edge over the photograph and orienting it to the line. The micrometer of the parallax bar is now set to the middle of the range of readings.

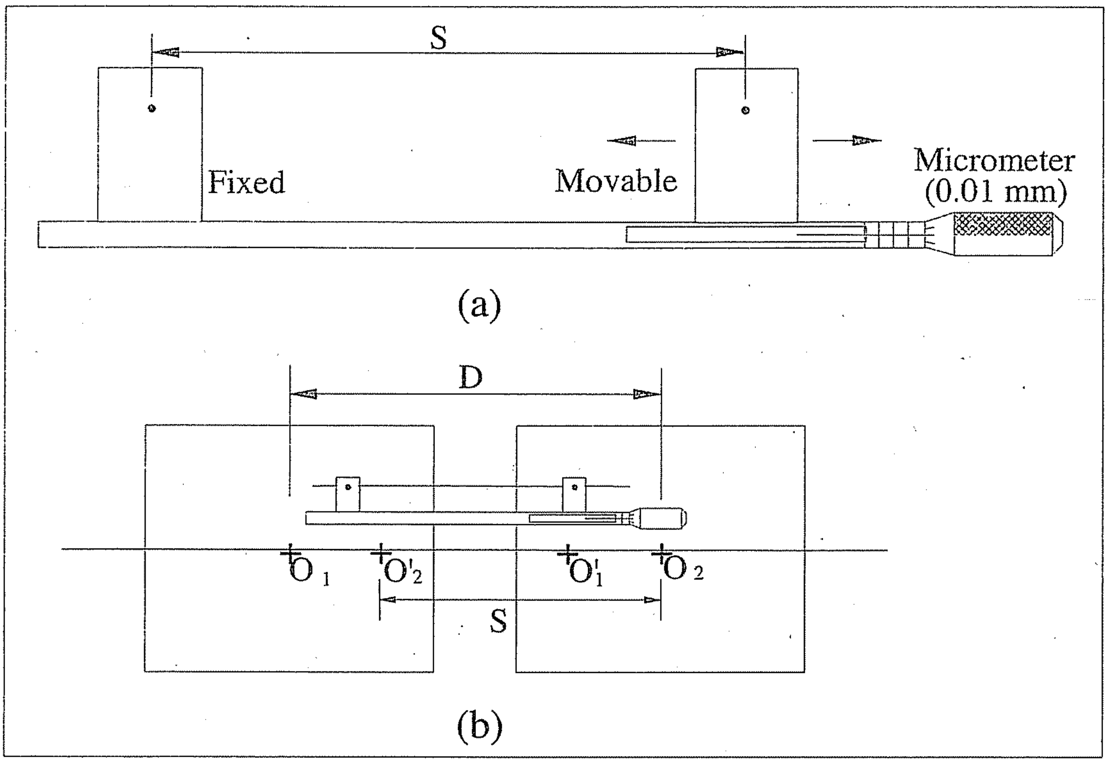
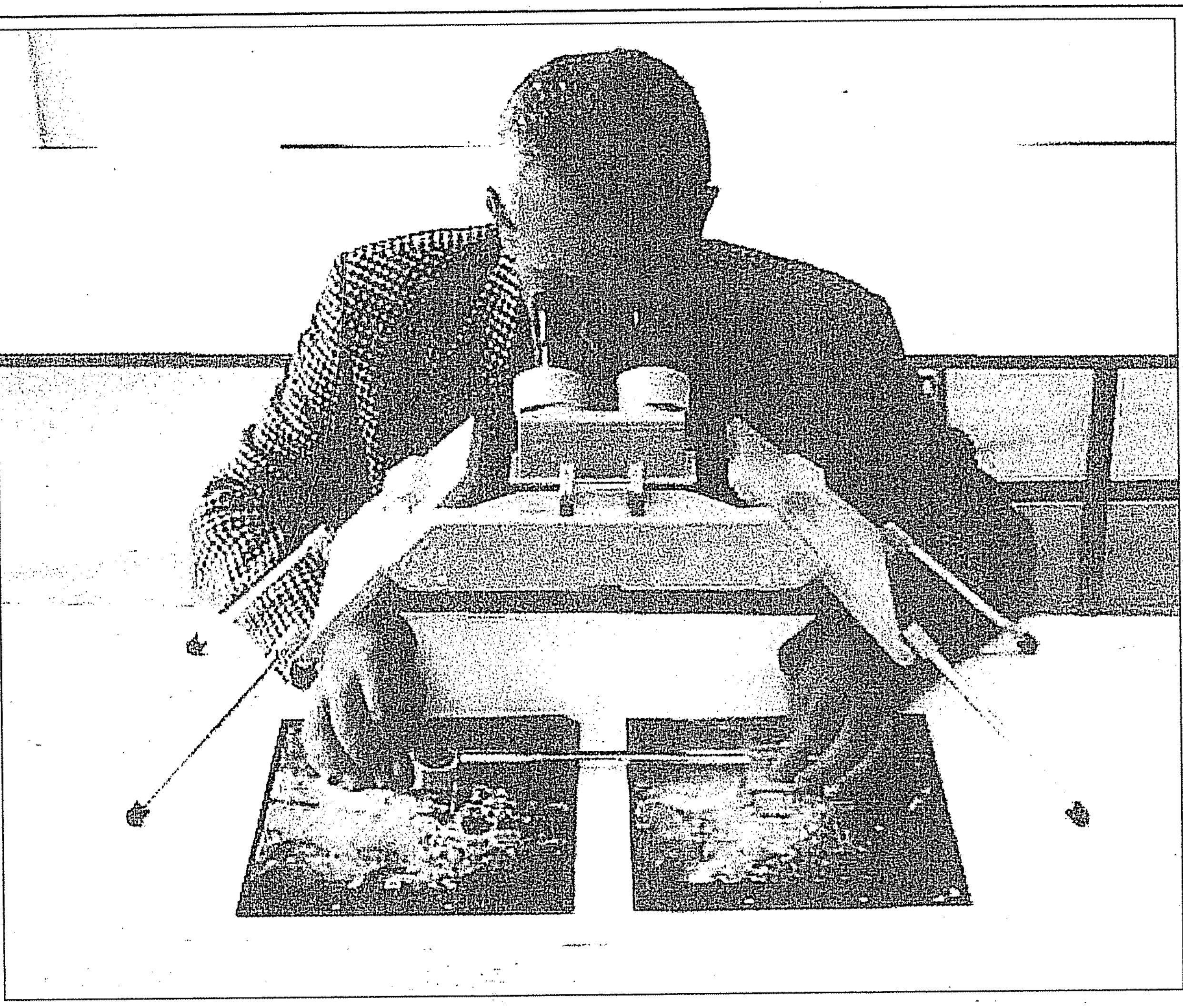


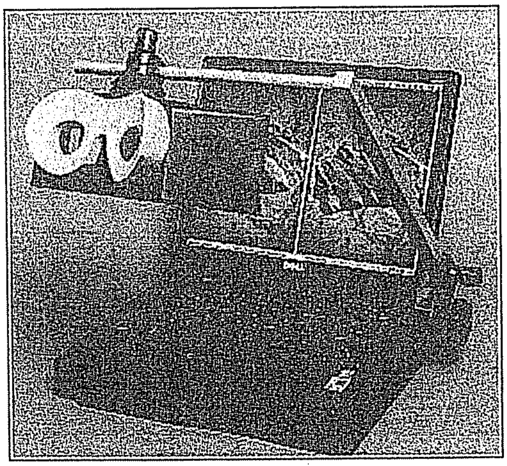
FIGURE 11.14: Diagram of parallax bar.

The distance between the centers of the two photographs, denoted by D in Figure 11.14(b), is obtained by having a comfortable stereoscopic view under the mirror stereoscope for paper photographs (Figure 11.15a) or in front of a screenscope for digital photographs (Figure 11.15 b & c). This distance varies from one observer to another.

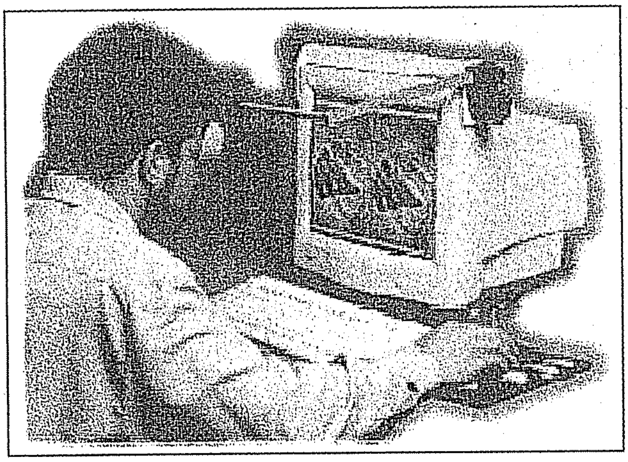
In Figure 11.16, let it be known that the parallax (p) of point m is 73.22 mm (see next section). The parallax bar of Figure 11.14(a) is placed over the photographs at m and m' which represent the same point. A micrometer reading r = 12.10 mm is taken. Now subtracting the reading r from the parallax p gives a constant, denoted by C, for this particular photograph setup and parallax bar. Thus,



(a) Parallax bar used under a mirror stereoscope.



(b) Laptop screenscope



(c) PC screenscope

FIGURE 11.15: Screen and mirror stereoscopes.

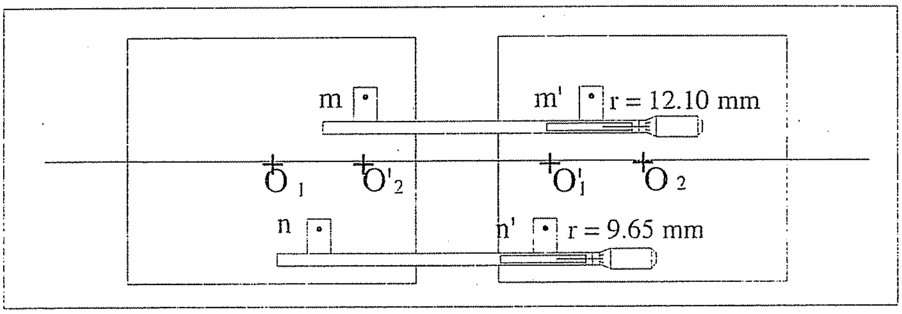


FIGURE 11.16: Micrometer readings of parallax bar.

$$C = p - r \qquad (11.12)$$

In the preceding situation,

$$C = 73.22 - 12.10 = 61.12 \text{ mm.}$$

The parallax of point n is obtained by setting the reference marks in apparent contact with the stereoscopic image at n, i.e. at n and n', reading the micrometer and combining the reading 9.65 mm shown in Figure 11.16 with the constant in accordance with the following relationship:

$$p = C + r \qquad (11.13)$$

For point n, $P_n = 61.12 + 9.65 = 70.77$ mm. It should be noted that the value of C will remain fixed for a given positioning of the photographs, a given parallax bar, and a given position of the fixed mark along the bar. If any of these three factors is changed, a new value of C must be determined, as just discussed. Once the constant is obtained, the parallaxes of all points to be measured are then determined using Equation (11.13).

11.10 PARALLAX OF THE PRINCIPAL POINTS

The direct determination of the parallax of a point by means of a parallax bar requires that the parallax of one point be known, as explained in the previous section. If for example, the image of a benchmark (whose elevation is known) appears in the photograph, then the parallax for this point can be calculated from Equation (11.10). This parallax can be used as explained in the previous section for measuring the parallaxes of other points.

The parallaxes of two other points, namely the two principal points, may be easily determined. In Figure 11.17, for example, the parallax of the left principal point O_1 is the x-coordinate of O_1 minus the x'-coordinate of O'_1. Since the x-coordinate of O_1 is 0 and the x'-coordinate of O'_1 is -b', the parallax of O_1 is 0 - (-b') = b'. Similarly, the parallax of the right principal point O_2 is the x-coordinate of O'_2 minus the x'-coordinate of O_2, or b" - 0 = b". Note that b" is larger than b' because the ground point O_2 lies at a greater elevation than does O_1.

Before the parallax bar is used, the distances b" and b' should be measured carefully. Two values of C may then be based on known parallaxes of the two principal points. Because of tilts of the photographs, unequal flying heights, differential paper shrinkage and scaling errors, the two values of the constant C will rarely be the same. Unless otherwise justified, an average value of the constant should be used in Equation (11.13).

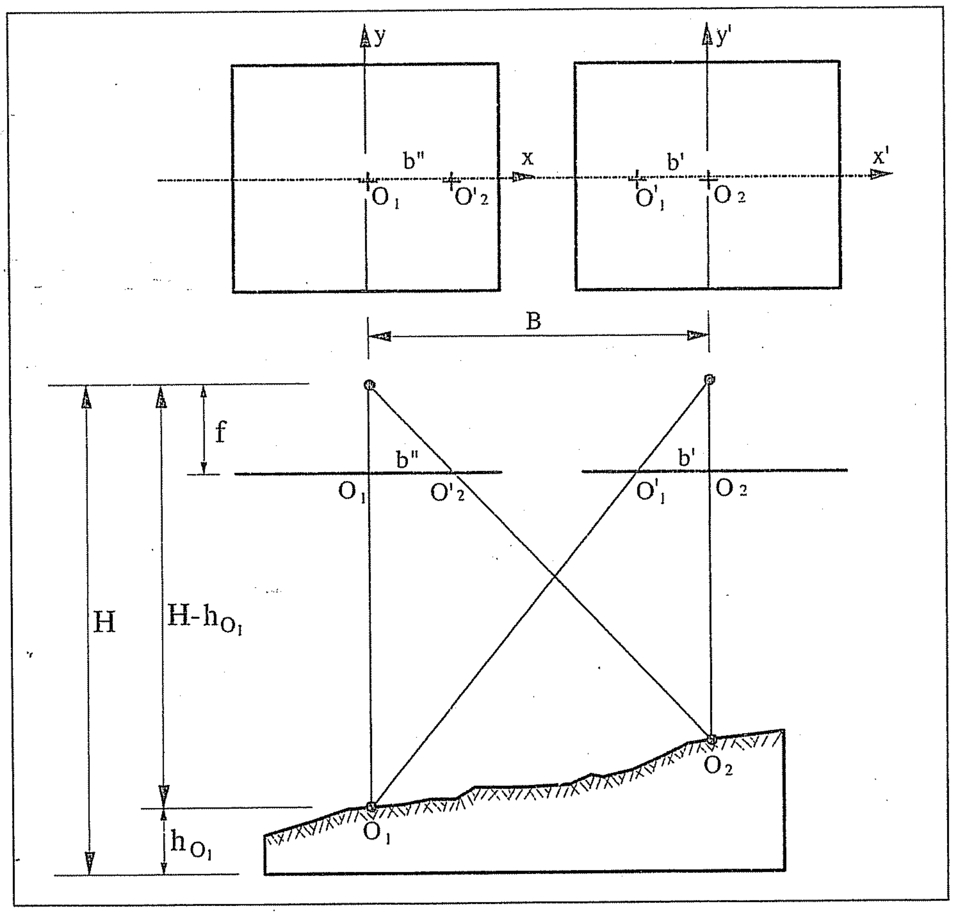


FIGURE 11.17: Parallaxes of the principal points.

11.11 ELEVATIONS FROM KNOWN AIR BASE AND ONE CONTROL POINT

Assume that a pair of overlapping vertical photographs, the air base of which is known, has been oriented for parallax bar measurements. The elevation of a control point designated as 224B has been determined by field surveys. The elevations of five points are to be determined from their parallax measurements. Table 11.1 gives the parallax bar readings on the points, together with other pertinent data.

The parallaxes of the points are determined by substituting the constant C and the readings in Equation (11.13). These results are shown in the right-hand column along with the data for the problem.

By using the parallax and the known elevation of the control point together with the air base and the focal length of the camera, the flying height above the datum (sea level) is determined using Equation (11.10). Thus,

This soft-copy scanned by Eng. Mohammed F. Al-Hallaq & Eng. Samer T. Saqallah

$$H = h + \frac{B}{p} \cdot f = 610.64 + \frac{530\text{m}}{90.82\text{mm}} \cdot 152.40\text{mm} = 1500.00 \text{ m AMSL}$$

By applying Equation (11.10) to the parallax of each point, the elevations of the points are obtained. The results of these calculations are arranged in Table 11.2.

TABLE 11.1: Parallax bar readings.

Data	Point	Parallax Bar Reading, r (mm)	Parallax p (mm)
Focal Length f = 6.0" = 152.4 mm	224B	12.62	90.82
Air Base (B) = 530.00 m	1	14.04	92.24
Elevation of 224B = 610.00 m	2	10.91	89.11
Constant C = 78.20 mm	3	11.02	89.22
	4	13.56	91.76
	5	9.76	87.96

TABLE 11.2: Computation of elevations from parallaxes.

Point	P (mm)	$\frac{B}{p} \cdot f$ (m)	$h = H - \frac{B}{p} \cdot f$ (m)
1	92.24	875.67	624.33
2	89.11	906.43	593.57
3	89.22	905.31	594.69
4	91.76	880.25	619.75
5	89.96	897.87	602.13

Now assume that the known elevation of the control point is h_N and it is required to determine the elevation h_K of an unknown point K. From Equation (11.10):

$$h_N = H - \frac{B}{p_N} \cdot f \qquad \text{.................................(11.14)}$$

$$h_K = H - \frac{B}{p_K} \cdot f \qquad \text{.................................(11.15)}$$

Where p_N and p_K are the parallaxes of points N and K respectively.

Subtracting Equation (11.14) from (11.15):

$$\Rightarrow h_K - h_N = \frac{B \cdot f}{p_N} - \frac{B \cdot f}{p_K} = B \cdot f\left(\frac{1}{p_N} - \frac{1}{p_K}\right) = B \cdot f\left(\frac{p_K - p_N}{p_N \cdot p_K}\right)$$

$$\Rightarrow h_K = h_N + B \cdot f\left(\frac{p_K - p_N}{p_N \cdot p_K}\right) \qquad \text{.......................(11.16)}$$

From Equation (11.10):

$$p_N = \frac{B \cdot f}{H - h_N}, \qquad p_K = \frac{B \cdot f}{H - h_K}$$

Substituting these two relations in Equation (11.16):

$$\Rightarrow \quad h_K = h_N + B \cdot f \left(\frac{p_K - p_N}{\dfrac{B \cdot f}{H - h_N} \cdot \dfrac{B \cdot f}{H - h_K}} \right)$$

In Figure 11.14, if $b = \dfrac{b' + b''}{2}$ = air base measured on the photographs, it is clear that:

$$B \cong \frac{H - h_K}{f} \cdot b \cong \frac{H - h_N}{f} \cdot b$$

Also $H - h_K \cong H - h_N$
Substituting,

$$\Rightarrow \quad h_K = h_N + \frac{H - h_N}{b}\left(p_K - p_N\right) \qquad \text{.................(11.17)}$$

The term $(H - h_N)$ can be approximated by $(H - h_{avg})$. Also Δp can be substituted for $(p_K - p_N)$. Then:

$$h_K = h_N + \frac{H - h_{avg}}{b} \cdot \Delta p \qquad \text{.............................(11.18)}$$

In equation (11.18), Δp can be found by subtracting the parallax bar reading on the known control point from the parallax bar reading on the unknown point K.

EXAMPLE 11.2:

In Figure 11.17, it was found that: $b'' = 105.42$ mm, $b' = 105.92$ mm.

The distance between two ground control points A & B which appeared on both photographs is L = 226.68 m. The corresponding distances measured on the two photos are: $\ell_1 = 87.63$ mm and $\ell_2 = 89.15$ mm. Also:

Ground elevation of control point N = $h_N = 224.20$ m

Average ground elevation (h_{avg}) = 222.00 m

Focal length of Camera (f) = 152.4 mm

Parallax bar readings at points N, K and G are:

At N = 12.62 mm

K = 14.04 mm

G = 10.91 mm

Compute the ground elevation of points K and G.

SOLUTION:

$$b = \frac{105.42 + 105.92}{2} = 105.67 \text{ mm}$$

$$\text{Average distance measured on the photographs} = \frac{87.63 + 89.15}{2} = 88.39 \text{ mm}$$

$$\text{Average photo scale} = \frac{\text{Photo Distance}}{\text{Ground Distance}} = \frac{88.39 \times 10^{-3}}{226.68} = \frac{1}{2565}$$

From Equation (11.3)

$$\frac{1}{\left(\dfrac{H - 222.00}{152.4 \times 10^{-3}}\right)} = \frac{1}{2565} \Rightarrow H = 612.84 \text{ m}$$

From Equation (11.17)

$$h_K = 224.20 + \frac{612.84 - 224.20}{105.67}(14.04 - 12.62) = 229.42 \text{ m}$$

$$h_G = 224.20 + \frac{612.84 - 224.20}{105.67}(10.91 - 12.62) = 217.91 \text{ m}$$

11.12 ORTHOPHOTOS

Orthophotos can be defined as photographs that have been processed in a way to remove the perspective aspect from them. As implied by their name, orthophotos are orthographic photographs. They do not contain the distortions of scale, tilt and relief characterizing normal aerial photographs (see section 11.2). In essence, orthophotos are "orthomaps". Figure 11.18a shows an example of an aerial photograph of a power line traversing over a hilly terrain, while Figure 11.18b shows its orthophoto with the true or actual path of the power line after the image displacement due to relief has been removed.

Like maps, orthophotos have a uniform scale (even in varying terrain), and like photographs, they show the terrain in actual detail (not by lines and symbols, but by images). Hence, orthophotos give the planner and analyst the "best of both worlds"—a product that can be readily interpreted like a photograph, but one on which true distances, angles, and areas may be measured directly. Because of these advantages, orthophotos make excellent base maps for compiling data to be input in a GIS (see Chapter 13) or overlaying and editing data already incorporated in a GIS. They also enhance the communication of spatial data, since data users can often relate better to an orthophoto than a conventional line and symbol map or display. Moreover, orthophotos are useful in land use planning, urban and regional developments, engineering feasibility studies, damage and hazard assessment as well as others.

Orthophotos are generated from overlapping conventional photos in a process called *differential rectification*. The process basically involves the division of an aerial photograph into rectangular patches measuring as small as a few millimeters in each dimension, and then each patch is individually corrected for tilts of the camera and slope of the terrain. It is beyond the scope of this book to describe how orthophotos are produced, but the reader can refer to photogrammetry books for more detail.

Orthophotomaps are orthophotos that have undergone cartographic treatments to add regular map information such as contours, place and street names, and map symbols. Much time is saved in the preparation of such maps because the instrument operator does not need to map the planimetric details in the map compilation process.

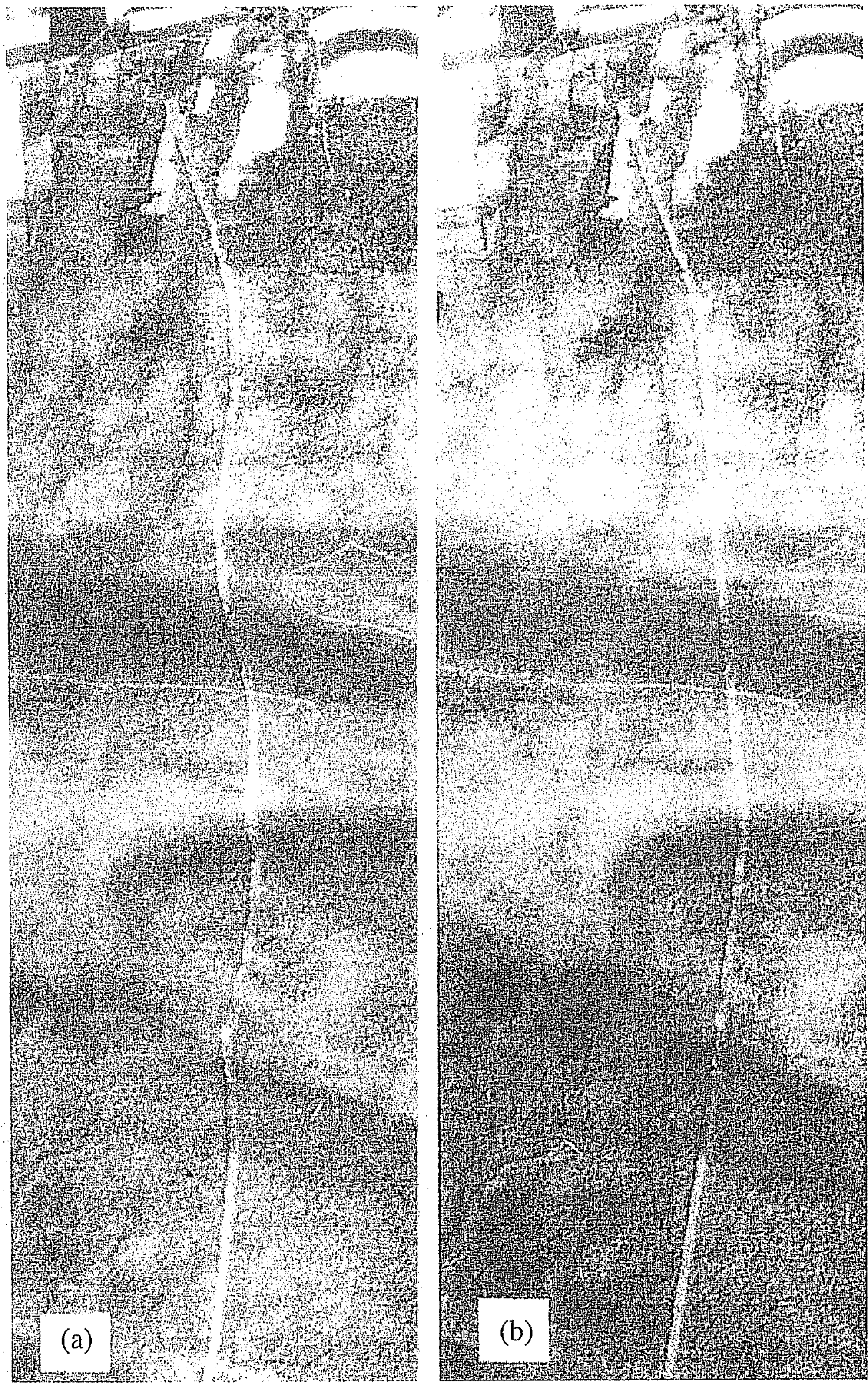


FIGURE 11.18: Portion of (a) a perspective aerial photograph and (b) an orthophoto showing a power line traversing hilly terrain.

11.13 FLIGHT MAP

A flight map, as shown in Figure 11.19, gives the project boundaries and flight lines the pilot must fly to obtain the desired coverage. Aerial photographs are usually taken in early spring or late autumn to minimize visual obstructions by trees and snow. Only clear, cloudless days are acceptable for aerial photography. To avoid long shadows caused by tall objects such as trees, towers and buildings, photography must be done during the part of the day when the sun is at least 30° above the horizon.

Figure 11.19 shows the normal geometric configuration for aerial photography in topographic mapping. The aerial photographs are taken along straight lines to form strips of photographs that overlap each other (Figure 11.20a). The flight lines are usually oriented along the longest dimension of the area in order to minimize the number of times the aircraft is required to turn around.

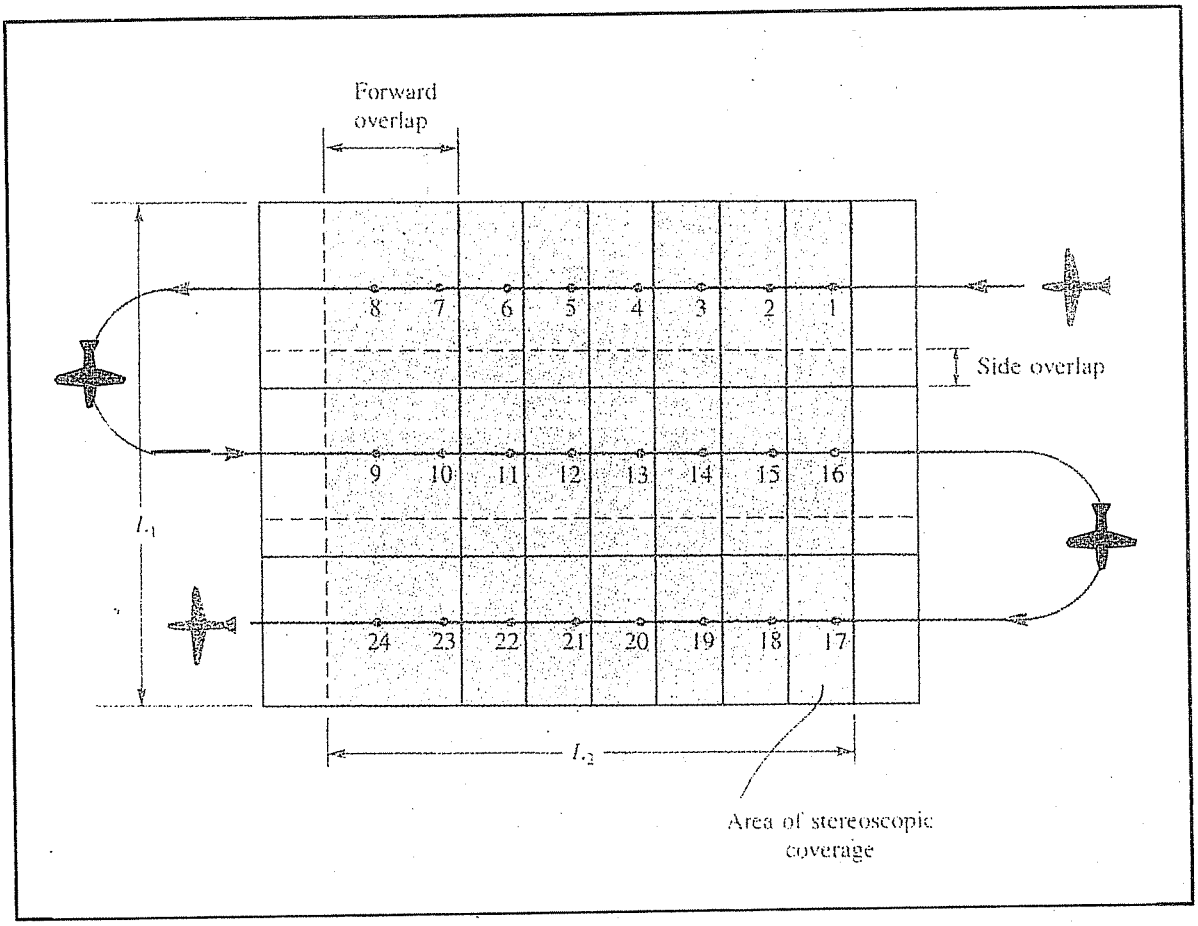
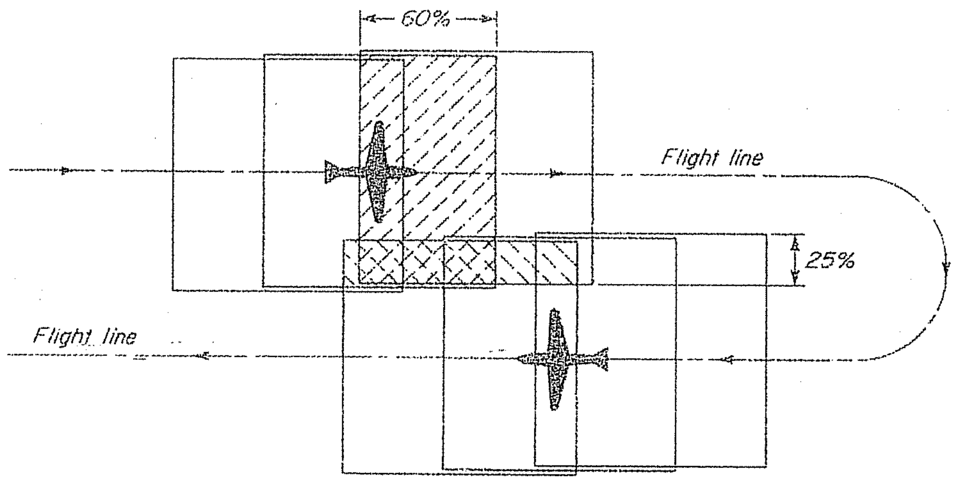


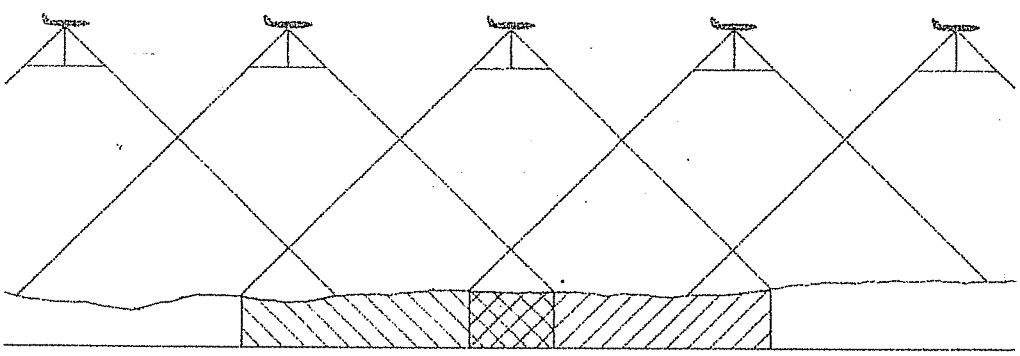
FIGURE 11.19: Geometric configuration for aerial photography.

Along each flight line, adjacent photographs overlap each other usually by about 60 to 65%. This is called the forward overlap (Figure 11.20b). The overlap between adjacent strips of photographs is called the side overlap, and usually ranges from 20 to 30%. The amount of overlap is measured with respect to the dimension of the photograph. Thus, for a 60% forward overlap, adjacent photographs along a strip would overlap each other by 9" x 0.6 = 5.4" for standard 9" x 9" photographs.

Referring to Figure 11.20, let O_f and O_s, respectively, represent the forward and side overlaps expressed as fractional parts of the dimension of the photo. For example, if the forward and side overlaps are 60% and 25% respectively, then $O_f = 0.6$ and $O_s = 0.25$. Furthermore, let W represent the ground distance covered by the width of a single photograph. Then,



(a) Photographic overlap.



(b) Forward overlap along flight line.

FIGURE 11.20: Side and forward overlap in aerial photography.

$$D_f = (1 - O_f)W \qquad (11.19)$$

And,

$$D_s = (1 - O_s)W \qquad (11.20)$$

Where D_f is the ground distance between adjacent exposures along a flight line (equal to the air base B), and D_s is the ground distance between adjacent flight lines.

The following expressions can also be derived from the geometry illustrated in Figure 11.20:

$$N_S = \frac{L_1 - W}{(1 - O_s)W} + 1 \qquad (11.21)$$

And,

$$N_p = \frac{L_2}{(1 - O_f)W} + 2 \qquad (11.22)$$

Where N_s denotes the number of flight lines (strips), N_p denotes the number of photographs per flight line, and L_1 and L_2 are the outside dimensions of the project area to be photographed. The 1 in Equation (11.21) is added to take into account that at least one flight line is needed for any photography. That is why W was subtracted from L_1 in the numerator. The 2 was added in Equation (11.22) to provide for the two extra photographs needed in each flight line to provide stereoscopic vision for the area at the beginning and end of the flight line.

EXAMPLE 11.3:

Aerial photographs are required to provide full stereoscopic coverage of a rectangular area that measures 7 km in the east-west direction and 5 km in the north-south direction. The desired average scale of the vertical photograph is 1:9600. The terrain is relatively flat and has an average elevation of 230 m above sea level. A 152.4 mm focal length camera with a 22.86 cm x 22.86 cm picture format is to be used. The flight lines are to be along the east-west direction. Forward and side overlaps are to be 60% and 25% respectively. Determine:

a) The flight height above sea level
b) The ground area, in hectares, covered by each photograph
c) The number of photographs required.
d) The time interval between exposures if the aircraft speed is to be 160 km/hr.

SOLUTION:

a) From Equation (11.3)

$$\frac{1}{\left(\frac{H - h_{avg}}{f}\right)} = \frac{1}{9600}$$

$$\Rightarrow H = 9600 \times 152.4 \times 10^{-3} + 230 = 1693 \text{ m}$$

b) Ground dimension of photo (W) = (22.86/100) x 9600 = 2194.56 m
Ground coverage per photo = W^2 = 2194.56 x 2194.56 m^2
= 481.61 Ha

c) $N_s = \frac{5000 - 2194.56}{(1-0.25)\cdot 2194.56} + 1 = 2.70 = 3$ lines

$N_p = \frac{7000}{(1-0.6)\cdot 2194.56} + 2 = 9.97 = 10$ photos

Total number of photographs required = 3 x 10 = 30 photos

d) Aircraft speed = 160 km/hr = 44.44 m/sec
Ground distance between exposures = (1 - 0.6) 2194.56 = 877.82 m
Time interval between exposures = (877.82/44.44) = 19.8 seconds

PROBLEMS

11.1 Compare between a map and a photograph.

11.2 Vertical photography having an average scale of 1:2400 is desired for highway design purposes. If the focal length of the camera lens is 6 in., at what height above average terrain elevation should the mission be flown?

11.3 The image of a radio tower is portrayed in a 1:7,500 scale vertical photograph, which was obtained with a 6-in. focal-length camera. The radial distances from the principal point to the images of the bottom and top of the tower are 70.36 mm and 79.87 mm respectively. Calculate the height of the tower.

11.4 The average height of an aerial camera above the terrain during a photographic mission is 8,200 ft and the air base of a stereo pair is 4.27 in. If the difference in parallax of the images of two features is 0.123 in., find the difference in elevation between them.

11.5 What are the ideal conditions for obtaining aerial stereoscopic views of the ground surface?

11.6 Describe how to obtain comfortable stereoscopic view under the mirror stereoscope for parallax bar measurements.

11.7 The distance between two ground control points A and B is 3337.8 m. The elevation of A is 208.9 m. When a pair of overlapping vertical photographs containing the images of the two control points are oriented under a stereoscope, the parallaxes of a and b found by parallax-bar measurements are p_a = 72.21 mm and p_b = 74.16 mm. The coordinates of a and b measured on the left-hand photograph with respect to the flight line as the x-axis are as follows: x_a = +42.28 mm and y_a = 50.60 mm, x_b = +67.66 mm and y_b = -70.15 mm. The focal length of the camera was 100.60 mm. Determine the length of the air base, the flying height, and the elevation of point B.

11.8 The difference in parallax between a point located at mean sea level and another point on a hill is measured and found to be 2.60 mm. The flying height is 2090 m above mean sea level, the air base is 895 m, and the focal length of the camera lens is 209.5 mm. Determine the elevation of the point on the hill.

11.9 Describe how to measure the x-parallax for the two principal points for a pair of overlapping photographs.

11.10 A parallax difference of 0.60 mm is measured between the image of the top of a water tank and the image of a point near the base of the tank. The flying height is 4875 m above the ground. The average of the distances between the principal points and the conjugate principal points, measured on the photographs is 77.6 mm. Calculate the approximate height of the tank.

11.11 Readings made with a parallax bar on four points appearing in the overlap area of a pair of vertical photographs are as follows:

Point	Parallax Bar Reading (mm)
a	20.25
b	21.15
c	21.50
d	18.20

The elevation of A is 294.1 m, and the parallax of *a* is 70.40 mm. The camera focal length is 153.08 mm. The scale of the photograph at the elevation of A is 1:14,000. Compute:

a) The flying height above sea level for the photographs.
b) The air base of the photographs.
c) The elevations of points B, C and D.

11.12 What are the factors that should be taken into consideration when planning aerial photography?

11.13 An aerial photography at a scale of 1:25,000 is required of an area whose average height above mean sea level is 700 m. Given that the camera lens focal length is 153 mm, the forward and side overlaps are 65% and 25% respectively, and that the picture format is 230 by 230 mm. Calculate:

a) The flight height above mean sea level.
b) The forward distance between exposures.
c) The distance between the flight lines.
d) The area covered by each photograph.
e) The ground area, expressed in Donums and Hectares, covered by a pair of overlapping photographs.

11.14 Aerial photographs are to be flown to provide stereoscopic coverage of a rectangular project area that is 11.25 miles long (north-south) and 6.50 miles wide (east-west). The desired average scale of the vertical photography is 1:10,000, the focal length of the lens is 6 in., and the picture format is 9 in x 9 in. A forward overlap of 60% and side overlap of 30% are specified. The average terrain elevation is 1,050 ft. Flight lines are to be directed in a north-south direction. Determine the following:

a) Flight altitude above sea level
b) Ground area in acres covered by each photograph
c) Number of flight lines required
d) Number of photographs per flight line.

12 GLOBAL POSITIONING SYSTEMS (GPS)

12.1 INTRODUCTION

The Global Positioning System (GPS) is a satellite-based technology that gives precise navigation and positional information, day or night, in most weather and terrain conditions. It is very much similar to the TV receiver where an antenna receives a signal from orbiting satellites which is, in turn, transformed into an image and sound that can be seen on the TV screen. An antenna in placed on the point for which the position (coordinates) is be measured. This antenna receives signals from orbiting satellites loaded with positional data, the signal being transformed into positional data by a GPS receiver that is connected to the antenna (Figure 12.1). Because it is becoming inexpensive, accurate and easy to use, GPS has significantly changed surveying, navigation, shipping, airline, transportation, and other fields including geographic information systems (Chapter 13).

The original intent of GPS was to provide military aircraft a method of world-wide navigation independent of ground station radio aids that are vulnerable to attack. As of 2001, there are two functioning satellite GPS systems and a third is in development:

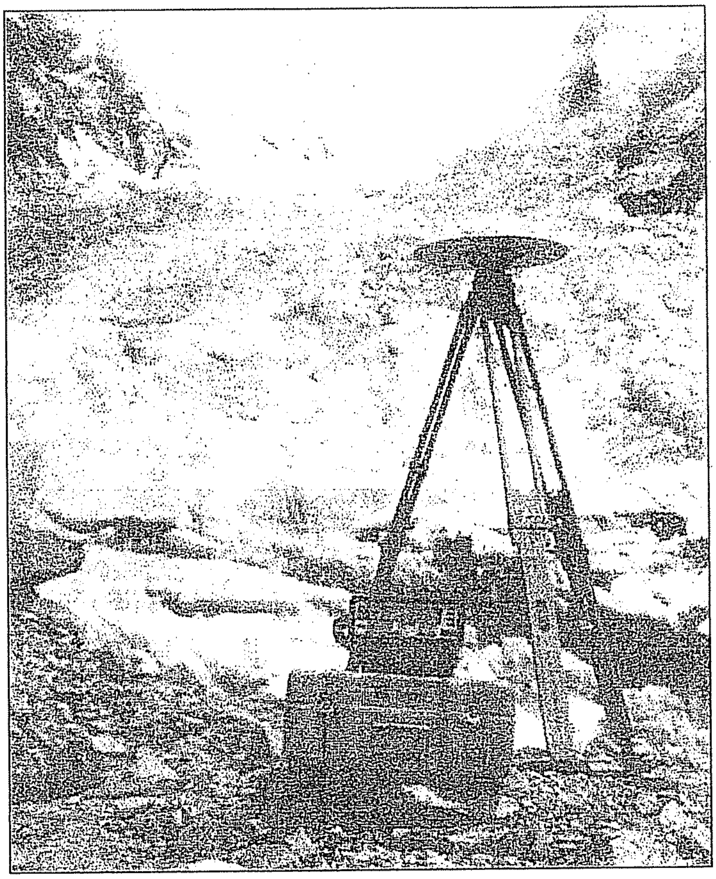


FIGURE 12.1: GPS receiver and antenna.

- NAVSTAR: this is the most commonly used system and has been developed by the US Department of Defense (DOD).
- GLONASS: this is a Russian system which is little used outside the former Soviet Union.
- Galileo: this is a non-military positioning system being financed, designed, and built by a consortium of European governments and industries.

Since NAVSTAR (NAVigation System with Timing And Ranging) is the most widely used system, it will be described as representative of the three above.

12.2 GPS COMPONENTS

The technical and operational characteristics of GPS are organized into three distinct components or segments: the space segment, the operational control segment, and the user equipment segment (Figure 12.2).

- The space segment. This segment consists of a constellation of 24 (21 active and 3 spares) satellites orbiting the earth at an altitude of approximately 20,000 km. These satellites are distributed among 6 offset orbital planes with 4 satellites in each plane (Figure 12.3). Every satellite orbits the earth twice daily and stays usually 8 or more hours above the flat horizon. Usually, 4 to 8 active satellites are typically visible from any unobstructed viewing location on earth.

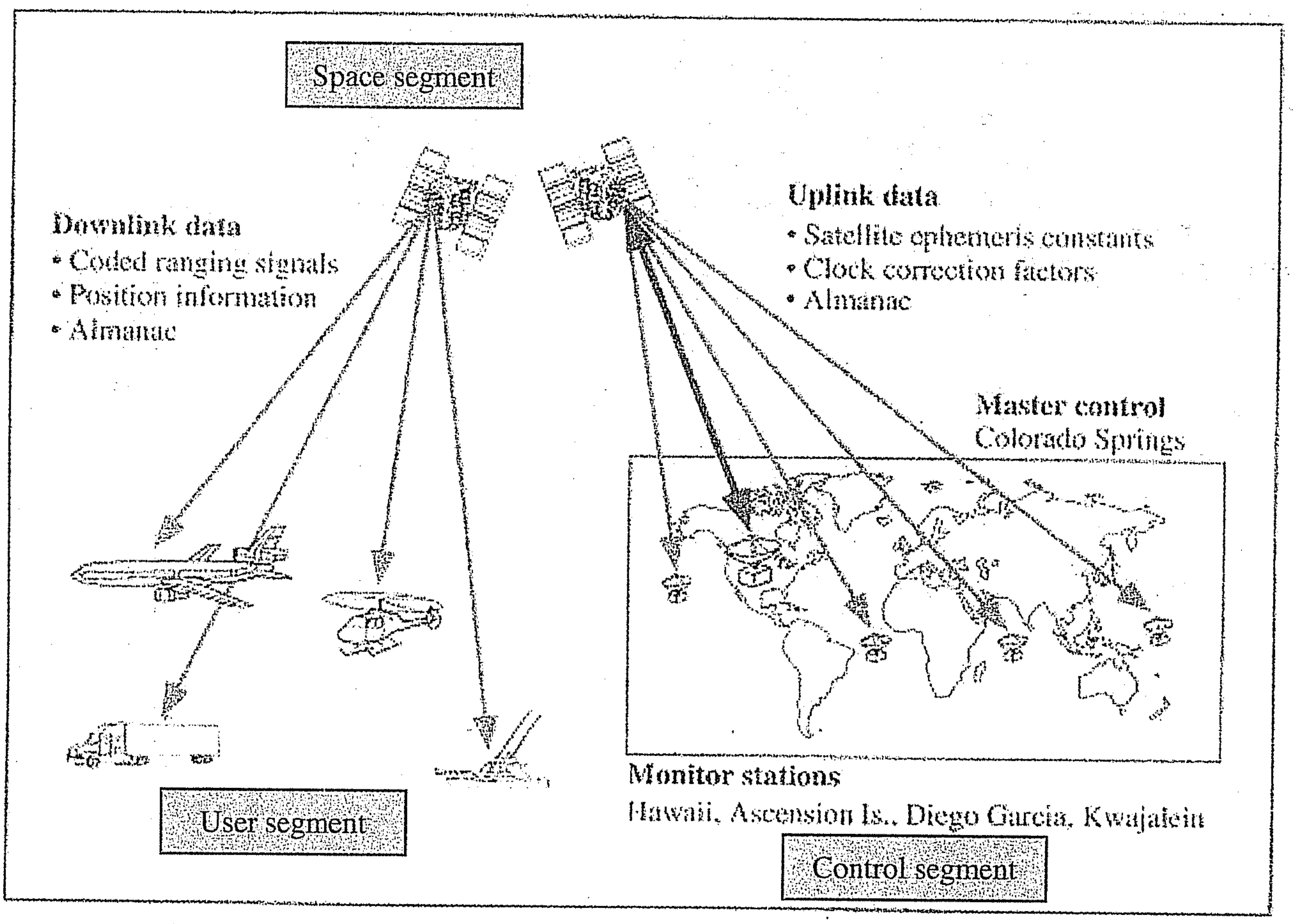


FIGURE 12.2: Segments of GPS system.

Satellite segment
21+ satellites
6 orbital planes
12 hour return interval for each satellite

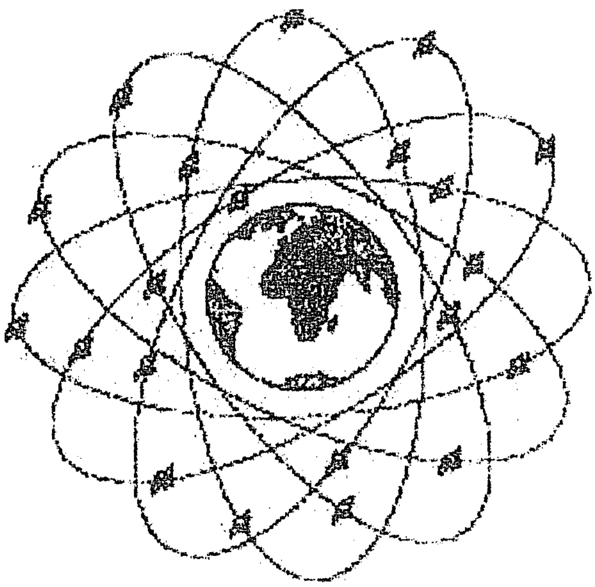


FIGURE 12.3: Satellite orbit characteristics for the GPS constellation.

- <u>The control segment.</u> This segment consists of the tracking, communications, data gathering, integration, analysis, and control facilities (Figure 12.2). There are 5 tracking stations spread across the earth, with a master control station in Colorado Springs in the US.

- <u>The user segment.</u> This segment comprises the set of individuals with one or more GPS receivers. A GPS receiver is a device that records data transmitted by each satellite and processes these data to obtain a 3-dimensional coordinates. These data include: satellite status and health, location in the GPS constellation, clock corrections as well as other data.

12.3 GPS BROADCAST SIGNALS

GPS positioning is based on radio signals broadcast by each satellite. The satellites broadcast at two fundamental frequencies. The first is 10.23 MHZ (called precision or P-code with a wavelength = 29.3 m), and is mainly used for military purposes. The second is 1.023 MHZ (called Coarse/Acquisition or C/A-code with a wavelength of 293 m) and is used for civil purposes.

These signals are modulated and carried on two high microwave frequencies denoted by L1 and L2, where L1=1575.42 MHZ and L2=1227.60 MHZ (Figure 2.4). The C/A is modulated onto the L1 carrier, whereas the P-code is modulated onto both L1 and L2. Such high frequencies help reduce the ionospheric effect among other benefits.

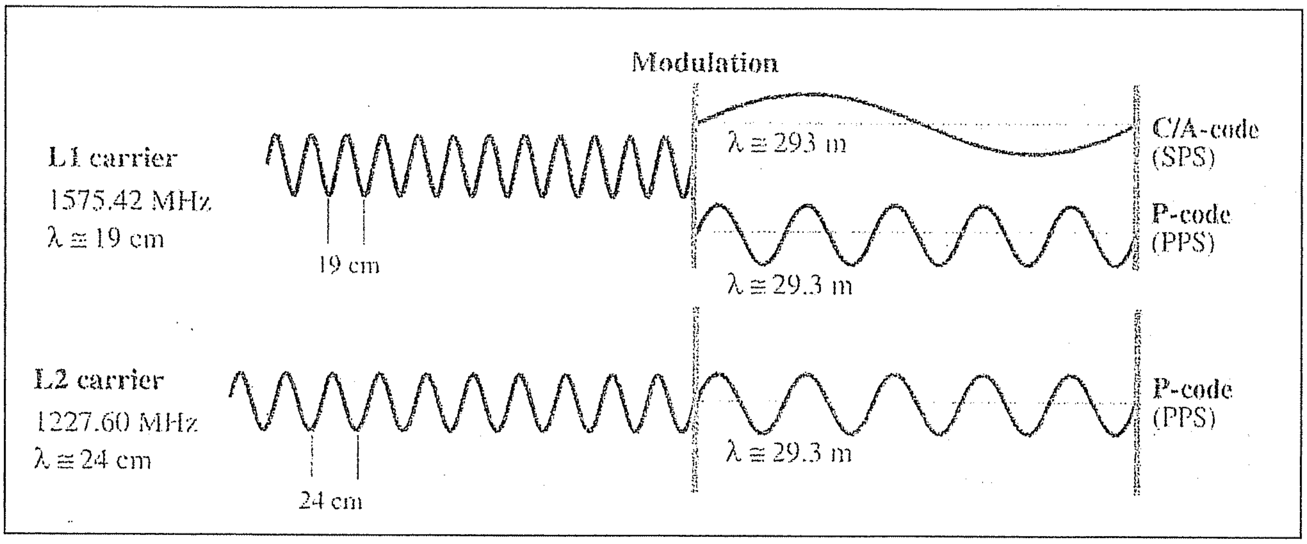
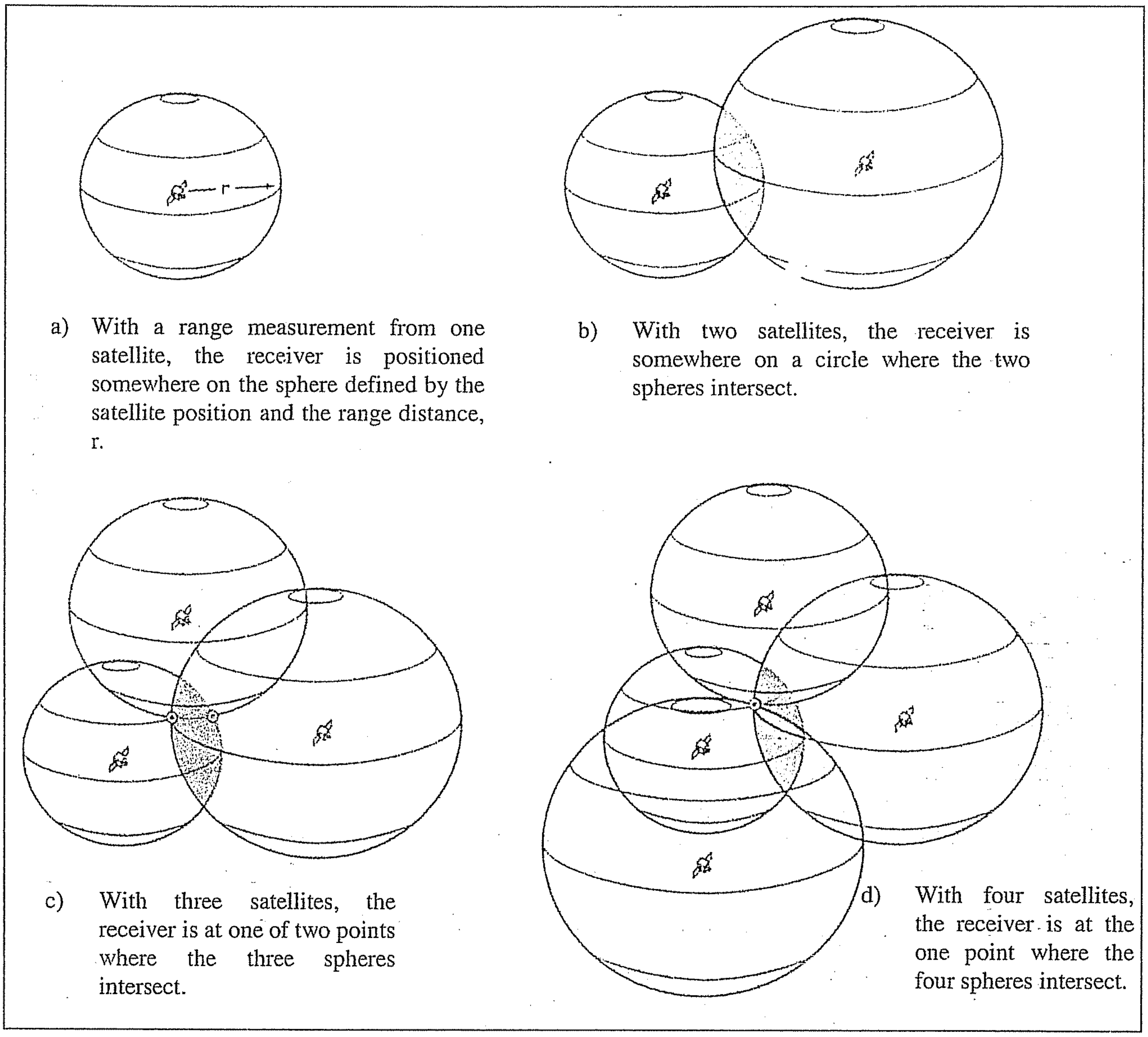


FIGURE 12.4: GPS satellite signals.

In order for the receiver to calculate the position of the point where it is located, it must know where the satellites are. To do this in real time requires that the satellites broadcast this information. Accordingly, there is a message superimposed on both L1 and L2 carriers along with the P and C/A-codes. Each satellite broadcasts its own message, which consists of orbital information (the ephemeris) to be used in the position computation, the offset of its clock from GPS system time, and information on the health of the satellite and the expected accuracy of the range (distance between the satellite and receiver) measurements. The message also contains almanac data from the other satellites in the GPS constellation, as well as their health status and other information. The almanac data are used by the receiver to determine the location of each satellite.

12.4 SATELLITE POSITIONING

The GPS satellites provide the user with the capability of determining his position, expressed in Cartesian coordinates (X_r, Y_r, Z_r), or longitude (λ), latitude (ϕ) and height (H), as well as velocity for moving objects. Fundamentally, this is accomplished using the principles of resection by measuring ranges from the receiver occupying the unknown point to satellites in space. Geometrically, the procedure can be envisioned to work as shown in Figure 12.5. A range measurement from a single satellite restricts the receiver to a location somewhere on the surface of a sphere centered on the satellite (Figure 12.5.a). Range measurements from two satellites identify two spheres, and the receiver is located on the circle resulting from the intersection of the two spheres (Figure 12.5.b). Range measurements from three satellites define three spheres, and these three spheres will intersect at two points (Figure 12.5.c). A sequence of range measurements through time from three satellites will reveal that one of the two points remains stationary, while the other point moves rapidly through space. If system and receiver clocks were completely accurate, it would be possible to determine the position of a stationary receiver by taking measurements from three satellites over a short time interval. Simultaneous measurements from four satellites (Figure 12.5.d) are usually required to reduce receiver clock errors and to allow instantaneous position measurement with a moving receiver, e.g., on a plane, in a car, or while walking. Data may also be collected from more than four satellites at a time and the position is obtained through a least squares adjustment solution. This usually improves the accuracy of position measurement.



a) With a range measurement from one satellite, the receiver is positioned somewhere on the sphere defined by the satellite position and the range distance, r.

b) With two satellites, the receiver is somewhere on a circle where the two spheres intersect.

c) With three satellites, the receiver is at one of two points where the three spheres intersect.

d) With four satellites, the receiver is at the one point where the four spheres intersect.

FIGURE 12.5: Range measurements from multiple GPS satellites. Range measurements are combined to narrow down the position of a GPS receiver.

Now think of the GPS satellites as frozen in space at a specific instant. The space coordinates (X_s, Y_s, Z_s) relative to the center of mass of the earth of each satellite are known from the ephemeris data received with the signal. If the ground receiver employed a clock that was set precisely to the GPS system time, the true distance (range) to each satellite could be accurately measured by recording the time required for the satellite signal to reach the receiver. Using this technique, ranges to only three satellites would be needed since the intersection of the three spheres whose radii are the respective ranges yields the three unknowns (X_r, Y_r, Z_r) and could be determined from three range equations. Figure 12.6 illustrates the problem and Equation (12.1) expresses the solution.

$$R_j = \sqrt{(X_{sj} - X_r)^2 + (Y_{sj} - Y_r)^2 + (Z_{sj} - Z_r)^2}\ ,\ j \geq 3 \quad \ldots\ldots\ldots \quad (12.1)$$

Unfortunately, there cannot be perfect synchronization between the satellite clocks and the receiver clock, and this clock offset or error introduces an unacceptable positional error. This error can be reduced or eliminated by receiving signals from four satellites which will provide us with four equations with four unknowns: the receiver three coordinates and the clock error (ΔT). Figure 12.7 illustrates the problem and Equations (12.2) express the mathematical model.

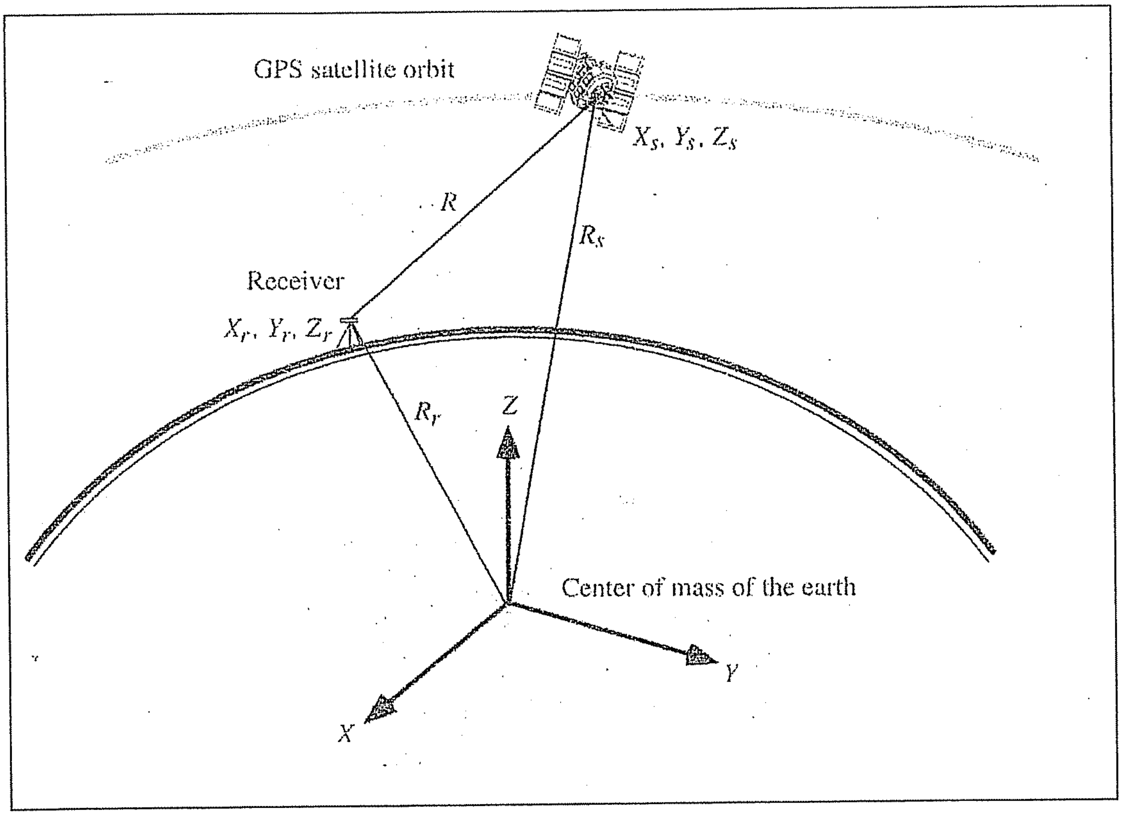


FIGURE 12.6: Range from satellite to ground receiver.

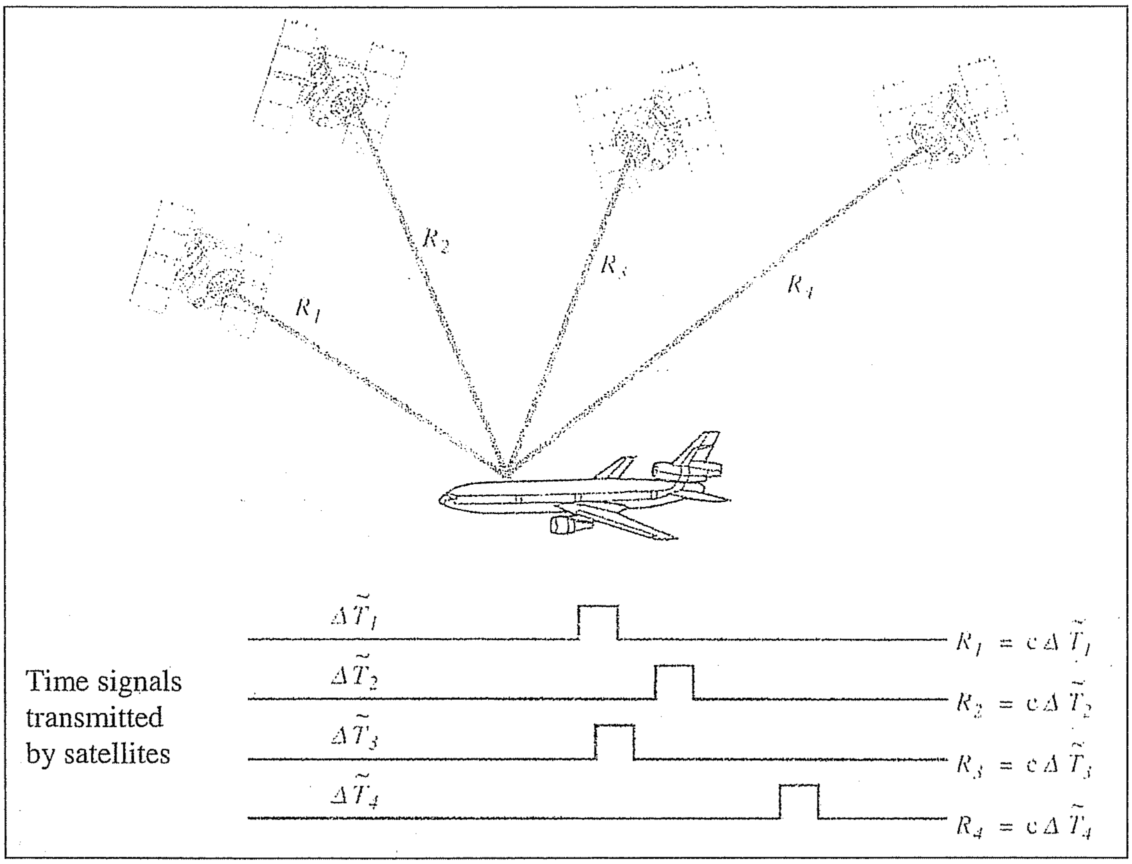


FIGURE 12.7: Receiver position solution using signals from four satellites.

$$\begin{aligned}
\sqrt{(X_1 - X_r)^2 + (Y_1 - Y_r)^2 + (Z_1 - Z_r)^2} &= R_1 - c.\Delta T_r \\
\sqrt{(X_2 - X_r)^2 + (Y_2 - Y_r)^2 + (Z_2 - Z_r)^2} &= R_2 - c.\Delta T_r \\
\sqrt{(X_3 - X_r)^2 + (Y_3 - Y_r)^2 + (Z_3 - Z_r)^2} &= R_3 - c.\Delta T_r \\
\sqrt{(X_4 - X_r)^2 + (Y_4 - Y_r)^2 + (Z_4 - Z_r)^2} &= R_4 - c.\Delta T_r
\end{aligned} \qquad \ldots\ldots\ldots\ldots (12.2)$$

Where c is the speed of light, and ΔT_r is the receiver clock error.

12.5 SOURCES OF RANGE ERROR AND POSITIONAL UNCERTAINTY

Errors in range measurements and satellite location introduce errors into GPS-determined positions. The sources of error can be simply classified into the following types:

1) Errors in ephemeris or orbital data of the satellite. This leads to erroneous estimate of the satellite position and hence an error in the calculated receiver position. Some of the ephemeris errors are deliberately imposed by the US DOD.

2) Ionospheric and atmospheric effect. The radio waves emitted by the satellites pass through the earth's ionosphere (25 to 250 miles high and is full of charged particles) and atmosphere (up to 25 miles high with varying temperature, pressure, humidity and density) and create a delay in the arrival time of the signals and hence an error in the range.

3) Receiver clock error. The receiver clock may contain biases and may not synchronize with the satellite clocks leading to an error in the calculated range. The receiver may also use algorithms that do not precisely calculate position.

4) Multi-path effect. Signals may reflect off of objects (such as buildings or vehicles) prior to reaching the receiver antenna. These signals travel further distance than direct GPS signals and hence leading to errors. This is a serious problem in urban populated areas and can be reduced using some screening techniques.

5) Satellite geometry. The geometry of the GPS satellite constellation in space is another major factor that affects positional accuracy. Given that each measured range to a satellite is associated with some uncertainty, these uncertainties combine together to form the shaded area in Figure 12.8 where the receiver position could be.

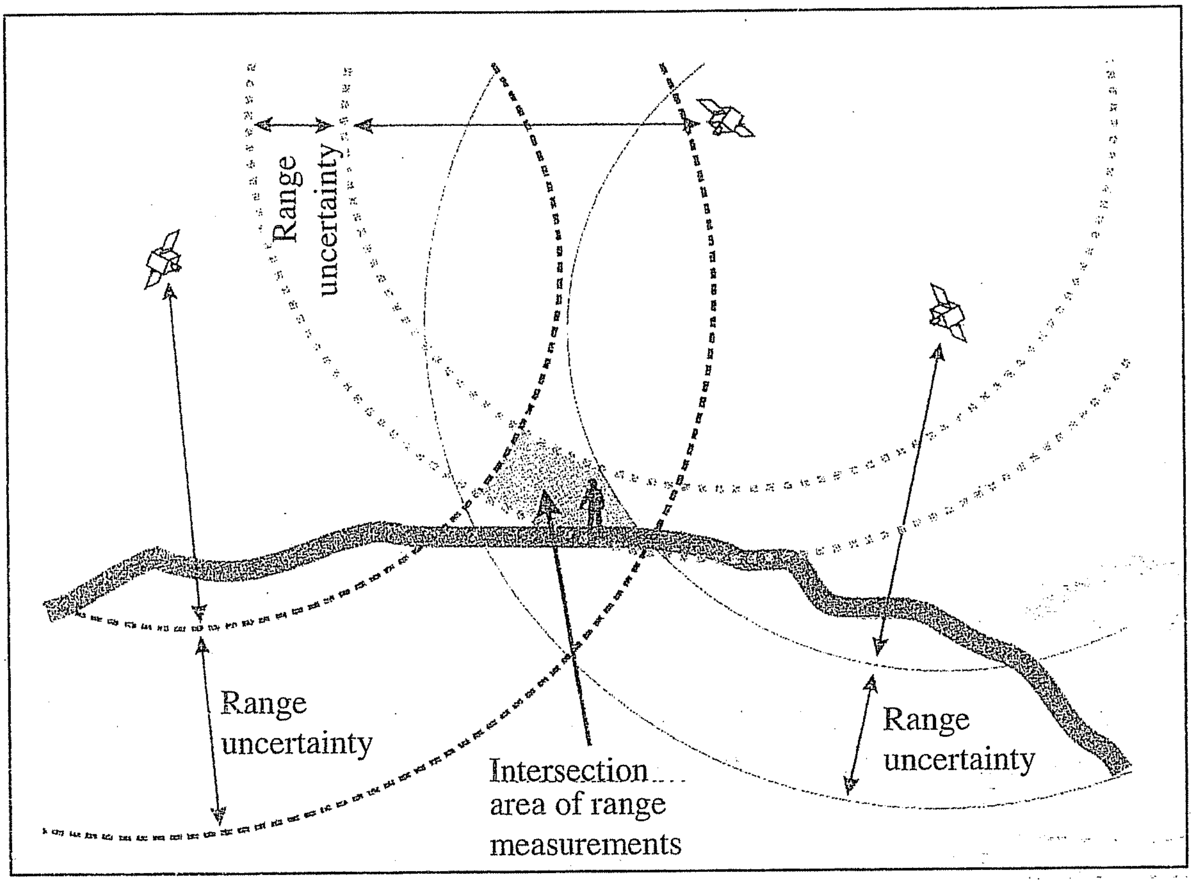


FIGURE 12.8: Effect of the GPS satellites distribution on positional accuracy.

Signals from satellites in close proximity overlap over a broad area, resulting in large areas of positional uncertainty. Widely spaced satellite constellations, on the other hand, provide more accurate GPS position measurements (Figure 12.9).

Satellite geometry is summarized in a number called the Positional Dilution of Precision (PDOP). This is defined as the ratio of an ideal tetrahedron to the actual volume of tetrahedron formed by 4 satellites. The ideal tetrahedron is formed by one satellite overhead and 3 satellites spaced at 120° intervals around the horizon. PDOP is equal to 1 or higher with 1 being the ideal. GPS receivers are capable of measuring the PDOP and accept the results only if it is within a threshold from 1.

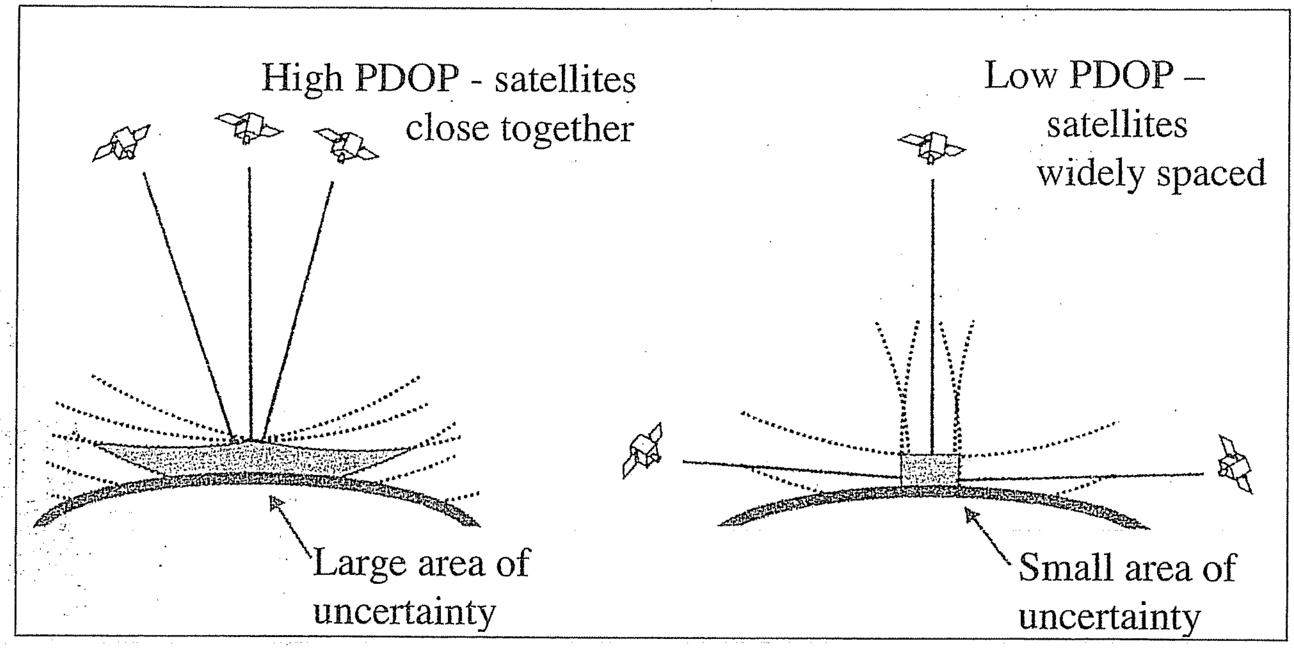


FIGURE 12.9: Effect of the spread of GPS satellites on the positional uncertainty.

The combined effect of the different types of error could lead to an error of about 50 m or more in the position of the receiver. This error may be acceptable for some applications such as when locating the position of a pilot whose airplane has been hit down by the enemy. Other applications, however, require much higher accuracy (on the order of few centimeters) such as when calculating the position of horizontal control points. This high accuracy comes at the cost of longer data collection period, and is most often obtained when using differential correction, a process described in the following section.

12.6 DIFFERENTIAL CORRECTION

In order to remove most of the range errors resulting in GPS positioning using a single receiver, an alternative method, known as *differential* GPS positioning is used. This technique employs two or more receivers. One receiver is located at a ground *base station* whose coordinates are precisely known using accurate survey methods. The other receiver (called the rover or the roving receiver) is located over the point whose coordinates need to be calculated (Figure 12.10).

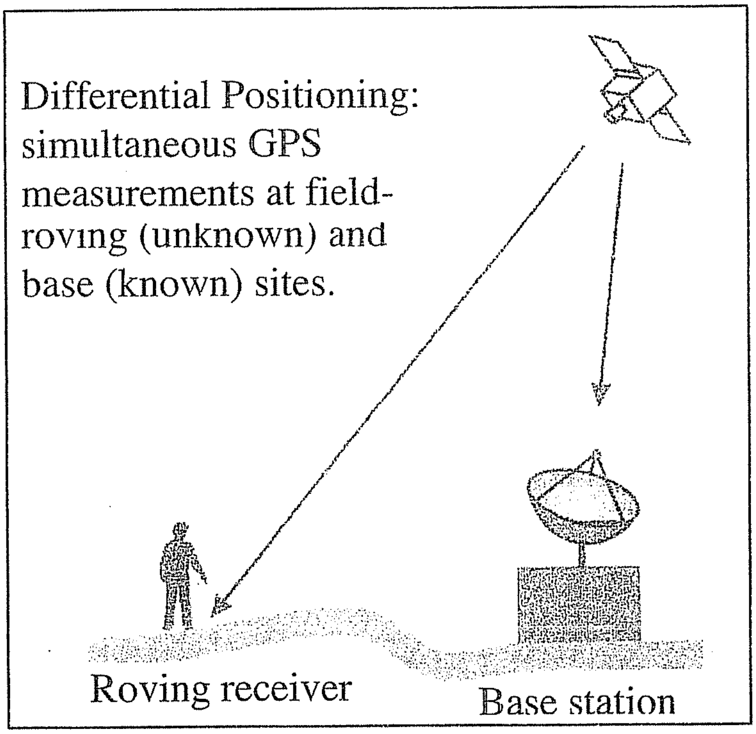


FIGURE 12.10: Differential GPS positioning.

The GPS position of the base station at a certain point in time is measured, and the resulting coordinates are compared to the known position. The difference between these positions defines an error vector which is characterized by a distance and direction. The correction vector is equal to the error vector in distance but opposite in direction. This correction measured at the base station is applied to the data collected by the rover to obtain corrected rover position (Figure 12.11).

There are limits on the application of differential correction. First, the base station and roving receivers must collect data from the same set of satellites. Errors from one set of satellites are different from errors from another set, even if the sets differ by just one satellite. In order to achieve this simultaneous viewing of the same set of satellites, the distance between the base station and the rover should preferably not exceed 300 km. Second, GPS measurements at both the base station and the rover should be performed at the same time.

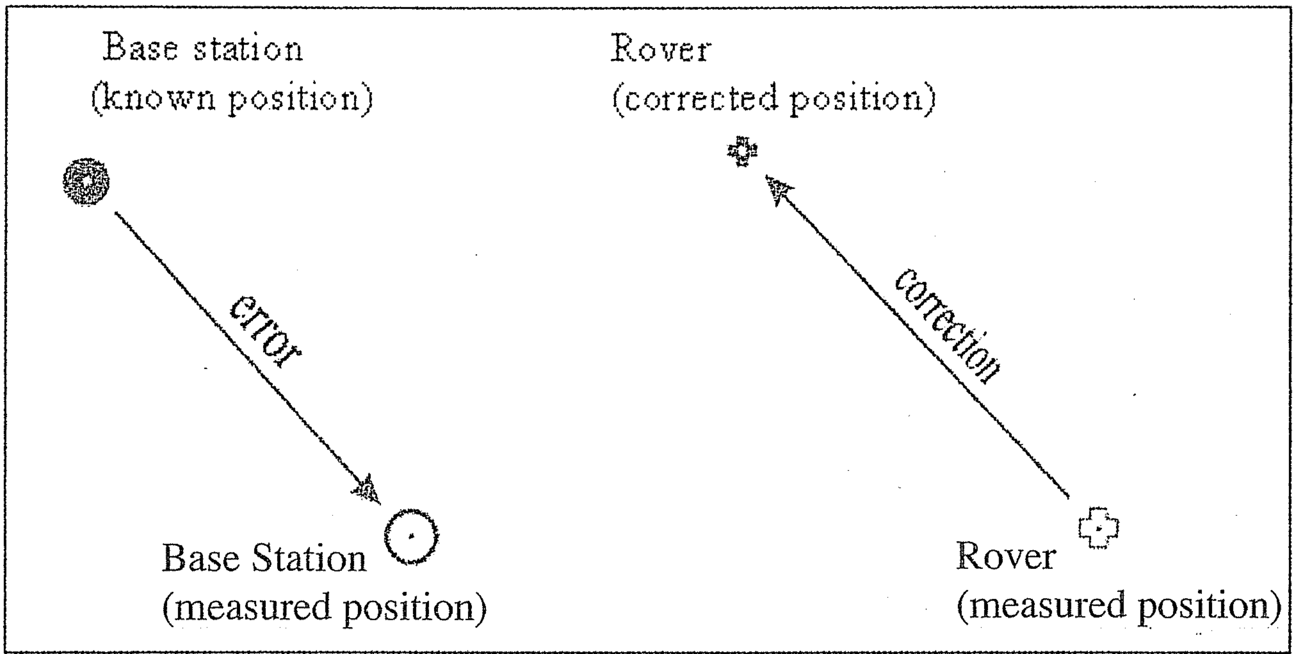


FIGURE 12.11: Differential correction is based on measuring a GPS position error at a base station, and applying the error as a correction to a simultaneously measured rover position.

In order to apply the differential correction, many receivers allow large amounts of data to be stored, either internal to the receiver, or to an attached computer. Files may be downloaded from the base station and roving units to a common computer. Software provided by most GPS system vendors is then used to combine the base and rover data and compute and apply the differential corrections. This is known as post-processed differential correction, as corrections are applied after, or post data collection (Figure 12.12).

Post-processing differential positioning has one serious limitation. Because precise positions are not known when the rover is in the field, post-processing technologies are useless when precise navigation is required. A surveyor, for example, recovering buried or hidden property corners often needs to navigate to within a meter of a position while in the field, so that monuments, stakes, or other markers may be recovered. Alternative methods, described in the next section, provide more precise in-field position determination.

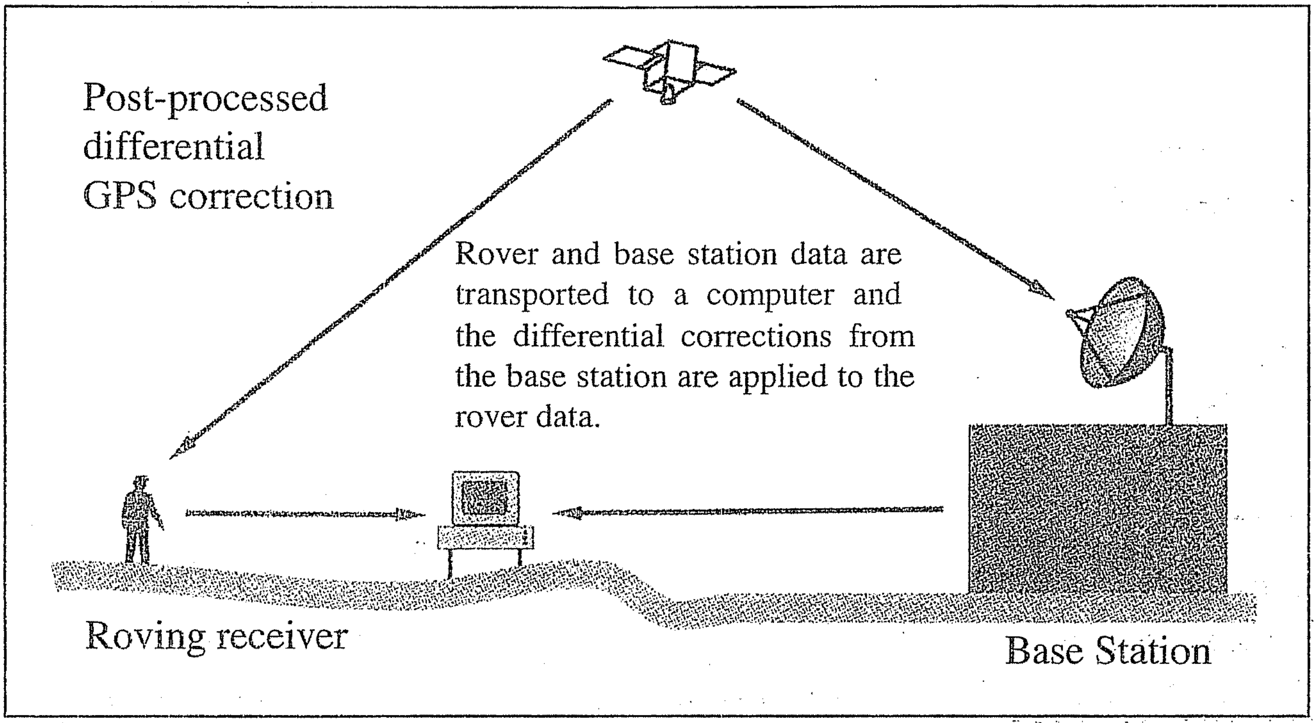


FIGURE 12.12: Post-processed differential GPS correction.

12.7 REAL-TIME DIFFERENTIAL POSITIONING

An alternative GPS method, known as real-time differential correction, may be appropriate when precise navigation is required. This method requires some extra equipment, and there is some cost in slightly lower accuracy when compared to post-processed differential GPS, but the gain is that accurate locations are determined while still in the field.

Real-time differential GPS positioning requires a communications link between the base station and the roving receiver (Figure 12.13). Typically, the base station is connected to an FM radio transmitter and antenna. The base station collects a GPS signal, calculates positional corrections (magnitudes and directions) and passes them to the radio transmitter, along with information on the timing and satellite constellation used. This continuous stream of corrections is broadcast via the base station radio and antenna.

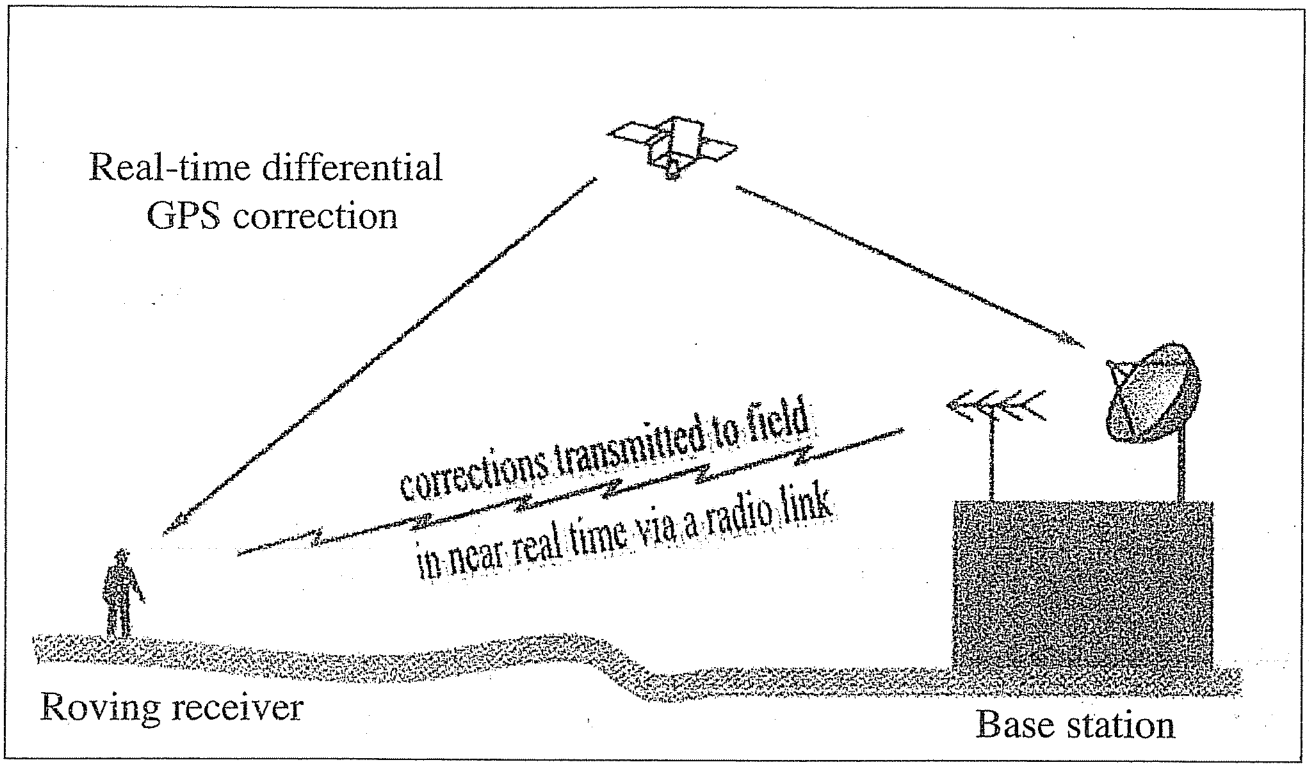


FIGURE 12.13: Real-time differential GPS correction.

Roving GPS receivers are outfitted with a receiving radio, and any receiver within the broadcast range of the base station may receive the correction signal. The roving receiver that is also collecting GPS data matches these data to the corresponding correction from the base station radio broadcast to obtain accurate field locations in real time.

12.8 TRANSFORMATION OF GPS COORDINATES

Denoting the geocentric Cartesian (rectangular) coordinates of a point in space by X, Y and Z, and assuming an ellipsoid of revolution with the same origin as the Cartesian coordinate system, the point can also be expressed by the ellipsoidal geographic coordinates ϕ, λ and h, as shown in Figure (12.14). The relation between the Cartesian and ellipsoidal geographic coordinates can be given by the following equations:

$$X = (R_N + h)\cos\phi \cos\lambda$$
$$Y = (R_N + h)\cos\phi \sin\lambda \qquad \ldots\ldots\ldots\ldots (12.3)$$
$$Z = \left(\frac{b^2}{a^2} R_N + h\right)\sin\phi$$

Where R_N is the radius of curvature in the prime vertical and is calculated from:

$$R_N = \frac{a^2}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} \qquad \ldots\ldots\ldots\ldots (12.4)$$

and a & b are the semi-major and semi-minor axes of the reference ellipsoid. The reference ellipsoid used by GPS is the World Geodetic System (WGS-84).

Equations (12.3) transform ellipsoidal coordinates ϕ, λ and h into Cartesian coordinates X, Y and Z. For GPS applications, the inverse transformation is more important because the Cartesian coordinates are known (measured) by the GPS, and the ellipsoidal coordinates are sought. This transformation is usually difficult to be done directly and needs an iterative solution. It is beyond the scope of this book to deal with such a transformation.

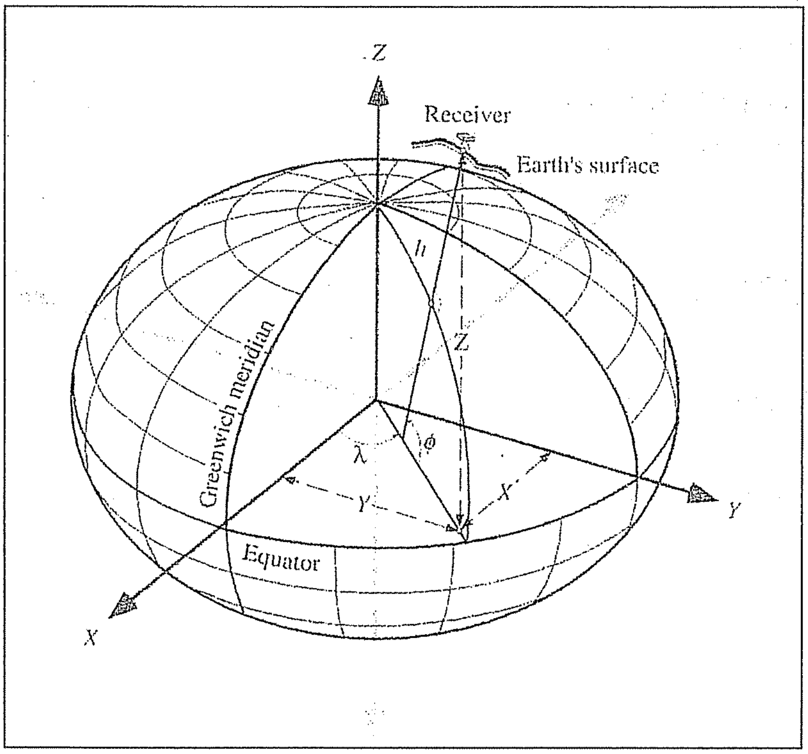


FIGURE 12.14: Geocentric Cartesian coordinate system of GPS and conversion to latitude, longitude and ellipsoidal height.

Once the ellipsoidal coordinates (ϕ, λ and h) are obtained for a point in a certain location, they can be used to calculate the local Cartesian coordinates of this point according to the suitable map projection used for the area under consideration.

12.9 HEIGHT DETERMINATION USING GPS

From the preceding section, it is clear that the ellipsoidal height h in Figure (12.14) can be obtained using Equations (12.3), but the geoid undulation N (the separation between the geoid and the ellipsoid) in Figure (12.15) must be known to determine the orthometric height H (above mean sea level). Orthometric heights are usually obtained by leveling. Once N is known, then H can be calculated from:

$$H = h - N \qquad (12.5)$$

The procedure for calculating N and obtaining orthometric heights above mean sea level can be summarized as follows:

1) Measure the X, Y and Z coordinates of a benchmark (BM) whose orthometric height (H) above mean sea level is known from leveling. Transform the GPS X, Y and Z coordinates of the BM using the inverse of Equations (12.3) to get ϕ, λ and h.

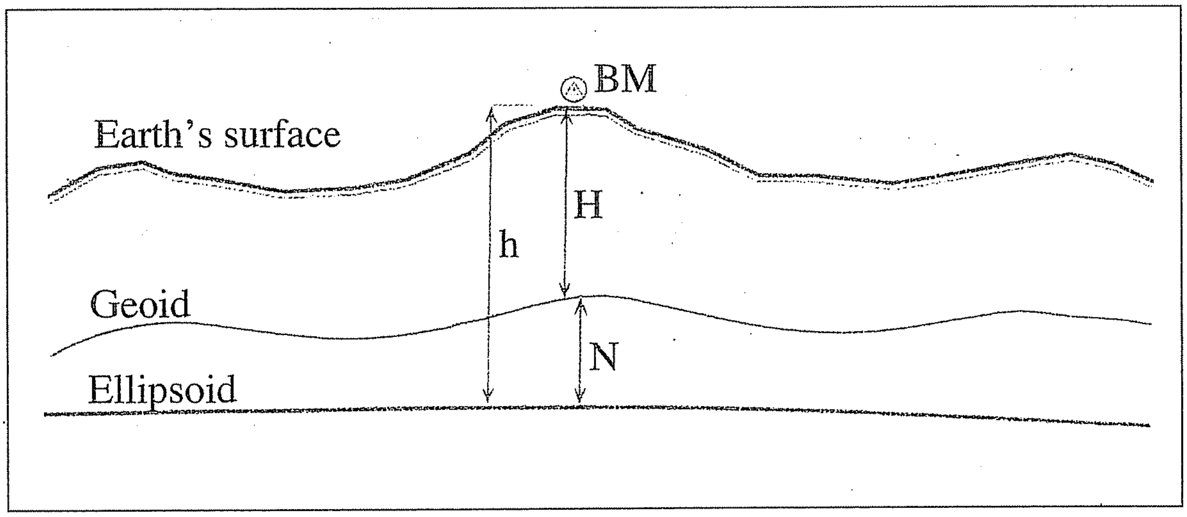


FIGURE 12.15: Relationship between ellipsoidal height h and orthometric height H.

2) Now that the ellipsoidal height h and orthometric height H of the BM are known, calculate the undulation $N = h - H$. This value of N is most likely to be the same for all neighboring points to the BM.

3) Measure the GPS X, Y and Z coordinates of the needed points in the area. Transform these coordinates using the inverse of Equations (12.3) to get ϕ, λ and h. Use Equation (12.5) to calculate the orthometric height H using the N value computed in step 2.

12.10 GPS APPLICATIONS

There are several important applications of GPS. These include:

1) ***Tracking***. This involves noting the location of objects through time. A common example is delivery vehicle tracking in real time. Large delivery and distribution organizations frequently require information on the location of their vehicles. This can be achieved by equipping the vehicles with a GPS receiver and a radio broadcast link. The vehicles will report back the location to a main office every few seconds. Moreover, these locations may be shown in real time on a digital map for the area where these vehicles may exist.

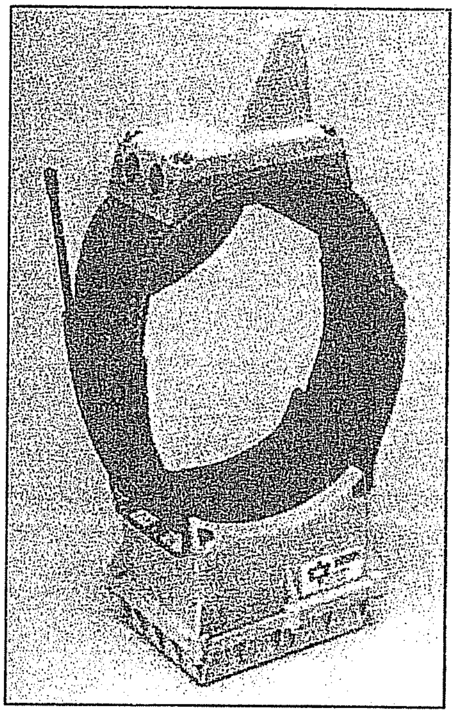


FIGURE 12.16: A GPS collar used in tracking animal movement.

Another new and innovative example is the GPS tracking of animal movement. A GPS-based animal tracking unit is fitted to animals, usually by a harness or collars (Figure 12.16). The animals are then released, and tracked in the same way as tracking vehicles above through position signals received by the GPS receiver on the animal and transmitted to the tracking office. This helps in the study of threatened, endangered, or important species.

2) *Navigation*. GPS receivers have been developed specifically for navigation, with digital maps set into on-screen displays (Figure 12.17). The location of the moving object (vehicle) appears on the screen in real time. Directions to the identified or destination points may be displayed, either as a route on the digital map or as a set of instructions, e.g., directions to turn at oncoming streets.

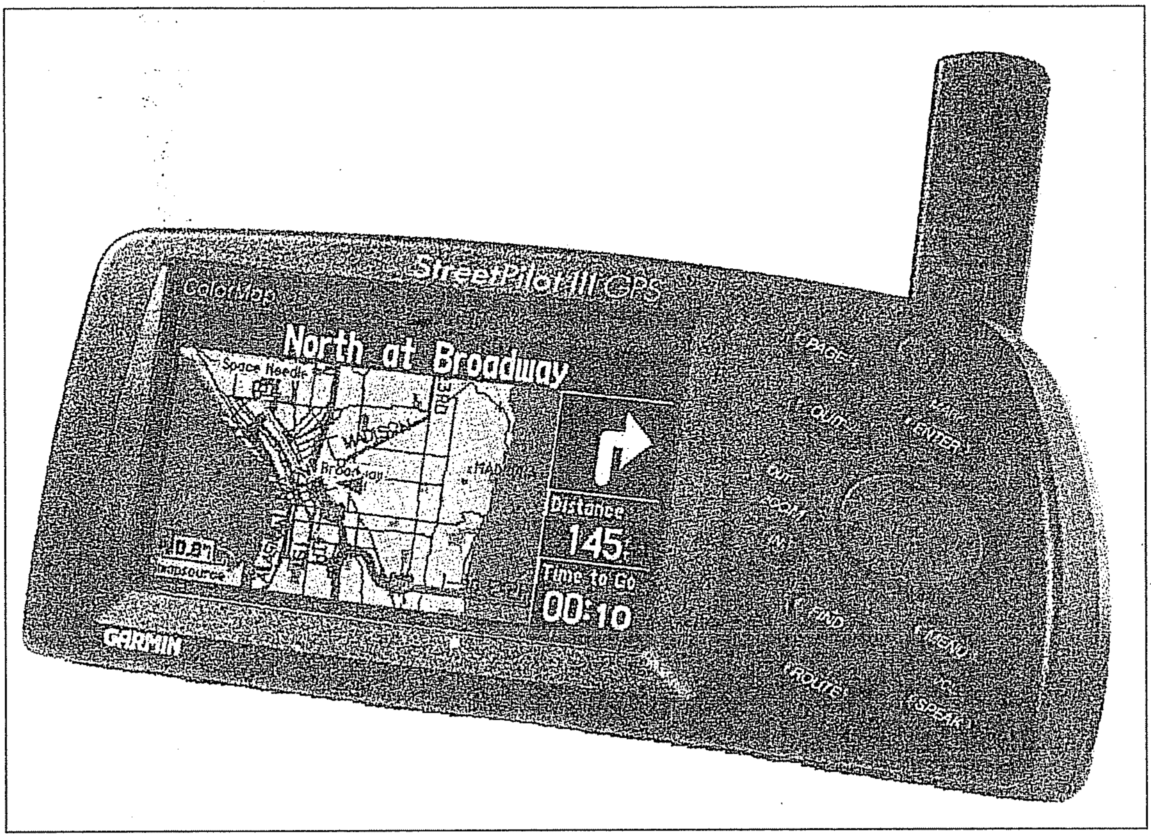


FIGURE 12.17: A GPS receiver developed for field navigation.

3) *Field digitization*. Positional data for point, line and area features may be recorded continuously in the field using GPS receivers carried in automobiles, on boats, bicycles, or helmets, or by hand. Coordinates are obtained at some fixed rates, e.g., every 2 seconds. The result of this process could be a map that can be used for several purposes including geographic information systems (GIS). The positional accuracy here is satisfactory for many applications and is higher than manual digitization from existing maps.

4) ***Establishment of horizontal control points.*** GPS offers an accurate direct method for measuring the coordinates of potential control points as compared to traditional surveying methods explained in Chapter 10. GPS significantly reduces the time and cost of control point collection.

PROBLEMS

12.1 Describe the main components of GPS, and what do they do.

12.2 What was the main intent for developing GIS?

12.3 Describe geometrically how GIS works to measure a point location.

12.4 Why do we need a minimum of four satellites viewed by the receiver to obtain accurate coordinates? What happens if the receiver is viewing more than four satellites?

12.5 Describe the main sources of error in GPS. How do we cancel or minimize the effect of these sources of error?

12.6 What is the positional dilution of precision (PDOP)? How does it affect GPS measurement?

12.7 Describe the basic principle of differential positioning.

12.8 What is the difference between post-processed and real-time differential positioning?

12.9 Describe the coordinate system that GPS uses.

12.10 Describe how heights above mean sea level are measured using GPS.

12.11 Discuss the main applications of GPS.

13 GEOGRAPHIC INFORMATION SYSTEMS (GIS)

13.1 INTRODUCTION

Researchers and practitioners in geography as well as in other disciplines have dealt for many years with problems relating to the analysis and manipulation of spatially related information. The most common medium for storing and displaying such coordinate-based information has traditionally been the analog (paper) map. The first map was apparently created before the first alphabet, so it is apparent that we have been working with these analog storage and display devices for spatial data over an extended period of time. During this time, these devices have evolved to a high level of sophistication and today's map combines high density data storage with complex color-based displays.

Spatial data elements, recorded on maps as points, lines and areas are commonly recorded on the basis of a coordinate system. Retrieval and analysis of these map elements normally involves visual inspection of the map document coupled with intuitive analysis, which is sometimes aided by simple measurement tools such as scales and planimeters. The information stored on maps is often of critical importance, but experience has shown that while it is easy to extract small amounts of data, the retrieval of a larger number of map elements or attempts to determine in a quantitative manner the complex relationships which exist among these elements is a very slow process.

For example, in attempting to find the best location for a chemical plant, several conditions need to be met. These might include: the land area for the plant should be between 30 and 40 donums, for sale, not agricultural, land slope less than 5%, at least 5 km from city boundaries, within 2 km from the closest road and at least 3 km from ground water and 2 km from surface water. Determining the answer to this simple problem manually requires the comparison and overlay of five maps: one that shows the city boundaries and displays land parcels, another which shows road network, a third that shows elevations, a fourth that shows zoning, and a fifth that shows surface and ground water boundaries. Other information regarding parcels that are for sale is also needed. It is very common in situations like this for the area of interest to cover several map sheets, thus requiring that maps be joined at the sheet edges; an operation which may be impeded by differences in the scales of maps and in the time at which data were compiled. Traditionally, integration of spatial data sets is carried out by transforming the spatial data sets to a common map scale, creating a transparent overlay for each data set, registering these overlays so that the coordinate systems are aligned, and then manually creating a composite overlay sheet that shows those locations where the needed conditions are met. The time involved in this process could be tremendously long.

Analog maps also display another major problem as a data storage device; they are expensive and time consuming to change and update. Updating these maps is difficult, expensive and requires that manual changes (re-stripping, cutouts, etc.) be made to the film master of the map sheet. Moreover, analog maps are subject to damage by frequent use. Attaching attribute (non-spatial) data to the features on an analogue map is also a difficult, if not an impossible process.

The technology involved in the creation of these analog devices for the joint storage and display of spatial data has reached a high level of development, but it has never succeeded in overcoming these, and other basic handicaps. Therefore, the advent of the digital computer as a data handling device soon raised the question of its applicability to the storage and manipulation of spatial data, and hence the development of geographic information systems (GIS).

13.2 THE DEFINITION OF A GEOGRAPHIC INFORMATION SYSTEM (GIS)

A geographic information system (GIS) can be defined as an assemblage of hardware, software, data and organizational structure for collecting, storing, manipulating, analyzing and presenting spatially referenced data and information. Other definitions can be found, but the important element which is common to all is that in a GIS, decisions are made based on spatial analyses performed on information that is referenced in a common geographical system. A close look at the software component of these systems shows that it contains the following major subsystems:

1. A data input subsystem which collects and/or processes spatial data derived from existing maps, remote sensors, etc.

2. A data storage and retrieval subsystem which organizes the spatial data in a form which permits it to be quickly retrieved by the user for subsequent analysis, as well as permitting rapid and accurate updates and corrections to be made to the database.

3. A data manipulation and analysis subsystem which performs a variety of tasks such as changing the form of the data through user-defined aggregation rules or producing estimates of parameters and constraints for various space-time optimization or simulation models. This subsystem is the main component that gives GIS its power and distinguishes it from other computer systems such as CAD and other graphics systems.

4. A data reporting subsystem which is capable of displaying all or part of the original database as well as manipulated data and output from spatial models in tabular or map form. The creation of these map displays involves what is called digital or computer cartography. This is an area that represents a considerable conceptual extension of traditional

cartographic approaches as well as substantial change in the tools utilized in creating the cartographic displays.

GIS helps improve the decision-making process by providing answers to questions that have been difficult to address before utilizing this technology. GISs are, by and large, a marriage of graphics software that deals with spatial data and database management software that deals with tabular data. Spatial data of different types are collected and stored in GIS in the form of layers registered to the same geodetic reference framework (Figure 13.1). This enables information from two or more different layers to be integrated.

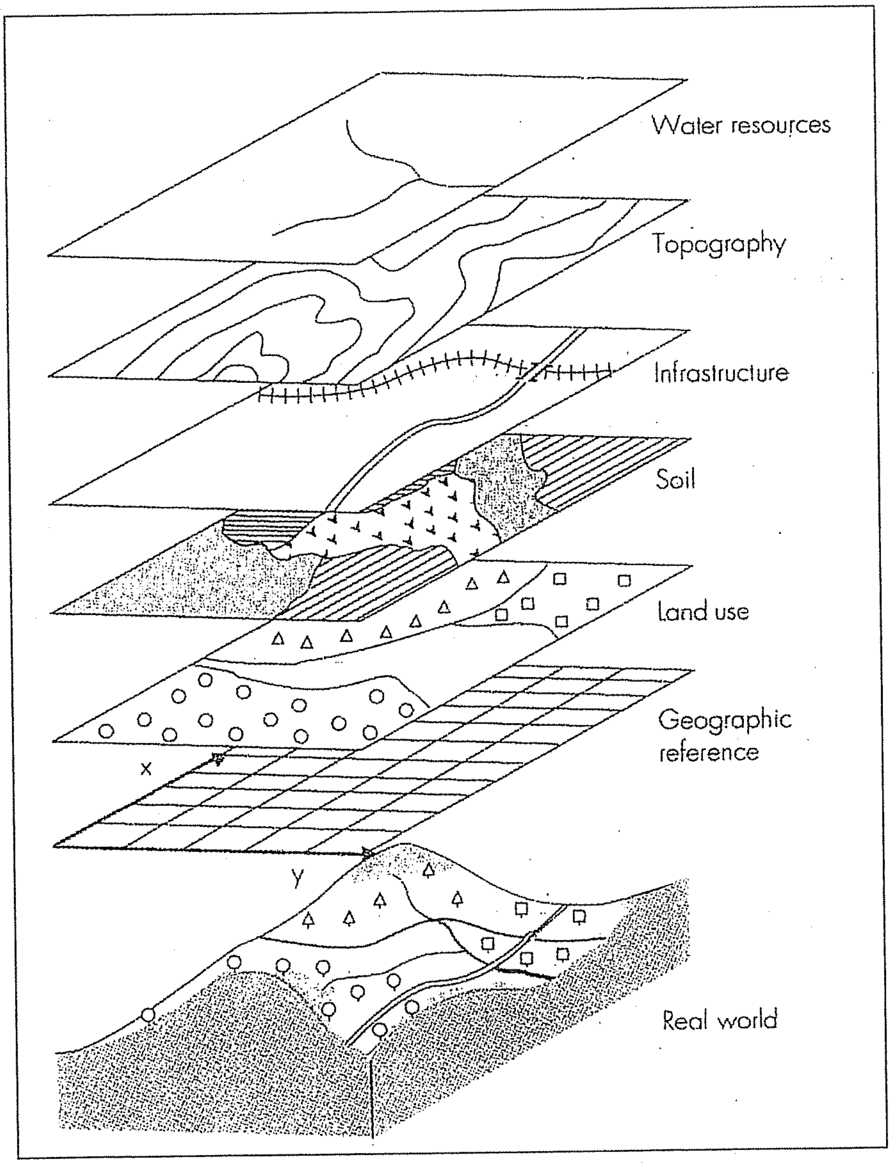


FIGURE 13.1: Concept of layers in a geographic information system.

13.3 SPATIAL DATA

The data in GIS are classified into two types: spatial (graphic) and attribute (non-spatial or textual). Spatial data consist in general of natural and cultural features that can be shown with lines or symbols on maps, or that can be seen as images on photographs. In a GIS, these data are represented, and spatially located in digital form, by using a combination of fundamental elements called *simple spatial objects*. The formats used in this representation are either ***vector*** or raster. The *relative spatial relationships* of the *simple spatial objects* are given by their ***topology***. These are described in the following sub-sections.

13.3.1 SIMPLE SPATIAL OBJECTS

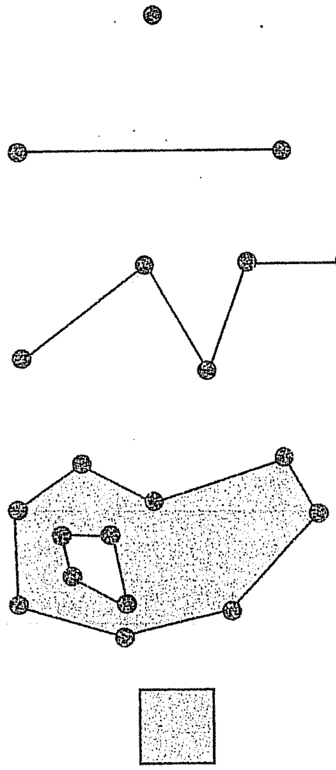
The simple spatial objects most commonly used for locating and describing spatial data are (Figure 13.2):

1) ***Points***. These define single geometric positions. They are used to locate features such as houses, wells, mines, poles, etc. and are assigned each a pair of coordinates (x, y).

2) ***Lines*** and ***strings***. These are obtained by connecting points. A line connects two points, and a string is a sequence of two or more connected lines. Lines and strings are used to represent and locate property lines, roads, streams, fences, etc.

3) ***Areas/polygons***. These consist of the continuous space within three or more connected lines or strings that form a closed loop. They can be separate polygons or nested within other bigger areas. Areas are used to represent and locate the limits of closed features such as land parcels, lakes, different types of land cover, etc.

4) ***Raster cells/Pixels***. These are usually tiny squares that represent the smallest elements into which a digital image or a continuous geographic variable is divided. Continuous arrays of pixels, arranged in rows and columns, are used to enter data from aerial photos, satellite images, etc.



Point: A zero-dimensional object that specifies geometric location specified through a pair of coordinates.

Line: A one-dimensional object that is a direct line between two endpoints.

String: A sequence of line segments.

Area/polygon: A two dimensional object bounded by at least three one-dimensional line segments.

Raster cell/pixel: A two-dimensional object (area) that represents an element of a regular tessellation of a surface.

FIGURE 13.2: The simple spatial objects: points, lines, areas and grid cells.

These pixels or cells may be used to represent terrain slopes, soil types, land cover, population density, etc. The distribution of a given data type within an area is indicated by assigning a numerical value to each cell, for example, showing soil types in an area using the number 2 to represent sand, 5 for loam, and 9 for clay. Pixel size is chosen carefully to give suitable representation of the area. A smaller size pixel gives a finer resolution, but leads to a larger size of data sets.

13.3.2 VECTOR AND RASTER FORMATS

The simple spatial objects described in the previous section show that two different data formats are used for storing and manipulating spatial data: *vector* and *raster*. In the vector format, points are used to specify locations of point objects such as utility poles, lines and strings are used to depict linear features such as rivers, and areas (polygons) are used to show regions having common characteristics such as land cover. An example illustrating the vector format is given in Figure 13.3 and Table 13.1. The figure shows two adjacent land parcels, one designated parcel 31, owned by Tamim, and the other identified as parcel 32 and owned by Hani. As shown, the spatial elements are points, lines and areas.

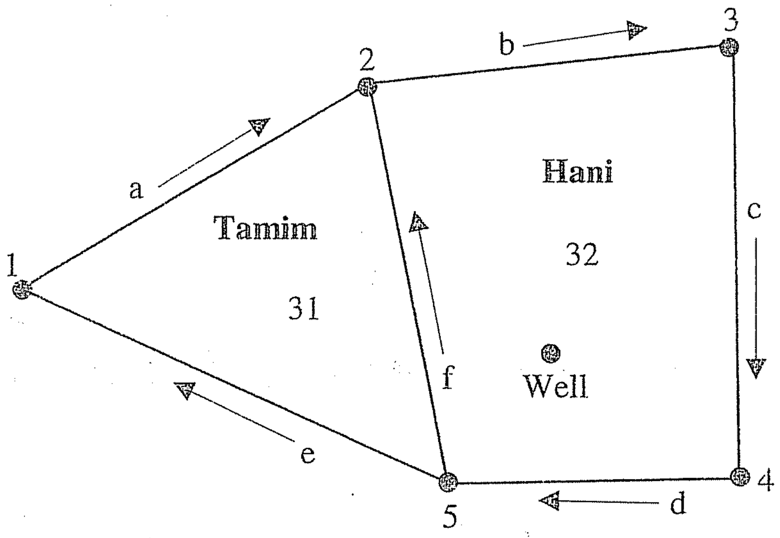


FIGURE 13.3: Vector representation of two land parcels.

TABLE 13.1: Vector representation of Figure 13.3.

(a)		(b)		(c)	
Point identifier	Coordinates	Line identifier	Points	Area identifier	Lines
1	$(X,Y)_1$	a	1,2	31	a, f, e
2	$(X,Y)_2$	b	2,3	32	b, c, d, f
3	$(X,Y)_3$	c	3,4		
4	$(X,Y)_4$	d	4,5		
5	$(X,Y)_5$	e	5,1		
Well	$(X,Y)_{Well}$	f	5,2		

Vector representation of data can be achieved by creating a set of tables that list these points, lines and areas (see Table 13.1). Data within the tables are linked using identifiers and are related spatially through the coordinates of points. As illustrated in column (a), all points in Figure 13.3 are indicated by an identifier. Similarly, each line is described by its endpoints, as shown in column (b), while areas (land parcels) are defined by the lines that enclose them as shown in column (c). Coordinates of endpoints locate the areas and enable the determination of their locations and magnitudes.

Other data types can also be represented in vector format. Consider, for example, the land cover map shown in Figure 13.4a. In this figure, the areas of different land cover (forest, marsh, etc.) are shown with standard topographic symbols. A vector representation of this region is shown in Figure 13.4b. Here lines and strings locate boundaries of regions having a common type of land cover. The stream in this figure consists of the string connecting points 1 through 11. By means of tables similar to Table 13.1, the data of this figure can be entered into a GIS using the vector format.

As an alternative to the vector approach, data can also be depicted in the raster format using pixels (grid cells). Each equal-size pixel is uniquely located by its row and column numbers, and is labeled with a numerical value or code that corresponds to the properties of the specific area it covers. In the raster format, a point would be indicated by a single pixel, a line with a sequence of adjacent pixels having the same code, and an area having common properties would be shown as a group of identically coded contiguous pixels. It should be clear; therefore, that in general the raster representation yields a coarser level of accuracy or definition of points, lines and areas than the vector representation. This accuracy can be usually increased by using a smaller cell size, yielding a finer resolution. This is clear in Figure 13.4 c and d which represent a coarse and fine representations respectively of the area shown in Figure 13.4 a and b. Each cell is assigned a code representing to one of the land cover classes, that is F for forest, G for grassland, M for marsh and S for stream. It is to be noted here that as the grid cell size becomes smaller, the size of the data set becomes tremendously larger.

Even though the raster format gives a coarser resolution of spatial features, and yields larger data sets, it is still often used in GISs. One reason is that a lot of data is available in raster format. Examples include digital aerial photos, orthophotos, satellite images and scanned maps. Another reason for the popularity of the raster format is that raster images can be easily processed and refined using available image-processing software. Moreover, many data sets such as wetlands and soil types, the boundary locations are rather vague and the use of the raster format does not significantly degrade the inherent accuracy of the data. Last but not least, some spatial analysis operations such as overlays are done faster and simpler using raster format.

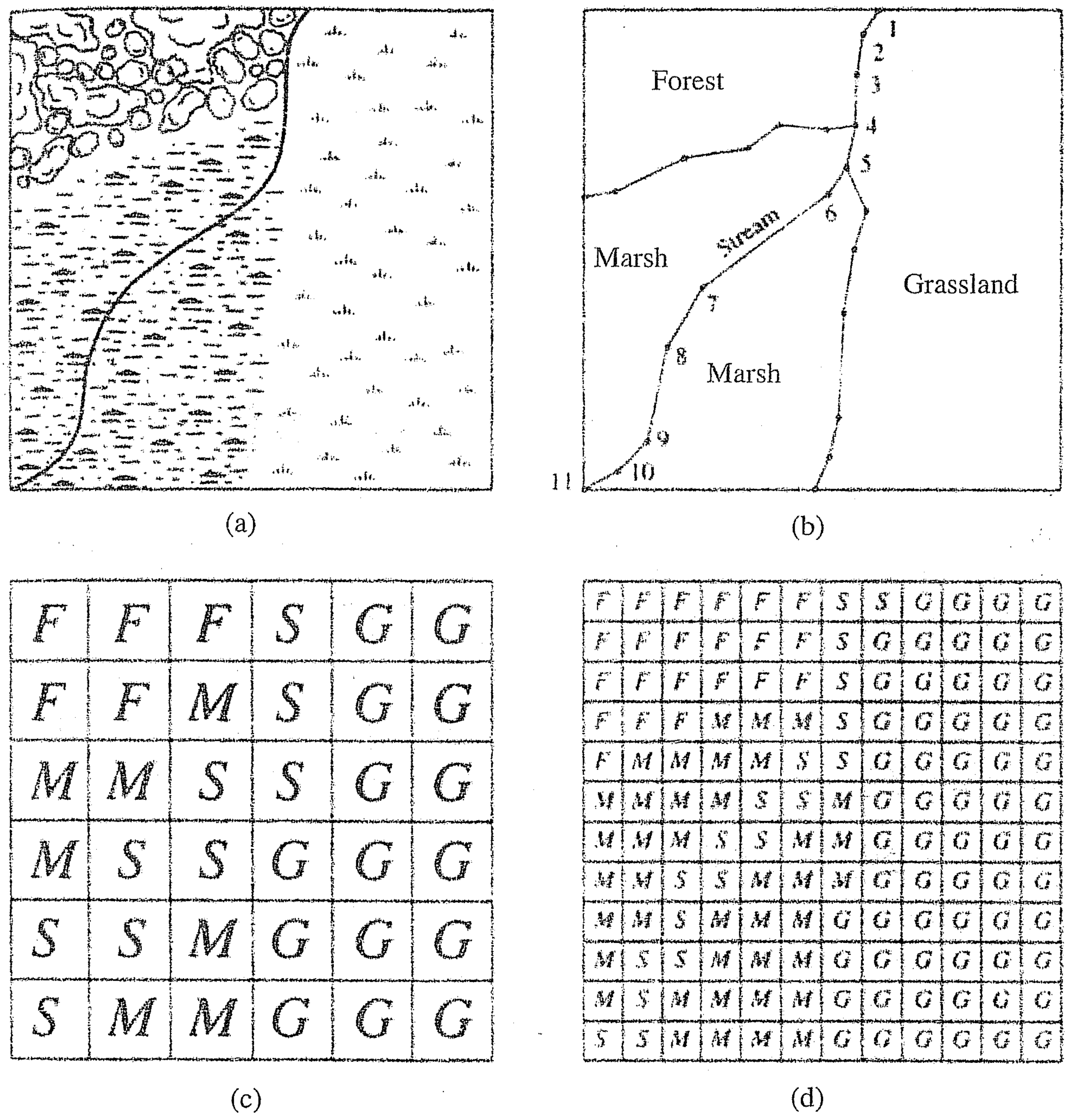


FIGURE 13.4: Land cover maps of a region. (a) Region using standard topographic symbols. (b) Vector representation of the same region. (c) and (d) are raster representation of the region using coarse and fine resolutions respectively.

13.3.3 CONVERSION OF DATA BETWEEN RASTER AND VECTOR FORMATS

Sometimes it is necessary or desirable to convert data from raster to vector format or vice versa. These processes are described briefly as follows:

A) Raster to Vector (Vectorization) Conversion:

Referring to Figure 13.5, point and line features are converted from raster to vector as shown below:

1. Point features. Each vector point feature is assigned to the coordinates of the corresponding raster cell center.

2. Line features. Raster cell centers are usually taken as the locations of vertices along the line. Lines may then be *smoothed* using a mathematical algorithm to remove the *stair-step (jagging)* effect.

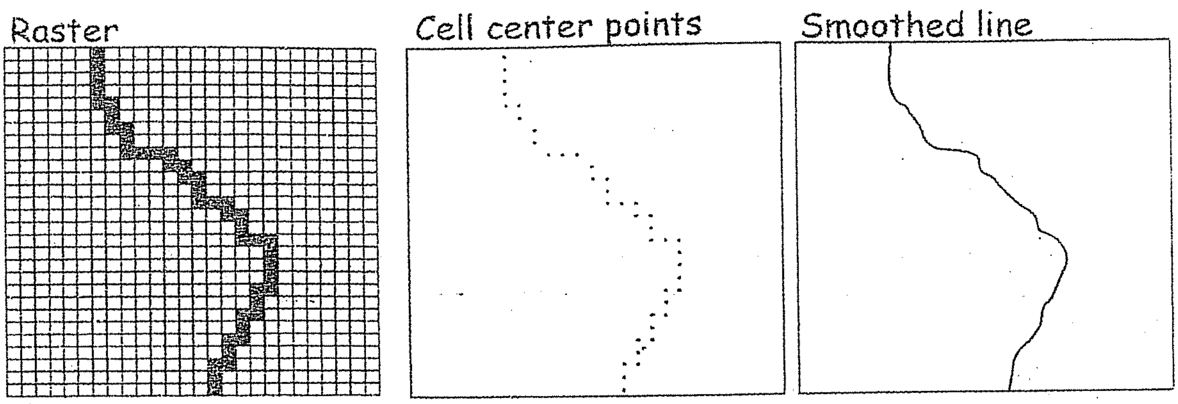


FIGURE 13.5: Raster to vector conversion.

B) Vector to Raster (Rasterization) Conversion:

Referring to Figure 13.6, point, line and area features are converted from vector to raster as shown below:

1. Point features. Each vector point feature is assigned a cell code depending at the cell where it falls. Usually a small cell size is chosen so that the diagonal cell dimension is less than the smallest distance

between two vector points to prevent two points from falling in the same cell.

2. Line features. two rules are used:

   a) Any cell rule: a value is assigned to a cell if a vector line intersects with any part of the cell. This rule ensures continuity of lines, but leads to wider lines.

   b) Near cell center rule: a value is assigned to a cell if the cell center is a certain predefined distance (like 1/3 cell width) from a vector line segment. This rule gives thinner lines, but does not ensure line continuity (Figure 13.6b).

3. Area features. Area boundaries are lines and therefore they are converted to raster in the same way as line features. One method is to consider the cell as part of the area if the vector boundary passes from it. Another method is when more than half the cell is falling within the area.

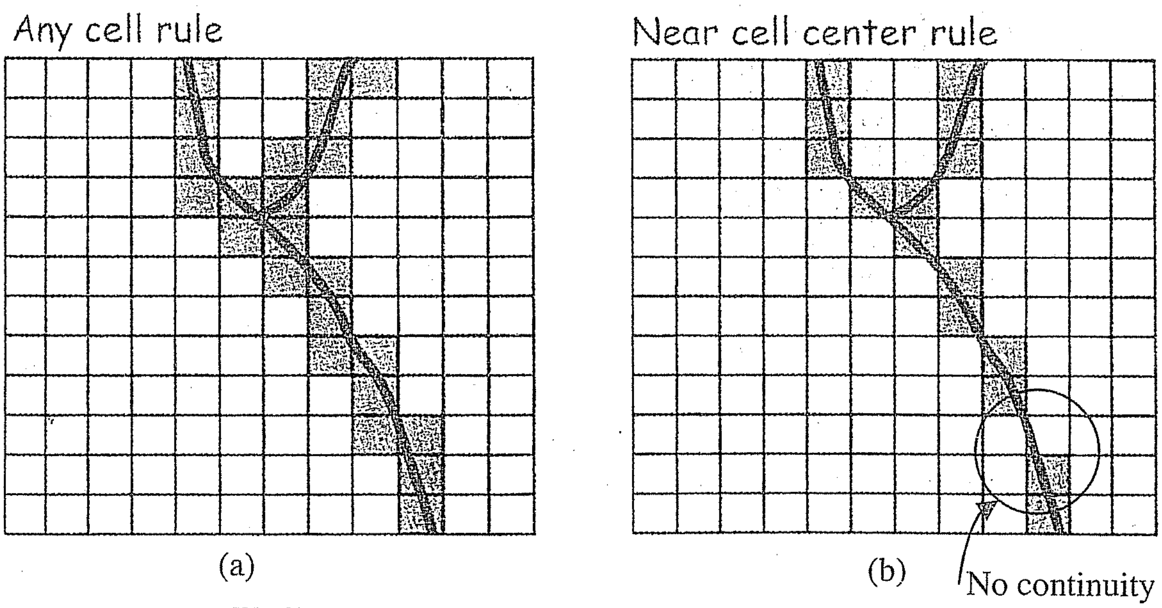


FIGURE 13.6: Vector to raster conversion.

13.3.4 TOPOLOGY

Topology is a branch of mathematics that deals with the geometric properties of figures, that do not change when the forms of these figures are bent, stretched or undergo similar transformations. The unique sizes, dimensions and shapes of individual objects are not addressed here. Rather, only their relative relationships are specified. Figure 13.7 illustrates the concept. Even though Figures 13.7 a and b look different, but they are topologically identical because they have the same connectivity and adjacency.

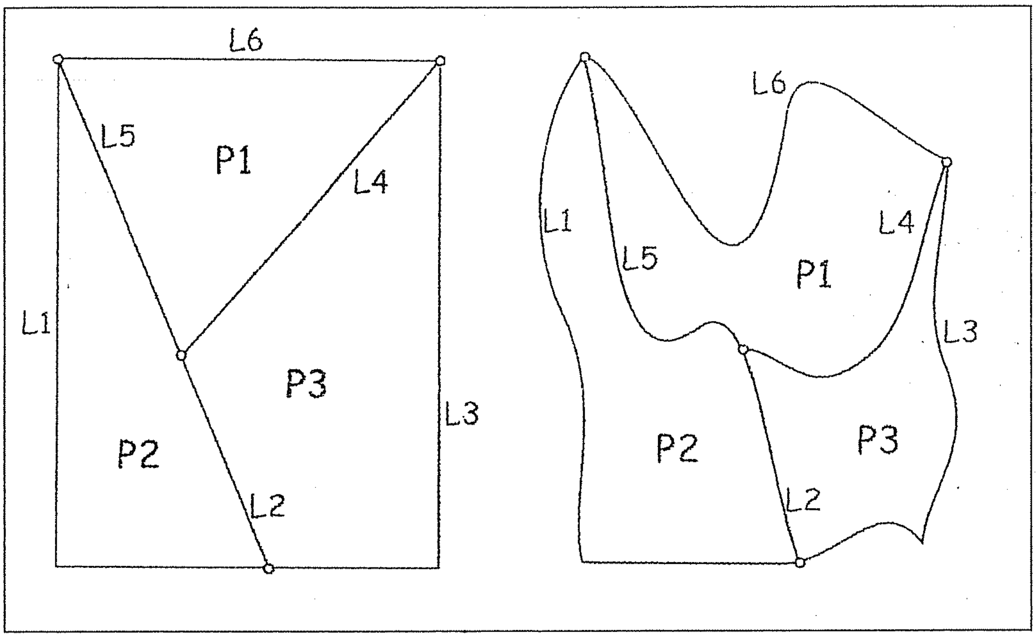


FIGURE 13.7: Examples of two different figures having identical topology.

It is easy for the human eye to look into a map and discover and realize the different topological relationships between objects. However, the computer does not have such eyes. That is why we use *nodes, chains (or links)* and *polygons* for representing topology in GIS. Nodes define beginnings and endings of chains, or identify the junctions of intersecting chains. Chains are similar to lines (or strings) and are used to define the limits of certain areas or delineate specific boundaries. Polygons are closed loops and are defined by a series of connected chains. In topology, sometimes single nodes exist within polygons for labeling purposes (e.g., the well in parcel 32 of Figure 13.3). The most important topological relationships dealt with in GISs are:

1) *Connectivity*: identifying which chains are connected at which nodes.

2) *Direction*: defining a "from node" and a "to node" of a chain. This is important in GIS for establishing such things as which way a river flows, or the direction traffic moves on one-way streets.

3) *Adjacency*: identifying which polygons are on the left and which are on the right side of a chain.

4) *Nestedness*: specifying what simple spatial objects are within a polygon. They could be nodes, chains or other smaller polygons.

These topological relationships are illustrated and described in Table 13.2 with reference to Figure 13.3. The relationships expressed through the identifiers of points, lines and areas of Table 13.1, and the topology of Table 13.2, conceptually yield a "map" (in the eyes through which a computer sees!). With these types of information available to the computer, the analysis and query processes of a GIS are made possible.

TABLE 13.2: Topological relationships of the graphical elements in Figure 13.3.

(a) Connectivity		(b) Direction			(c) Adjacency			(d) Nestedness	
Nodes	Chain	Chain	From node	To node	Chain	Left Polygon	Right polygon	Polygon	Nested node
1-2	a	a	1	2	a	0	31	32	Well
2-3	b	b	2	3	b	0	32		
3-4	c	c	3	4	c	0	32		
4-5	d	d	4	5	d	0	32		
5-1	e	e	5	1	e	0	31		
2-5	f	f	5	2	f	I	32		

13.4 ATRRIBUTE DATA

As mentioned in the previous section, data in a GIS are of two types: spatial and attribute data. Attribute data usually describe the non-spatial properties of the spatial objects. These data are usually stored in tables and that is why they are sometimes referred to as tabular data. They are alphanumeric in nature and provide information such as color, texture, quantity, quality, date, etc. Tamim and Hani, as property owners of parcels 31 and 32 in Figure 13.3, and the land cover classifications of forest, marsh, grassland and stream in Figure 13.4 are examples of attribute data. Other examples could be the route numbers for highways, pavement type, number of lanes, lane widths, and year of last resurfacing. Table 13.3 shows an example of an attribute table for the two land parcels of Figure 13.3.

The two types of data (spatial and attribute) in GIS should be linked by some means or another. Usually, this is achieved with a common identifier which is stored with both the graphical and the non-graphical data. Identifiers such as a unique parcel identification number may be used.

TABLE 13.3: An example of an attribute table for the parcels of Figure 13.3.

Object_ID	Parcel_ID	Object_Type	Owner_Name	Area_m2	Date_Purchased	Assessed_Value
1	31	Polygon	Tamim	500	20/5/1981	12000
2	32	polygon	Hani	800	16/11/1984	17000

13.5 DATA COLLECTION AND DATA SOURCES FOR GIS

Data collection is the most expensive and time-consuming aspect of setting up a major GIS facility. Experience shows that:

- 60 to 80% of the total cost (time and money) of fully operational GIS is for data collection.
- 10 to 30% for the purchase of equipment and software, and
- The rest for training, development and administration.

There are many sources for spatial data to be used in GIS (see Figure 13.8), but these sources are conveniently grouped into two main categories. These are:

- Hardcopy forms. These are any drawn, written, or printed documents, including hand-drawn maps, manually measured survey data, legal records, and coordinate lists with associated tabular data.

- Digital forms. These are the spatial data provided in a computer-compatible format. They include text files, lists of coordinates, digital images (satellite images, scanned maps) and coordinate and attribute data in structured file format (GPS, data from modern surveying equipment).

There are several methods for producing digital map data. These include:

1. Digitizing existing maps manually using a digitizing tablet.
2. Scanning existing maps.
3. On-screen digitizing.
4. Keyboard entry and coordinate geometry.
5. Transfer from existing digital sources.

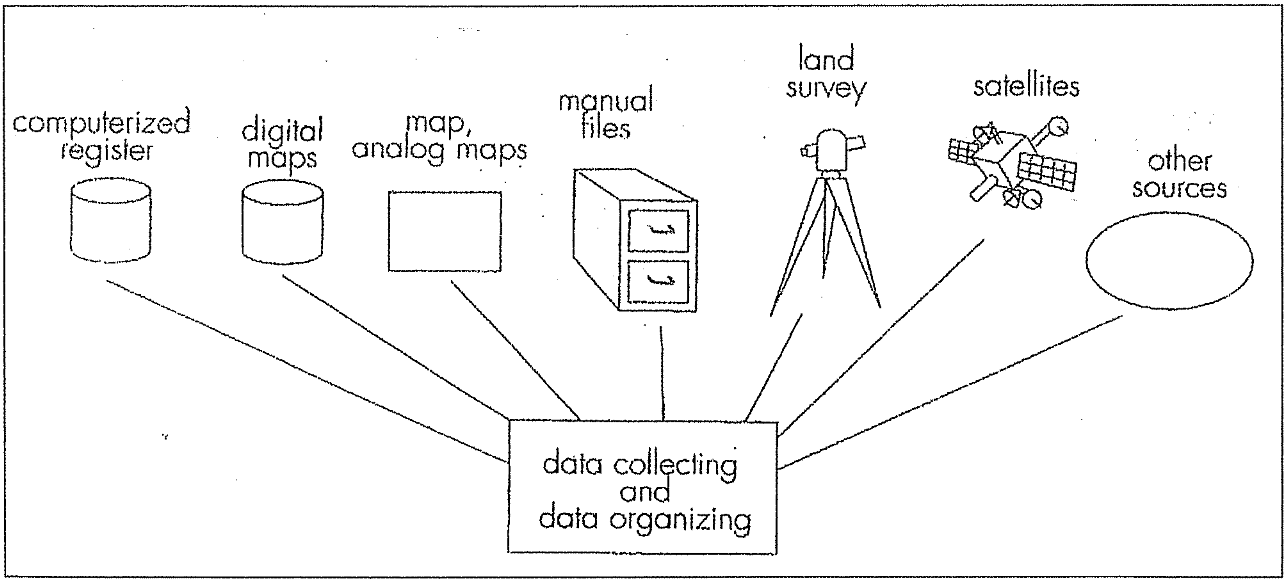


FIGURE 13.8: Sources of data for GIS.

13.5.1 DIGITIZATION OF EXISTING GRAPHICAL MATERIALS

Graphical documents such as maps, orthophotos, construction plans and other similar documents are conveniently and economically converted to digital files using a flat digitizing tablet (Figure 13.9) by a method called manual digitization. This is a human-guided coordinate capture of graphic features (points, lines and areas). As shown in Figure 13.9, a digitizing tablet contains an electronic grid and an attached cursor. Movement of the cursor across the grid creates an electronic signal unique to the cursor's position. At any desired point, the coordinates can be relayed to the computer for storage by pressing the Record button on the cursor. Point features are digitized by clicking on them. Lines and area features are captured by tracing over them and clicking at frequent intervals so that the geometry will be preserved. Data identifiers or attribute codes can be associated with each point through the computer's keyboard, or by pressing additional buttons on the cursor. Using this process, both planimetric features and contours can be digitized and the result is usually in vector format.

Manual digitization may be done in point mode (especially for point features), or stream mode (for line and area features) where points are automatically sampled at a fixed time or distance frequency. Lines usually have a starting point, often called a *starting node*, a set of *vertices* defining the line

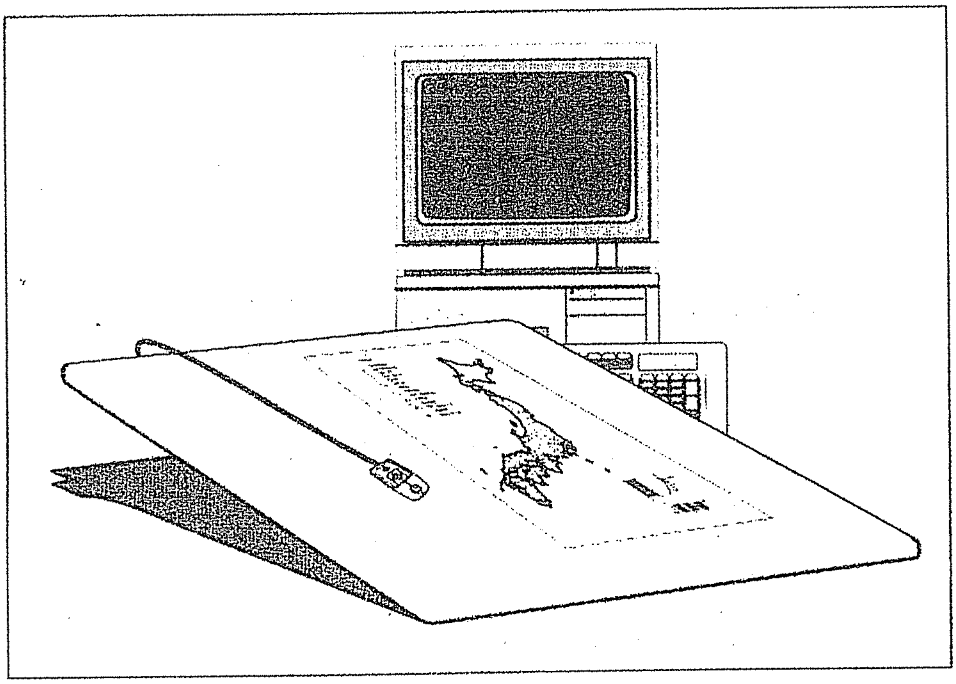


FIGURE 13.9: Digitizing tablet.

shape, and an ending *node*. Rapid collection in stream mode usually causes redundancy in picked points, while slow collection may result in the loss of important spatial detail.

Manual digitization may be characterized as being slow, labor-intensive, tedious and leads to inconsistent results and accuracies among different human operators, but it provides sufficient accuracy for most applications, needs lower initial capital outlays than other input techniques, and needs shorter training periods for operators. Also humans are better than machines in interpreting information contained in faded, multicolor or poor quality maps as compared to scanning as explained in the next section.

Data sets generated by digitizing will usually need to be checked carefully to ensure that all desired features have been included. Also the data must be corrected or cleaned before being used in a GIS. In this process, undesired points and line portions must be removed, and unclosed polygons, which result from imprecise pointing when returning to the polygon's starting node, must be closed. Finally, thin polygons or silvers, created by lines being inadvertently digitized twice, but not exactly in the same location, must be eliminated. This editing process can be performed by the operator, or with a program that can find and remove certain features that will fall within a set of user-defined tolerances.

13.5.2 SCANNING

To speed up the digitizing process, a scanner is used. Scanners pass a sensing element over the map which measures both the location of the point being sensed and the strength of light reflected or transmitted from that point (see Figure 13.10 which shows both types of scanners). Scanning results in a raster representation of the map. Each equal-sized cell is located by its row and column numbers and is coded with a numerical code that represents attributes of the area it covers. A point, therefore, is represented by a single cell, a line by a series of continuous cells having the same code, and an area having common properties would be shown as a group of identically coded contiguous cells. The size of data produced is determined by how finely (pixel size) the scanner scans the document.

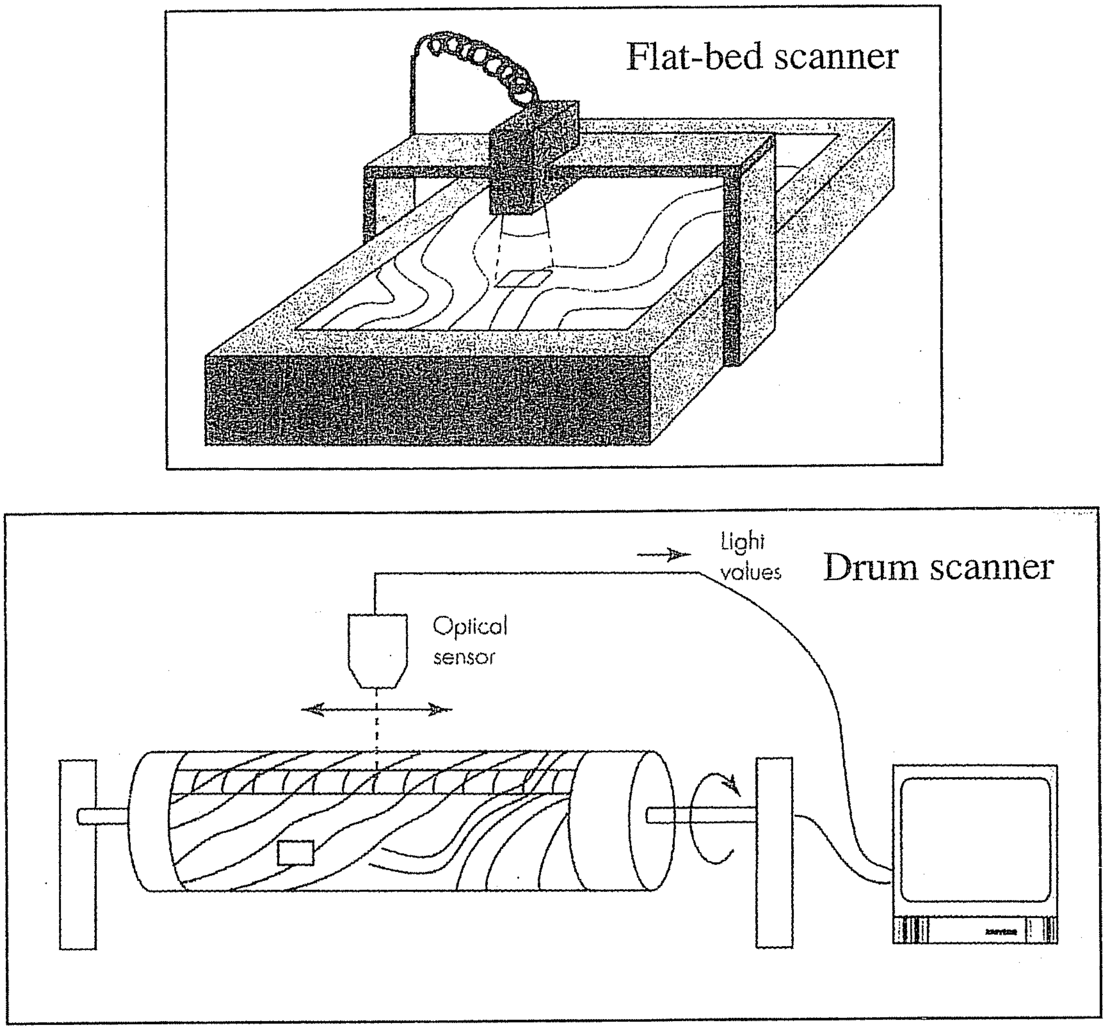


FIGURE 13.10: Flat-bed and drum scanners.

Even though scanning is easily performed and provides rapid compilation of digital map images, it works best when very clean map materials are available (no marks, folding, or wrinkling). It works even better when maps are available as map separates, with one thematic feature type on each map.

13.5.3 ON-SCREEN DIGITIZING

On-screen digitizing is a combination of manual digitizing and scanning. It involves manually digitizing on a computer screen, using a scanned map, aerial photograph, digital photograph, or a satellite image as a backdrop. This method may be used to limit operator-induced positional error when manually digitizing because of the ability to zoom in to larger scales on the screen.

13.5.4 KEYBOARD ENTRY AND COORDINATE GEOMETRY

Data can also be entered into a GIS database directly using the keyboard on a computer. Often data input by this method are non-spatial, such as map annotations or numerical or tabular data. To make the process easier and faster, data are usually manually keyed in using a database application program on a laptop or a PC, and subsequently exporting the database file to GIS.

Data for use in a GIS can also be generated directly from field survey data using coordinate geometry software. Surveying is used when highly accurate coordinates are needed such as in the case of property lines. With the use of modern surveying equipment, survey data are transferred directly to spatial data formats which can be read in GIS.

13.5.5 EXISTING DIGITAL DATA

Many spatial data currently exist in digital forms. These data include: roads, political boundaries, water bodies, land cover, soils, elevation as well as others. Most of these data have been produced by governmental offices to provide basic public services such as safety, health, transportation, water and energy. These digital data provide a means for rapidly and inexpensively populating a GIS database and should be available at little or no cost, often via the internet.

Often existing digital data must undergo conversion of file structures and formats to be usable with specific GIS software. Because of variations in the way data are represented by different software, it is possible that information can be lost or that spurious data can creep in during the process, something that should be taken into consideration.

13.6 SPATIAL ANALYSES

Spatial analyses can be defined as the application of computer-based operations on coordinate and related attribute data for the purpose of solving problems and/or creating an understandable image of reality. GISs are usually equipped with a menu of basic analytical functions which enable data to be manipulated, spatially analyzed and queried. When accompanied with appropriate databases, these functions provide GISs with powerful capabilities for supplying information that can significantly aid in planning, management and decision making.

Even though technology might help to a great extent, stating problems and delineating approaches to solutions together comprise one of the most difficult steps in GIS analyses. This has to be solved by the GIS user (operator) before the technology can be put to use, based on his/her professional knowledge in the fields of agriculture, environmental protection, planning, etc., supplemented with knowledge of GIS.

Operations used in the analyses of spatial data may be *logical, arithmetic, geometric, statistical*, or a combination of two or more of these four types (using Boolean algebra). These operations may be applied on both attribute data (tables) and geometric data (points, lines and areas). Operations may be applied sequentially to solve a problem. Each operation may create an output that will be used as an input in the next operations. Part of the challenge here is the choice of the appropriate operations to be applied in the appropriate order.

The specific functions available within the software of any particular GIS system will vary, but some of the more common and useful spatial analysis and computational functions are:

1) Spatial Selection:

Selection operations involve identifying features that meat one to several conditions or criteria set on the attributes and/or geometry of features. Features that satisfy the conditions are selected and then written to a new output data layer, or manipulated in some manner. Three types of selection criteria are used in GIS:

- On-screen query: a data layer is displayed, and features are selected by a human operator (identify tool).

- Selection by attributes through conditions applied on the attribute table using set and Boolean algebra.

- Selection by location using spatial relationships such as distance, containment, intersection and adjacency.

2) Classification:

Classification (also known as reclassification or recoding) categorizes geographic objects based on a set of conditions applied on the attributes. It helps to see where attribute values (quantitative) lie in relation to one another on a continuous scale. The goal here is to display similar objects with a uniform color or symbol so that they are identified as a group. Dividing values into groups (classes) requires that we choose both the number of classes and a method to determine where one class ends and another begins.

Classification may be done *manually or automatically*. Manual classification provides greater control over class assignment. Automatic classification, on the other hand, may save considerable time. A manually controlled automatic classification may be the best compromise.

Example: Land parcels in an area can be categorized according to their price ranges, or the countries of Africa divided into classes according to their populations.

3) Dissolve and Merge Operations:

Dissolve has the primary purpose of combining and merging alike features with a common attribute value together within a data layer. In the dissolve process, each line that serves as a boundary between two polygon features is assessed. The values for the dissolve attribute are compared across the boundary line. If the values are the same, the boundary line is removed, or dissolved away. If the values for the dissolve attribute differ across the boundary, the boundary line is left intact.

Example: An attribute table of cities in the West Bank and Gaza might have a field of the location of the city being in the northern part of the West Bank, middle, south or Gaza strip. If the Dissolve function is applied on this field, four regions will result.

4) Proximity Functions and Buffering:

Many important questions in spatial analysis hinge on proximity, the distance between features of interest. Examples include:

- Which schools are within 1.5 km from an earthquake fault?
- Which land parcels are more than 5 km from the nearest highway?
- Which customers live within 2 km from a certain store?

Buffering is one of the most commonly used proximity functions. A *buffer* is a region that is less than or equal to a specified distance from one or more features. Buffers may be determined for point, line, or area features (Figure 13.11), and for raster or vector data (Figure 13.12). They typically identify areas that are "outside" some given threshold vs. those "inside" some threshold distance.

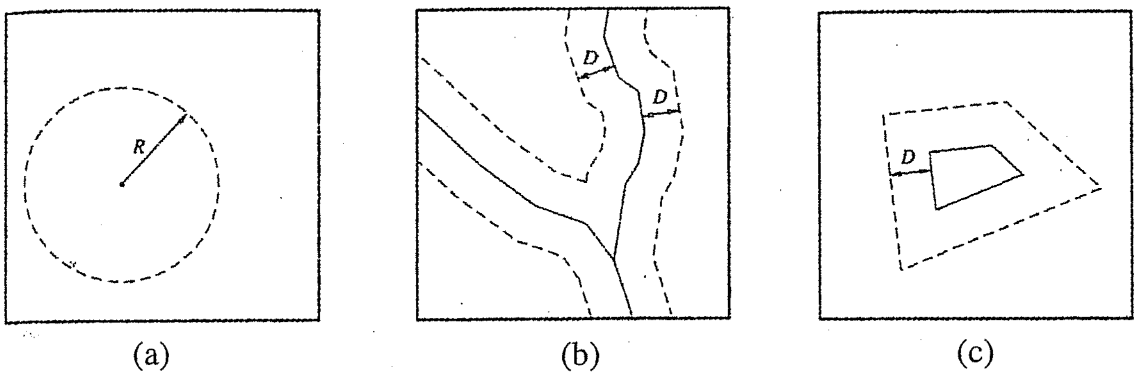


FIGURE 13.11: Vector buffering around points (left), lines (middle) and polygons (right).

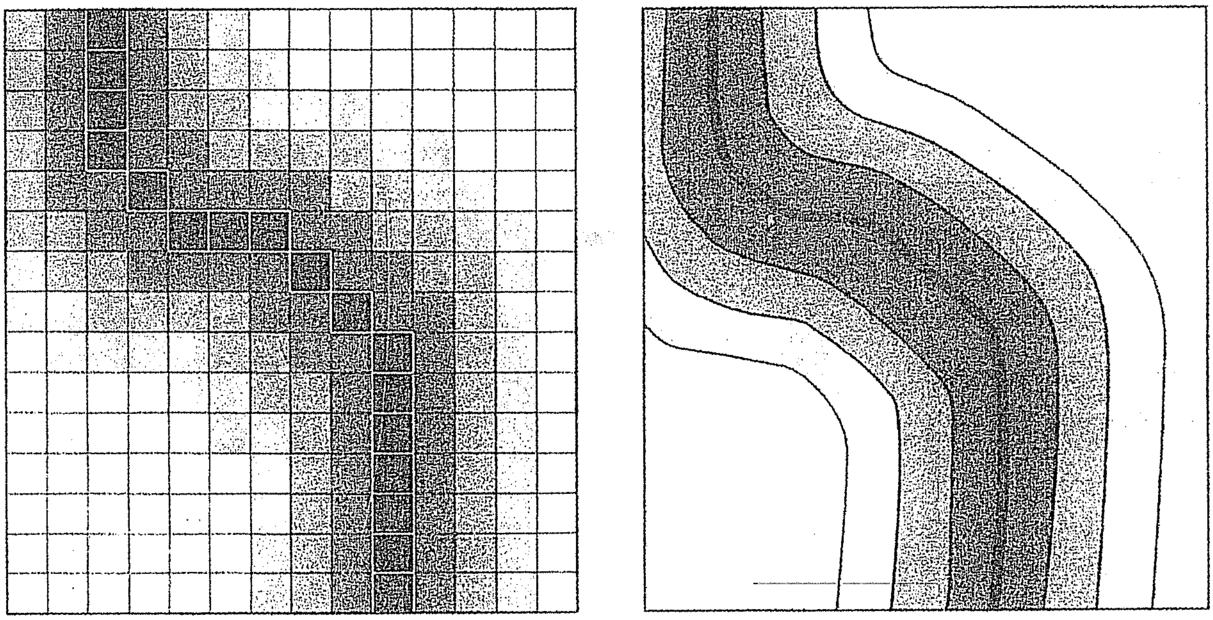


FIGURE 13.12: Raster (left) and vector (right) examples of line buffering.

5) Overlay:

Overlay is a spatial operation in which a thematic layer containing spatial features is superimposed onto another to form a new thematic layer, usually with new features and attributes (Figure 13.13). This operation is one of the most powerful applications of GIS technology. It requires that data layers use a common coordinate system, and could be performed on both raster and vector data.

Raster data overlay involves the cell-by-cell combination of two or more data layers. Cell values from different raster layers are combined in some manner (using what is known as map algebra) and an output value is assigned to the corresponding cell in the output layer (Figure 13.14). There is no need here to distinguish between polygons, lines and points because all raster data comprise cells.

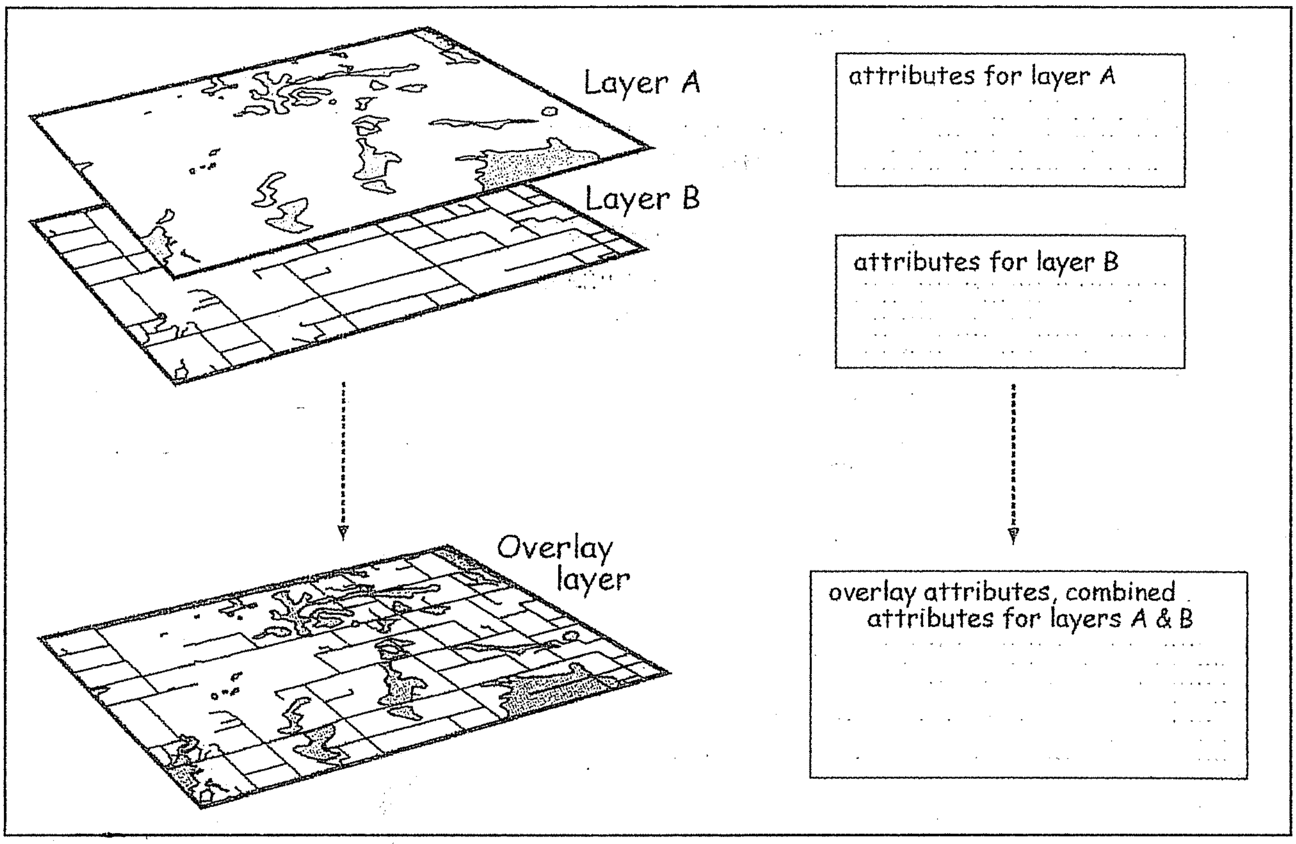


FIGURE 13.13: Concept of overlay in GIS.

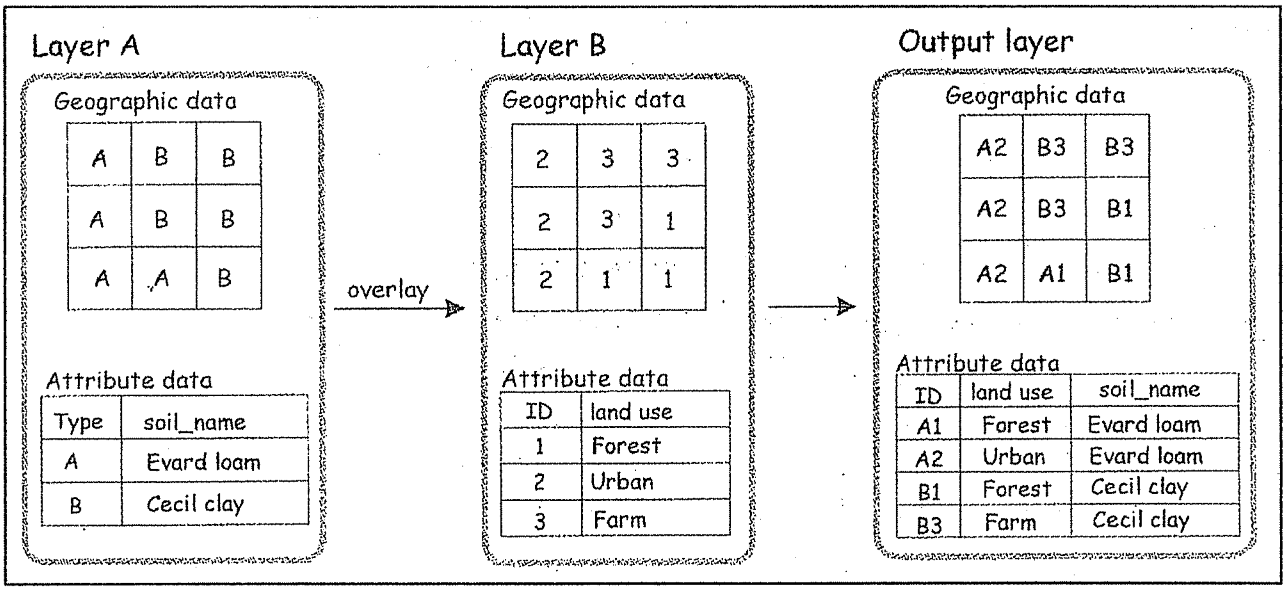


FIGURE 13.14: Cell-by-cell overlay of raster data.

Vector data overlay, on the other hand, involves combining the point, line and polygon geometry and associated attribute data, and thus, it creates new geometry. The process might require the calculation of intersection points of features from both layers, as well as the splitting of lines or areas and the creation of new features. The overlay also entails the combination of attribute data during polygon overlay (Figure 14.15). Moreover, the topology of vector overlay output will likely be different from that of the input data layers. It must be re-created if it is needed in further processing.

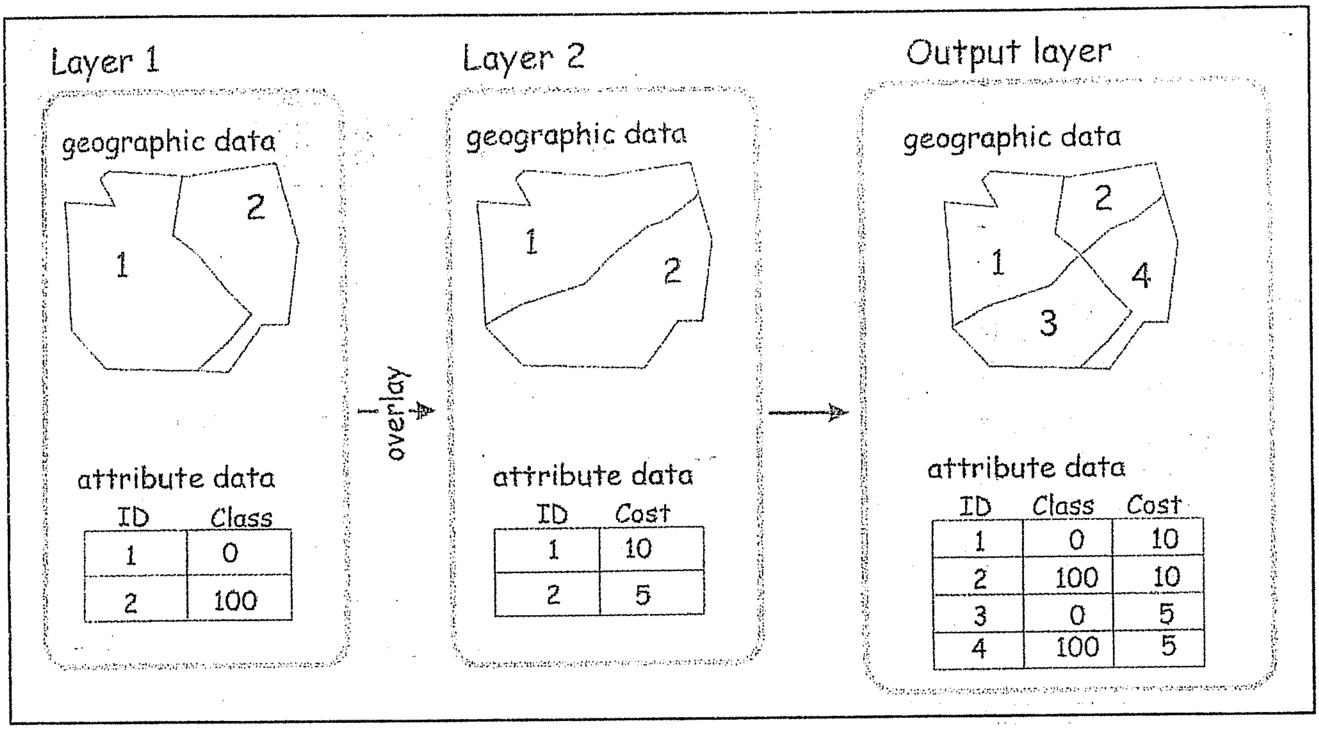


FIGURE 13.15: Overlay of vector data.

Raster overlay is faster and much more efficient than vector overlay. That is why some GISs support functions for manipulating both raster and vector data, and overlay can be done in the following sequence:

Vector → Raster → Overlay → Back to Vector.

Examples: Two examples on overlay operations will be described here:

1) When evaluating land suitability for development in an area, two polygon layers are used: the slope and soil stability layers (Figure 13.16). When these two layers are superimposed on top of each other, four polygons result with different characteristics of slope and soil

stability. The one with 5% slope and stable soil is the most suitable for development.

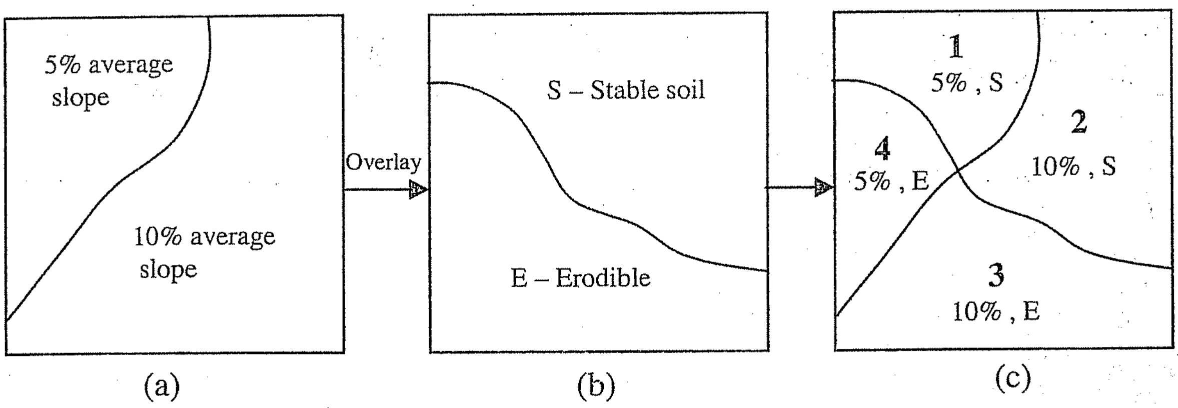


FIGURE 13.16: Example of GIS overlay used to evaluate land suitability.

2) Overlaying road network with soil, hydrology, and topography will yield important information about where a landfill would be best placed in a certain area (Figure 13.17). This application is one of the traditional uses of GIS.

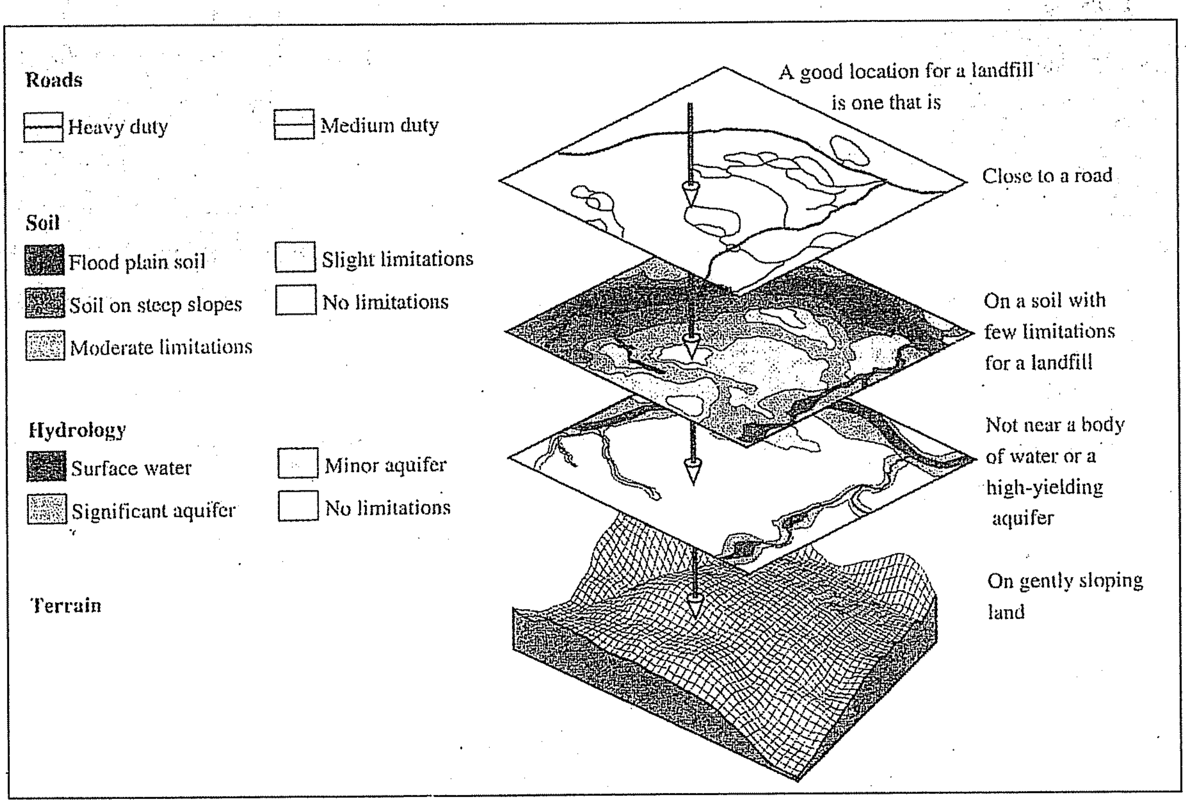


FIGURE 13.17: Example on site selection for a landfill using overlay operations.

6) Terrain Analysis:

Most GISs are also capable of performing several different digital terrain analysis functions. Some of these (1) calculate profiles along designated reference lines, (2) determine cross-sections at specified points along a reference line; (3) generate perspective views where the viewpoint can be varied; (4) analyze visibility to determine what can or cannot be seen from a given vantage point; (5) compute slopes and aspects; (6) make sun intensity analysis; (7) derive hydrologic networks; and (8) locate optimum transportation routes, as well as others. Most terrain analyses are usually performed using a raster data model (called digital elevation model - DEM). Raster data sets are simple, easy to understand, manipulate and program.

7) Network Analysis:

One of the major applications of GIS is in Network analysis. A network may be defined as a set of connected line segments. Network models include roads, power-lines, telephone and television cables, water distribution systems as well as others. The network model is different from other regular GIS models in that it is not based on entities/layers, but on links and nodes and built-in topology that defines which links are connected to which links at what nodes. Once the model is constructed, it is possible to represent and analyze the cost, time, delivery, and accumulation of resources along links and between the connected nodes. It is also possible to simulate the quickest and/or the shortest route between points A and B based on the route with lowest accumulated resistance.

Example: An example of this kind of application is finding the best routes for fire-fighters to follow when there is a fire, or selecting routes to improve public transport through the network, or most suitable routes for school buses, etc.

8) Other GIS Functions:

In addition to the previous spatial analysis functions just described, many other functions such as mapping functions are available with most GIS software. These include: (1) changing the map projection and the reference coordinate system, (2) rotating the reference grid, (3) making map scale changes, and (4) changing the contour interval used to represent elevations. Also, single layers of information or any combination of layers can be mapped.

Output from GISs can be provided in graphical form as maps, charts, or diagrams; in numerical form as statistical tabulations; or in other files that result from computations and manipulations of the geographic data. These materials can be supplied in either hardcopy or softcopy form.

13.7 APPLICATIONS OF GIS

As can be seen from the preceding section, the areas of GIS applications are very numerous and widespread. GIS technology is being used worldwide, at all levels of government, in business and industry, by public utilities, and in private engineering offices. Some of the more common areas of application are:

- Land-use planning,
- Natural resource management,
- Environmental impact assessment,
- Census, population distribution, and related demographic analysis,
- Agriculture,
- Analysis of the geographic distributions of disasters such as hurricanes, floods and fires,
- Routing of buses or trucks in a fleet,
- Tax mapping and mapping for other purposes,
- Infrastructure and utility mapping and management, and
- Urban and regional planning.

Concerning the work of the surveyor, the following are some operations that can be made in a GIS environment:

- Easements or rights-of-way can be added or removed,
- An entire section of a land subdivision can be added,
- Ownership and dimensional changes can be noted,
- Land parcels can be combined when land consolidation takes place,
- Streets can be removed (vacated),
- The scale can be changed,
- Real-estate work can be improved through building a suitable database.

13.8 LIST OF GIS SOFTWARE PACKAGES

There are many public domain and commercially available GIS software packages. According to many surveys, ArcGIS produced by ESRI (Environmental System Research Institute) and GeoMedia produced by Intergraph corporation have dominated the market. Table 13.4 lists some of these systems and their web sites.

TABLE 13.4: A List of some GIS software packages.

Name of GIS	Producer	Web Site
ArcGIS	ESRI	http://www.esri.com
GeoMedia	Intergraph Corporation	http//:www.intergraph.com
MapInfo	MapInfo Corporation	http//:www.mapinfo.ccom
IDRISI	Clark Labs	http//:www.clarklabs.org
ERDAS	Leica Geosystems, LLC.	http//:www.erdas.com
Autodesk Map	Autodesk, Inc.	http//:www3.autodesk.com
Microstation	Bently Systems, Inc.	http//:www2.bently.com

PROBLEMS

13.1 Define GIS and list its fundamental components.

13.2 Compare between analog maps and GIS.

13.3 What are the main subsystems of a GIS software? Which one of these distinguishes GIS from other computer systems?

13.4 Discuss the importance of the reference coordinate system layer in a GIS.

13.5 What are the two main types of data that a GIS database contains?

13.6 Name and describe the different simple spatial objects used for representing spatial data in digital form. Which ones are used in the vector format?

13.7 Describe the differences between vector and raster formats for storing data in a GIS. Give the advantages and disadvantages of each. What type of data or analysis is each of the two formats suitable for? What format do you recommend for cadastral mapping?

13.8 Even though raster representation yields less spatial accuracy, it is still widely used in GIS. What do you think the reasons for that?

13.9 Explain how data is converted from raster to vector and vice versa.

13.10 Define the term *topology*, and discuss its importance in GIS.

13.11 What are the most important topological relationships dealt with in GIS? Give practical examples on analyses that need these relationships.

13.12 What are the two main sources of spatial data to be used in GIS? Give examples.

13.13 Compare between manual digitization and scanning for converting maps and other graphic documents to digital form.

13.14 What is meant by on-screen digitizing?

13.15 Give examples on situation where we need in spatial analysis to do (a) point buffering, (b) line buffering and (c) polygon buffering.

13.16 What does the dissolve and merge operation in GIS do? Give a practical example other than that mentioned in the book.

13.17 What is meant by overlay in GIS? Give examples on both raster and vector overlay operations.

13.18 How does a network model differ from other regular GIS models?

13.19 Give examples other than spatial analyses operations that a GIS can do.

13.20 List five areas of applications of a GIS.

13.21 How can you as a surveyor benefit from GIS in your work?

13.22 Compile a list of data layers and attributes that would likely be included in a GIS for:

(a) Selecting the optimum corridor for constructing a new highway that connects two major cities.
(b) Select the optimum location for a new airport in a large metropolitan area.
(c) Selecting the fastest routes for reaching locations of fires from various fire stations in a large city.
(d) Selecting the best location for a new mosque in the city.
(e) Selecting the most suitable roads to transport hazardous and chemical materials throughout the country.

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